A Primer on Gains From Openness*

Costas Arkolakis
Yale and NBER

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Abstract

This primer is designed for undergraduate students studying international economics. It presents three simple frameworks to calculate gains from openness to international trade, multinational production and migration. The backbone of our analysis is the simple Armington (1969) model where trade arises because the representative consumer loves variety and demands a positive amount of the good produced in each location.

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1 Introduction

Over the past several decades, international trade has grown tremendously as both a cause and an effect of globalization. In Figure 1, it is shown the share of total spending on domestic goods, namely the value of all goods and other market services produced domestically as a fraction of total GDP. The trend is decreasing for all the selected countries; the average share dropping from 87.5% in 1960 to 71.7% in 2013. This is just one of many examples that display how much the economic linkages between countries have been growing over time. Similar trends can also be observed for developing economies. The increasing liberalization of trade and capital markets as well as the establishment of a number of international institutions has had an important role in promoting trade in place of protectionism. Moreover, technological advances lowered the costs of transportation, communication, and computation in such a way to make it profitable to import more and to sometimes locate different phases of production in different countries. This phenomenon is known as multinational production. In fact, multinational production has become one of the most important channels through which countries exchange goods, capital, ideas, and technologies. In Figure 2 it is shown the share of production from domestic firms of some selected countries over the last thirty years. Data availability significantly restricts the observations. Nevertheless, we can observe a common negative trend which testifies to the increasing importance of production outside the borders for the world economy. Technological and institutional developments have not only eased mobility of goods and services, but also

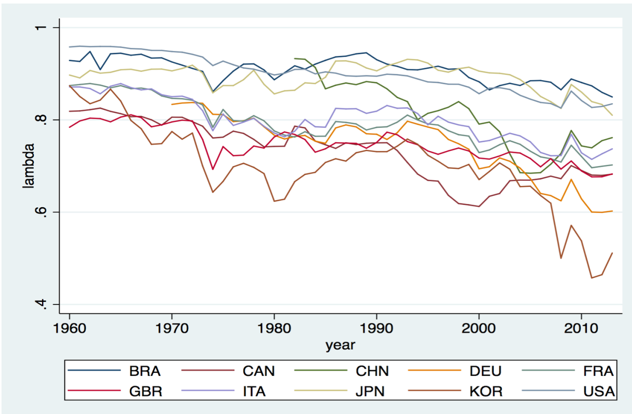

Figure 1: Share of spending on domestic goods

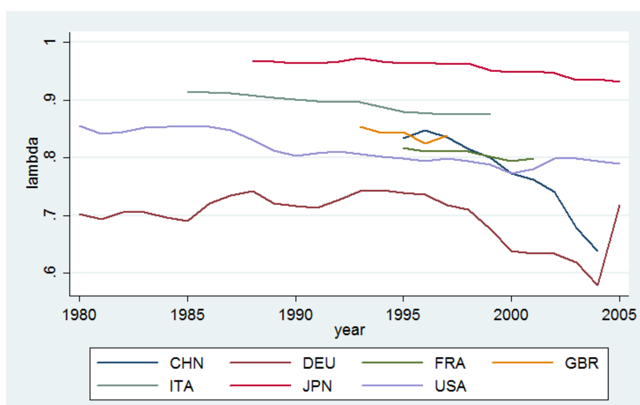

Figure 2: Share of production from domestic firms

that of people. Moving across the globe has become increasingly easier as common barriers hindering migration (language, racism, political differences) have been lifted. Figure 3 shows the evolution of the share of native born population in a given country over the last decades.

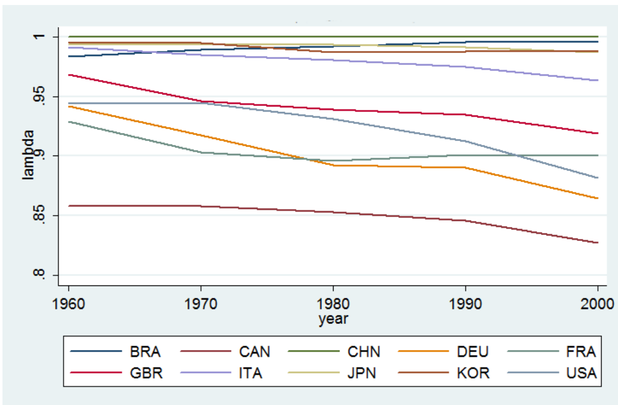

Figure 3: Share of native born population

The most developed economies exhibit again a decreasing trend, with an average drop of about 10% in the home-born population. The less developed countries are distinguished by a trend that is flat, and in some countries, such as Brazil, it is even increasing. Globalization is by no means always a win-win game, as it entails potential benefits but also risks for participating countries. Among examples of benefits of globalization and openess we include: technological spillovers, the fact that trade allows countries to exploit their competitive advantage, economies of scale in production, and the ability of individuals to seek employment in countries with higher wages. On the other hand, increased competition due to trade or the presence of multinational companies could force small firms out of business. Additionally, migration might constitute a threat to local employment.

