

Is trade good? Lessons from an Exchange Economy*

Costas Arkolakis, Yale University

September 2008

Abstract

This note describes the potentials for gains from trade for a simple exchange economy with two consumer. It is designed for the needs of the course of International Trade Theory and Policy at Yale 'Econ 300'

1 A Barter Equilibrium

We consider a very stark society.

- No production
- Two goods $j = A$ (Apples) and B (Bananas)
- Two consumers $i = 1, 2$, endowed with (e_A^i, e_B^i) of the two goods
- Utility of consumers is a function of the amount of each good they consume (x_A^i, x_B^i) . Assume preferences are strictly concave in each element
- Voluntary exchange, no coercion

This economy can be analyzed using the famous edgeworth box

- Each corner is the origin for one consumer.
- Each point of the box has 4 coordinates. 2 indicating the consumption of each of the consumers.
- Only the allocations inside the box are **feasible**.

*I am grateful to Olga Timoshenko for various suggestions. All remaining errors are mine.
email: costas.arkolakis@yale.edu

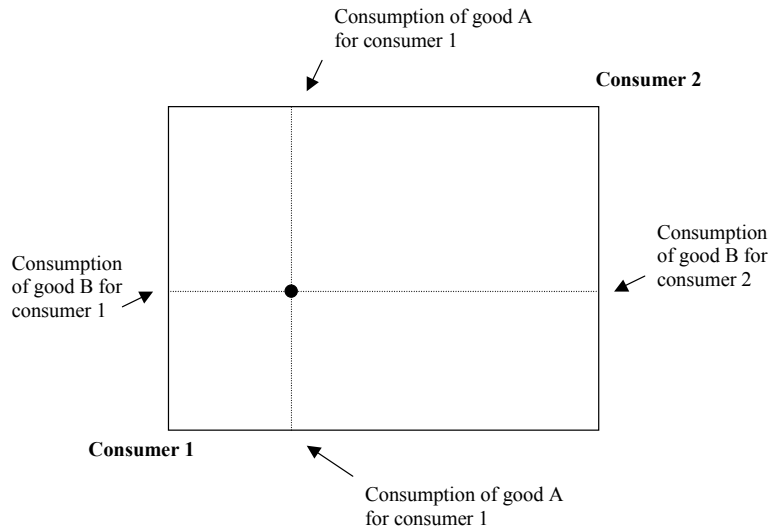


Figure 1:

1.1 Equilibrium in the Barter Economy

The indifference curves are convex (What is an indifference curve?)

- A contract curve is the set of points where the indifference curves of the consumers are tangent. It is the set of allocations where none of the agents can be improved upon with redistribution without making the other worse off (see Pareto efficiency below)

An allocation $x = (x_A^i, x_B^i)_{i=1,2}$ is a barter equilibrium if it satisfies the following three requirements:

- Feasible (in the box)
- Has to be INSIDE the lens shaped area formed by the indifference curves of the consumers that pass through their initial endowments. Since this is a voluntary exchange economy a consumer would never accept an exchange that would reduce its welfare
- Has to be on the contract curve.

Any point satisfying these three requirements is an equilibrium in this Barter economy. Clearly in all the equilibria there is no way of improving one consumer without making the other worse off. This brings us to the concept of Pareto optimality, fundamental when thinking of the welfare gains from trade.

Definition 1 (Pareto efficient Allocations) *A feasible allocation x is Pareto efficient if there is no other feasible allocation such that it strictly improves the welfare of at least one consumer without strictly reducing the welfare any of the other.*

2 A competitive Equilibrium

- Perfectly competitive market system: consumers, producers insignificant to the market, no market power on prices
- Markets clear (and consumers, producers expect them to do so).

Notice: Consumer, producer needs no other information than the prevailing price. If consumer preferences are strictly increasing and concave such a competitive equilibrium exists.

Let's be a bit more specific. Denote the utility function of consumer 1 (home consumer) as $U^1(x_A^1, x_B^1)$ and of consumer 2 (foreign consumer) as $U^2(x_A^2, x_B^2)$. We will first analyze what happens under autarky and then under free trade.