In this note, we want to shed more light on this trade off. In particular, we are going to study the gains from trade, multinational production and migration in a simple Armington (1969) model with perfect competition and no capital markets. The Armington model is built on the idea that international trade reflects consumers desire for goods from different countries. Hence, we assume that the force for trade comes from consumer preferences, unlike other models of international trade which predict that patterns of trade and production are based on differences in technologies between trading partners (e.g. Heckscher-Ohlin (H-O) model). The nice feature of this simple setting is that it shares the main insights with more complicated and realistic models of international trade, while keeping things simple and tractable.[1]

[1] To avoid the technical details we will not discuss the similarities of the Armington model and other known models in international trade, multinational production and economic geography. Suffices to say that expressions related to the ones derived in this note appear in many contexts with and without: firm heterogeneity (e.g. Melitz (2003) and Chaney (2008) for trade and Arkolakis, Ramondo, Rodríguez-Clare, and Yeaple (2013) for multinational production), comparative advantage across sectors (e.g. Eaton and Kortum (2002) for trade and Ramondo (2012) for multinationals and Ramondo and Rodríguez-Clare (2013) for trade and multinationals), and labor mobility (Krugman (1991), Helpman (1998), Redding (2012), and

The next section describes the main features of the general framework which will be used throughout. In Section 3 we derive an expression for welfare, and compare welfare in this economy to a counter-factual scenario where trade is not allowed and individuals can only consume the domestic good. In so doing, we can measure how much each country gains from opening to trade. We also derive an explicit formula for the gains from trade as a function of trade openness. We then develop two extensions of this setup in the following sections. The first extension, in Section 4, allows for multinational production. In this case, firms are able to choose where to locate production of the domestic good. We explicitly derive welfare in this economy as a function of openness to foreign production. In Section 5 and 6 we extend the baseline setup by assuming that individuals can decide where to work, and analyze welfare gains from trade in a model with free factor mobility. Finally, in Section 7, we study wage inequality within countries in our simple setup, and discuss potential implications of migration and trade policies.

2 The Baseline Setup

In the baseline setup model, two countries domestically produce a differentiated good and trade with each other. Trade arises because the representative consumer loves variety and demands a positive amount of the good produced abroad.

2.1 The Baseline Model

Assume a world of two countries, US and Greece. Firms in each country produce a differentiated good. We will adopt the convention that the US is the home country, and Greece is the foreign country. The good produced in the US is corn, which we will denote as good c. The "foreign" country Greece produces feta, which we will denote as good f. We will use asterisk notation whenever we refer to variables specific to Greece. For example, US and Greek consumption of corn will be denoted as c_c and c_c^*, respectively. Each country is populated by a representative consumer who receives utility from consumption of both goods and earns labor income. The only input of production is domestic labor, which means that individuals can only work in their own countries.

Markets are assumed to be competitive and the price for each unit of the good produced equals the marginal cost of producing it, i.e. the marginal cost of labor. Let w and w^* be the nominal wages in the US and Greece, respectively. It follows immediately that the price of the local good in the US (i.e. corn) is $p_c = w$ and the price of the local good in Greece (i.e.

Allen and Arkolakis (2014)).

feta) is $p_f^* = w^*$. If one country wants to export its own good to the other country, it incurs trade costs τ (τ^*). In other words, the final at-the-dock price of corn in Greece is $p_c^* = w\tau$ and the price of feta in the US is $p_f = w^*\tau^*$.

2.1.1 The Consumer's Problem

Let us consider the optimal choice of consumption of the representative consumer in the US. She maximizes the following Constant Elasticity of Substitution (CES) utility function[2] over the two goods:

$$U(c_c, c_f) = \left(c_c^{\frac{\sigma-1}{\sigma}} + c_f^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

where c_c is consumption of the domestic good, i.e. corn, c_f is consumption of feta, and $\sigma > 1$ is the elasticity of substitution across good varieties. The parameter σ measures the degree of substitutability between the two goods for the consumer. A high value of σ means that the two goods are more substitutable; if σ is low, the consumer loves variety more. The individual is endowed with L units of labor, and labor income is $y = wL$.

The problem of the US consumer can thus be written as

$$\begin{aligned} &\max_{c_c, c_f} \left(c_c^{\frac{\sigma-1}{\sigma}} + c_f^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\ s.t \qquad &p_c c_c + p_f c_f = wL, \end{aligned} \tag{1}$$

with prices taken as given. The problem of the representative consumer in Greece is similar, with the relevant variables marked with asterisks.

As is standard, consumption optimization implies that the marginal rate of substitution (MRS) between the two goods equals their relative prices. The MRS equals the ratio of the marginal utilities of the two goods, i.e.

$$MRS_{c,f} \equiv \frac{U_c(c_c, c_f)}{U_f(c_c, c_f)} = \left(\frac{c_c}{c_f}\right)^{-\frac{1}{\sigma}}, \tag{2}$$

where U_c and U_f are the partial derivatives of the utility function with respect to the first and second argument, respectively. Optimal consumption can thus be derived as[3]

$$\left(\frac{c_c}{c_f}\right)^{-\frac{1}{\sigma}} = \left(\frac{p_c}{p_f}\right) \tag{3}$$

$$\Rightarrow \frac{c_c}{c_f} = \left(\frac{p_c}{p_f}\right)^{-\sigma}. \tag{4}$$

[2] This function is sometimes called the "love of variety" utility function

[3] See Appendix for a formal derivation.

Therefore, relative consumption in each country is a function of the relative price of the consumption goods, and the elasticity of substitution σ. Among other things, this important result implies that trade costs matter just as long as they affect the relative price of goods (recall that $p_f = w^*\tau^*$).

Rewrite (4) as

$$c_c = \left(\frac{p_c}{p_f}\right)^{-\sigma} c_f. \tag{5}$$

Plug it in the budget constraint, equation (1), to get

$$\begin{aligned} p_c^{1-\sigma} c_f + p_f^{1-\sigma} c_f &= p_f^{-\sigma} wL \\ P^{1-\sigma} c_f &= p_f^{-\sigma} wL \\ c_f &= \left(\frac{p_f^{-\sigma}}{P^{1-\sigma}}\right) wL \end{aligned} \tag{6}$$

and similarly

$$c_c = \left(\frac{p_c^{-\sigma}}{P^{1-\sigma}}\right) wL, \tag{7}$$

where we defined the CES price-index as $P \equiv \left(p_c^{1-\sigma} + p_f^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$, a weighted-mean over prices. Equations (6) and (7) are the consumer's optimal demand for each good expressed as a function of prices and parameters. Note that both optimal consumption and price index depend on the elasticity of substitution σ. The elasticity of substitution is a measure of consumption sensitivity to prices. It measures the percentage change of relative consumption when relative prices change by 1%. By taking logs on both sides of (4) and rearranging terms, we can write

$$\log \frac{c_c}{c_f} = -\sigma \log \frac{p_c}{p_f}. \tag{8}$$

Equation (8) says that for a given price change, the higher σ the more consumption shifts across goods, i.e. the goods are more substitutable for the individual.

The elasticity of substitution is closely related to another important parameter of interest: the price elasticity of demand. The elasticity of demand measures the percentage change of the expenditure on each good, given a 1% change in the price of that good. We can formally derive an expression for the elasticity of demand as follows. First, use equation (7) to derive total expenditure on good c, namely

$$p_c c_c = \left(\frac{p_c}{P}\right)^{1-\sigma} wL. \tag{9}$$

Ignoring the effect on P and General Equilibrium effects on w (i.e. effects of price changes on labor demand, and thus wages), we can then easily derive the price elasticity of demand as:

$$\frac{\partial \log (p_c c_c)}{\partial \log p_c} = 1 - \sigma. \tag{10}$$

The elasticity of substitution across goods produced in different countries is one of the key parameters in Armington models, and more generally for international economics. Recall that an important assumption of this model is that products traded internationally are differentiated by country of origin. Hence, substitutability across goods is related to how much countries trade with each other. For this reasons, the elasticity of substitution is key to understanding many features of the global economy. These include the role of international prices in trade balance adjustment and, as we will see more in detail in the next three sections, the welfare benefits of expanding world trade.