2.1 Demand

To derive the demand function we have to solve the maximization problem of the consumer which is taking the prices p_A and p_B as given. Thus the consumer solves

$$\begin{aligned} & \max U^i(x_A^i, x_B^i) \\ p_A x_A^i + p_B x_B^i & \leq p_A e_A^i + p_B e_B^i \end{aligned}$$

which gives the following three conditions

$$\begin{aligned} \frac{\partial U^i}{\partial x_A^i} &= \lambda^i p_A \\ \frac{\partial U^i}{\partial x_B^i} &= \lambda^i p_B \\ p_A x_A^i + p_B x_B^i &= p_A e_A^i + p_B e_B^i \end{aligned}$$

where λ^i is the Lagrange multiplier for consumer i .

2.2 Autarky

The solution of the autarky is straightforward and the equilibrium involves the prices that solve

$$\frac{\frac{\partial U^i(e_A^i, e_B^i)}{\partial x_A^i}}{\frac{\partial U^i(e_A^i, e_B^i)}{\partial x_B^i}} = \frac{p_A}{p_B}, \quad (1)$$

and where given that $x_A^i = e_A^i, x_B^i = e_B^i$ the budget constraint is trivially satisfied:

$$p_A x_A^i + p_B x_B^i = p_A e_A^i + p_B e_B^i$$

The ratio of the marginal utilities also represents the marginal rate of substitution (MRS)

$$\begin{aligned} dU^i &= \frac{\partial U^i}{\partial x_A^i} dx_A^i + \frac{\partial U^i}{\partial x_B^i} dx_B^i \implies \\ 0 &= \frac{\partial U^i}{\partial x_A^i} dx_A^i + \frac{\partial U^i}{\partial x_B^i} dx_B^i \implies \\ \frac{\frac{\partial U^i}{\partial x_A^i}}{\frac{\partial U^i}{\partial x_B^i}} &= -\frac{dx_B^i}{dx_A^i} \equiv \text{MRS} \end{aligned} \quad (2)$$

The MRS represents the rate at which consumers are willing to trade good apple with bananas, i.e. how many bananas are they willing to give away in exchange for one apple given their current consumption.

2.3 Trade

Assume now that the autarky prices for the consumer (country) i are

$$\frac{p_A^i}{p_B^i}.$$

Claim 2 *There is room for trade as long as*

$$\frac{p_A^1}{p_B^1} \neq \frac{p_A^2}{p_B^2}.$$

The intuition for the result is simple. If the prices are different, say

$$2 = \frac{p_A^1}{p_B^1} > \frac{p_A^2}{p_B^2} = \frac{1}{3}$$

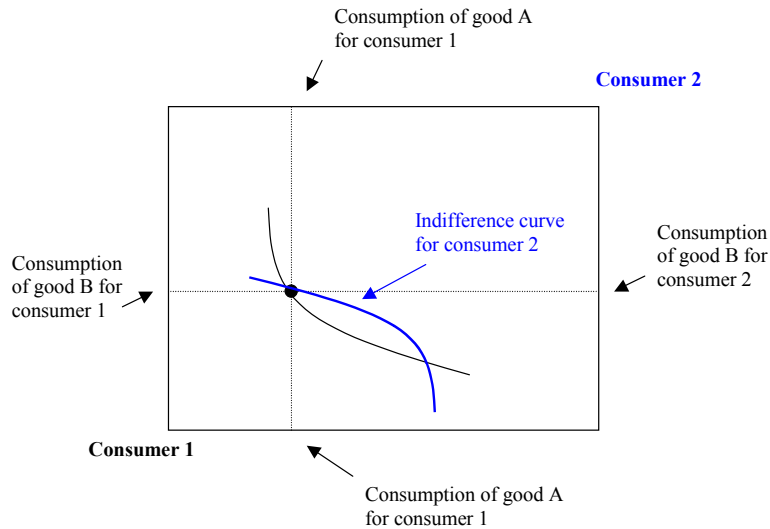


Figure 2:

then consumer 1 is willing to trade 2 bananas for 1 apple while the domestic consumer is willing to trade 3 apples for 1 banana. The autarky prices for each country are the slopes of the lines tangent to the indifference curve of each consumer.