3 A Simple Model to Count Gains from Openness

We now move to discuss gains from international trade, while maintaining the same assumptions of the baseline model.

We start by discussing an old rhetoric claiming that free trade causes nations to compete like corporations in a global marketplace and that productivity gains of one country are losses for the other. This is a popular, albeit misguided, opinion. The previous paragraph made it clear that international trade is just another economic activity. Consumers in each country maximize utility across different goods. How and where these goods are produced affect price and final demand, but nothing fundamentally distinguishes a closed and an open economy. World trade is beneficial, as it allows individuals to consume more varieties; high productivity is also good as it allows countries to produce more, and thus consume more. In our context, world trade should not be viewed as competition, but as potentially mutually beneficial exchange.[4]

The goal of this section is to use the simple model to persuade the reader that the idea of global trade as a win-lose struggle between countries, when markets function close to competitive – which they are in our simple setup–, is a myth to dispel.

3.1 Welfare and Productivity

Consider the baseline model, and assume that there are no costs of trade, namely $\tau = \tau^* = 1$, and countries differ in terms of productivity. In other words, the two countries have different production technologies such that the marginal product of a worker is different

[4] See Krugman (1997), Chapter 8.

across countries. Specifically, assume that productivity in the US is indexed by A, and productivity in Greece by A^*. You may think of productivity as the number of labor units which are necessary to produce a unit of good. Perfect competition implies that prices equal marginal costs, and so we can write

$$p_c = p_c^* = \frac{w}{A}, \tag{11}$$

$$p_f = p_f^* = \frac{w^*}{A^*}. \tag{12}$$

Note from (11) and (12) that the absence of trade costs implies that the basket of prices is the same across countries so that $P = P^*$.

Consider now the labor market clearing conditions, which say that aggregate demand in each country must equal aggregate production. For the general model with trade costs, they read

$$c_c + \tau c_c^* = L, \tag{13}$$

$$\tau^* c_f + c_f^* = L^*. \tag{14}$$

Use condition (13), together with (7) and $\tau = 1$ to write

$$\begin{aligned}
\left(\frac{p_c^{-\sigma}}{P^{1-\sigma}} wL + \frac{p_c^{*-\sigma}}{(P^*)^{1-\sigma}} w^* L^*\right) &= L \\
\left(\left(\frac{w}{P}\right)^{1-\sigma}\left(\frac{1}{A}\right)^{-\sigma} + \left(\frac{w}{P}\right)^{1-\sigma}\left(\frac{1}{A}\right)^{-\sigma}\left(\frac{P}{P^*}\right)^{1-\sigma}\left(\frac{w}{w^*}\right)^{-1}\frac{L^*}{L}\right) &= 1 \\
\left(\frac{w}{P}\right)^{1-\sigma}\left(\frac{1}{A}\right)^{-\sigma}\left(1 + \left(\frac{w}{w^*}\right)^{-1}\frac{L^*}{L}\right) &= 1 \qquad (15)
\end{aligned}$$

By symmetry

$$\left(\frac{w^*}{P^*}\right)^{1-\sigma}\left(\frac{1}{A^*}\right)^{-\sigma}\left(1 + \left(\frac{w}{w^*}\right)\frac{L}{L^*}\right) = 1 \tag{16}$$

Combining these two equations yields

$$\begin{aligned}
\left(\frac{w}{P}\right)^{1-\sigma}\left(\frac{1}{A}\right)^{-\sigma}\left(\frac{wL + w^*L^*}{wL}\right) &= \left(\frac{w^*}{P^*}\right)^{1-\sigma}\left(\frac{1}{A^*}\right)^{-\sigma}\left(\frac{wL + w^*L^*}{w^*L^*}\right) \\
\left(\frac{w}{w^*}\right)^{1-\sigma} &= \left(\frac{A}{A^*}\right)^{-\sigma}\left(\frac{w}{w^*}\right)\left(\frac{L}{L^*}\right) \\
\frac{w}{w^*} &= \left(\frac{A}{A^*}\right)\left(\frac{L}{L^*}\right)^{-\frac{1}{\sigma}}. \qquad (17)
\end{aligned}$$

One immediate result from the above expression is that the relative wages depend on the

productivity of the two countries. More productive countries will be able to pay higher wages in this competitive model. This raises a crucial question at this point. Given this differential in returns that saliently reflects the rhetoric of competitiveness, would an increase in foreign productivity negatively affect home welfare under free trade?
We next illustrate that the answer is no, at least through the lenses of our simple model. Substitute (17) in (15) and (16) to get closed form expressions for welfare, i.e.

$$\frac{w}{P} = \frac{A}{\left(A + A^* \left(\frac{L^*}{L}\right)^{\frac{\sigma-1}{\sigma}}\right)^{1/(1-\sigma)}}. \tag{18}$$

Welfare in one country is increasing in the productivity of all countries, and decreasing in the labor ratio. This expression simply stresses the point that trade is not a zero-sum game. Instead, it is a "win-win" arrangement. Despite the resulting inequality of wages, an increase in productivity in one country also benefits its trading partners. The intuition is that along with the effect on nominal wages, there is a negative effect of trade on the price level which implies an increase in domestic and foreign real wages, and thus welfare.

3.2 Measuring the gains from trade

What are the gains from trade in this simple two-countries economy? In other words, how much does welfare in each country increase (or decrease) when countries are allowed to trade with each other, as opposed to being in autarky?
In order to talk about welfare gains from trade, we start by introducing the concept of domestic trade share. The domestic trade share λ is the ratio between spending on goods produced by the home country over total spending. We can write λ for the US as

$$\lambda = \frac{p_c c_c}{p_c c_c + p_f c_f}. \tag{19}$$

From (9), recall that total spending on each good is proportional to the income of the individual. The weight depends on the relative price of the good and on the elasticity of demand, and in particular it coincides with λ. Use (1) and (9) in (19) to write

$$\begin{aligned} \lambda &= \frac{p_c c_c}{wL} \\ &= \left(\frac{p_c}{P}\right)^{1-\sigma}. \end{aligned} \tag{20}$$

We now need a measure of welfare, or well-being of a country. In this simple setup, where the total income of the individual coincides with labor income, we can measure welfare as real wage, i.e $W = \frac{w}{P}$. Since $p_c = w$, this means

$$\begin{aligned} W &= \frac{p_c}{P} \\ &= \left(\frac{p_c^{1-\sigma}}{P^{1-\sigma}}\right)^{\frac{1}{1-\sigma}} \\ &= \lambda^{\frac{1}{1-\sigma}} \end{aligned} \tag{21}$$

We just showed that in this setting, welfare is a function of only two elements: the domestic trade share λ and the elasticity of demand $1-\sigma$. This striking result was first derived by Eaton and Kortum (2002) and was generalized by Arkolakis, Costinot, and Rodríguez-Clare (2012) for a large class of trade models, including the Armington model (see also Arkolakis, Demidova, Klenow, and Rodríguez-Clare (2008)).

We can now use this formula to compute gains from trade. Let welfare in autarky in the US be denoted by W, and consider a counter-factual situation in which countries are allowed to trade. Denote US welfare in this second scenario as W'. The gains from trade are measured as the change in welfare before and after trade is allowed. We adopt the convention to denote a change in any variable x as $\hat{x}$. We can thus write

$$\hat{W} = \left(\frac{\hat{w}}{P}\right) = \left(\hat{\lambda}\right)^{\frac{1}{1-\sigma}} = \left(\frac{\lambda'}{\lambda}\right)^{\frac{1}{1-\sigma}} \tag{22}$$

Therefore, to measure gains from trade, we only need to know changes in trade shares $\hat{\lambda}$ and an estimate of $\varepsilon \equiv 1-\sigma$. What are the plausible ranges for gains from trade? Let us consider a practical example. In 2000, the import penetration ratio, i.e. the share of imports over total spending in the US, was 7%. It follows that the domestic trade share of the US was λ_{trade}= 0.93. Obviously, when trade is not allowed the entire income is spent in the domestic market, i.e. $\lambda_{autarky} = 1$.

Estimates of demand elasticity $\hat{\varepsilon}$ can be found in the literature. Anderson and Van Wincoop (2004) conduct a comprehensive review of the existing studies on trade elasticity and conclude that $\varepsilon \in [-10, -5]$. Applying the formula above, it follows immediately that gains from openness range from 0.7% to 1.4% of national income. GDP in the US in 2000 was about US$ 10,300 billion. If this was GDP in autarky, simple algebra tells us that by opening to trade, US GDP could have increased by US$ 72.1 to 144.2 billion.