In this situation there is clearly room for mutual gains from trade. In the following graph we can see the consumption under autarky. The new equilibrium allocation is the point where the indifference curves are tangent and are also tangent to the straight budget line that passes through the initial endowment. The equilibrium relative prices under trade will be in between the initial autarky prices. The following pictures illustrates this equilibrium.

Claim 3 *Consumers (weakly) gain from trade*

Proof. Trivial. In the maximization problem of each consumer under trade, consuming their endowment is clearly affordable given that

$$0 \leq p_A (e_A^i - x_A^i) + p_B (e_B^i - x_B^i). \quad (3)$$

This means that any possible choice the consumer will have with trade will clearly give him at least as much utility as they had in autarky. ■

Theorem 4 (First Welfare Theorem) *Under the above assumptions (and actually even a bit weaker) every competitive equilibrium is Pareto efficient.*

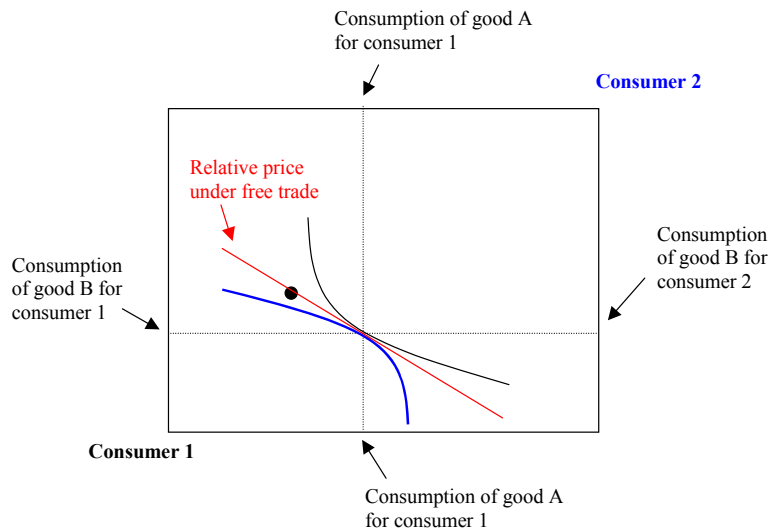


Figure 3:

We see that in this standard benchmark the predictions about gains from trade are stark: people should always (weakly) prefer trade than nontrade. Why some people oppose free trade then?

The first thing to note is that efficiency does not necessarily mean social justice. In the equilibrium of free trade or autarky, some people might end up being very rich. In fact, while trade (weakly) improves everyone it could be the case that the gains of some of the participants would be much higher while of some others relatively very small (or none).

Here is another obvious reason: some of the assumptions of the model are not representative of the real world. For example the assumptions related to competition structure, the existence of demand or production externalities (e.g. environmental pollution) etc.

Finally, notice that the quantitative magnitude of the gains from trade would be affected by demand and production structure, the extend of heterogeneity across consumers and firms etc.

In this course, and after we study variations in the technology structure inside the perfectly competitive environment we will consider:

- Alter the assumption of Perfect competition → Monopolistic competition
- Alter the assumption of Homogeneous Firms → Firm Heterogeneity

The interesting thing is that in the main benchmark models that economists use up to now the gains from trade are almost the same (see Arkolakis, Demidova, Klenow and Rodriguez-Clare, 2008). Still, these gains are not that large

however. But very big gains can be achieved for countries that can increase substantially their trade/GDP ratio. We will go through this last argument towards the end of this class.

3 References-Sources

Arkolakis C., S. Demidova, P. Klenow, A. Rodriguez-Clare, (2008), “Endogenous Variety and the Gains from Trade,” American Economic Review, Papers and Proceedings, May

Guido Lorenzoni, (2006), “course materials for 14.54 International Trade,” MIT OpenCourseWare (<http://ocw.mit.edu/>)

Jehle G. A. and P. J. Reny, (2000), “Advanced Microeconomic Theory, 2nd edition” Addison Wesley Longman, Boston, Massachusetts