4 A Simple Model to Count Gains from Multinationals

In this section, we build on the same general framework described above and assume that firms in each country are Multinational Enterprises (hereafter MNE). That is to say, we assume that they can decide to produce their good either domestically or abroad. Our analysis wants to emphasize the analogies between multinational production and international trade. These two activities are intimately related since trade and multinational production are alternative ways by which firms can serve foreign markets.

4.1 The Model

We want to think of goods as differentiated both by country of origin of their producing firms and by location of production. Since we are working with a model with two countries, it follows that there exist four different goods. We are going to adopt the following notation: c denotes corn that US firms produce domestically, while c^M denotes the corn produced by US firms in Greece. Similarly, good f is feta produced by Greek firms in Greece, while good f^M is feta produced by Greek firms in the US.

When the firm produces its good abroad, it must pay a MNE cost. We denote this cost as γ $(\gamma^*) > 1$. The final price of each good will thus depend on trade costs, on MNEs costs, or both. For example, consider the goods traded in the US. The price of domestically produced corn is $p_c = w$, as before. The price of corn produced abroad, i.e. good c^M, is instead $p_{c^M} = \gamma w^* \tau^*$. This expression reflects the fact that the good is produced far from the country of origin (MNE cost) and must also be imported (trade cost). By the same token, the price of good f is $p_f = w^* \tau^*$, whereas the price of good f^M is $p_{f^M} = \gamma^* w$.

4.1.1 The Consumer's Problem

We maintain the assumption that the representative consumer wants to maximize a CES utility function over goods. A CES utility function is a weighted sum of consumption goods. The functional form does not depend on the number of goods the consumer can choose from.[5] Since there are four different goods in this section, this means that the utility function reads

$$U = \left((c_c)^{\frac{\sigma-1}{\sigma}} + (c_{c^M})^{\frac{\sigma-1}{\sigma}} + (c_f)^{\frac{\sigma-1}{\sigma}} + \left(c_{f^M}\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{23}$$

where c_c, c_{c^M}, c_f and c_{f^M} are the consumption of each of the four goods, and σ is the elasticity of substitution across varieties.

[5] In the general case with N goods, the utility function reads $U = \left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$.

As before, we define the price index P as a weighted mean over prices, i.e.

$$P = \left((p_c)^{1-\sigma} + (p_{c^M})^{1-\sigma} + (p_f)^{1-\sigma} + \left(p_{f^M}\right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} . \tag{24}$$

By symmetry with the previous problem (where country of origin and production coincided), we can write optimal expenditure on each good as

$$p_i c_i = \frac{p_i^{1-\sigma}}{P^{1-\sigma}} wL \quad i = c,\ c^M,\ f,\ f^M \tag{25}$$

The consumption share of the home good produced at home is

$$\tilde{\lambda} \equiv \frac{p_c c_c}{wL} = \frac{p_c^{1-\sigma}}{P^{1-\sigma}} \tag{26}$$

What is the interpretation of $\tilde{\lambda}$ in this model? Let us start from the simple case where trade costs are infinite, so that the only feta purchased by the US, is the one produced in the US by Greek MNEs, i.e. good f^M.[6] Similarly, Greek consumers will only consume corn which is produced in Greece, so that $c_f,\ c_c^* = 0$. In this simple case $\tilde{\lambda}$ is the home share of production, depicted in Figure 2.

In the more general case, both goods c and c^M are produced by the US, which means that the home share of consumption and production do not coincide. In particular, for a given level of trade, we have that $\tilde{\lambda} < \lambda$ where $\tilde{\lambda}$ is the share of *domestic* goods produced and consumed domestically. This is a subset of λ, the share of goods (domestic or foreign) produced and consumed domestically (see equation (20)).

4.2 Gains from Multinationals

How large are the gains from hosting foreign firms? This is an important question, as the gains from opening to trade for a country could be even larger in the presence of multinational enterprises.

Let W^{MNE} denote welfare in the economy with MNEs. Recall that $p_c = w$. We have

$$\begin{aligned} W^{MNE} &= \frac{w}{P} = \frac{p_c}{P} \\ &= \left(\tilde{\lambda}\right)^{\frac{1}{1-\sigma}} \end{aligned} \tag{27}$$

[6] This is the case studied explicitly by Ramondo (2012). Her model yields an equation similar to (26).

Note that when $\sigma > 1$ we have that $W^{MNE} = \tilde{\lambda}^{\frac{1}{1-\sigma}} > \lambda^{\frac{1}{1-\sigma}} = W$. Thus gains from openness also depend on multinational activity. In particular, if firms are MNEs, welfare gains from trade are amplified.

In section 6.2, in the Appendix, we discuss an alternative, more general formulation of the problem with MNEs. The consumer's problem can be thought of as a two-stage problem. In a first stage the consumer decides how much of each good he wants to consume, e.g. how much of the Greek good versus the American good. Then, given the optimal expenditure on each good, he decides how much to consume of the home produced good versus the good produced abroad. It is shown that welfare can be written as

$$\left(\frac{w}{P}\right) = \left(\widetilde{\lambda}\right)^{\frac{1}{1-\nu}} (\lambda)^{\frac{\theta-\nu}{(1-\theta)(1-\nu)}} . \tag{28}$$

Welfare depends on both the home share of production and the home share of consumption. In equation (28), θ is the elasticity of substitution between goods (i.e. Greek vs American), whereas ν is the elasticity of substitution between home versus foreign-produced variety of each good. The case analyzed here is a special case of this more general formulation of the problem, where the two elasticities coincide, i.e. $\theta = \nu = \sigma$.

This analysis is closely related to a recent literature on multinational production (hereafter MP). Ramondo (2012) evaluates the gains from engaging in multinational activities, both from autarky and a situation in which the activity of multinational firms is liberalized. In her model, multinational production is the main activity and there is only trade of a homogeneous freely traded good and she derives an expression for the gains from trade similar to (27). It can be shown that a related expression can be derived in the MP model of McGrattan and Prescott (2010). Ramondo and Rodríguez-Clare (2013) study a model that features both trade and MP with perfect competition. Arkolakis, Ramondo, Rodríguez-Clare, and Yeaple (2013) present a related analysis in a model with imperfect competition. Both these papers derive analytical relationships for the gains from trade and MP related to expression (28).

5 A Simple Model to Count Gains from Migration

We now want to see how the simple framework outlined above can be used to think about migration. So far, we have (implicitly) assumed immobility of factors of production. In particular, we have assumed geographical immobility, namely the impossibility for people to move from one country to another in order to work. In the real world, there are several reasons why geographical immobility might exist, such as family and social ties, high transportation costs, as well as differences in the cost of living between regions and countries. However, one

could argue that these barriers to migration are binding only in the short-run. In the long run, if income is different across countries, it is likely that people choose to relocate.
In this section, we use the approach of Allen and Arkolakis (2014) to relax the assumption of geographical immobility and study the welfare implications of free factor mobility. In a model where the only input of production is labor these implications coincide with welfare gains from migration.

5.1 Welfare in a Model with Migration

If people are free to decide where to work, as long as there are wage differences people from the low-paying country will move to the high-paying one. Migration will happen until real wages (and then welfare, $\bar{W} \equiv \frac{w}{P}$) are eventually equalized. This means that in equilibrium we have

$$\begin{aligned} \frac{w}{P} &= \frac{w^*}{P^*} \\ \Rightarrow \left(\frac{w}{w^*}\right)^{1-\sigma} &= \left(\frac{P}{P^*}\right)^{1-\sigma} \end{aligned} \tag{29}$$

The first case we want to analyze is one in which there are no trade costs ($\tau = \tau^* = 1$) and countries differ in terms of productivity. In so doing we will be able to draw a parallel with the baseline model studied in section 3.1. It turns out that even when workers can freely locate themselves, welfare in one country is positively related to productivity of each country. In particular, in the model with migration, welfare can be written as[7]

$$\left(\frac{w}{P}\right) = \frac{1}{\left(A^\sigma + A^{*\sigma}\right)^{\frac{1}{1-\sigma}}} \tag{30}$$

We now consider the more general framework with trade costs. Rewrite the price index as

$$P^{1-\sigma} \equiv p_c^{1-\sigma} + p_f^{1-\sigma} = (w)^{1-\sigma} + (w^*\tau^*)^{1-\sigma} \tag{31}$$

$$(P^*)^{1-\sigma} = (w\tau)^{1-\sigma} + (w^*)^{1-\sigma} \tag{32}$$

and use (31) and (32) to get

$$\left(\frac{P}{P^*}\right)^{1-\sigma} = \frac{(w)^{1-\sigma} + (w^*\tau^*)^{1-\sigma}}{(w\tau)^{1-\sigma} + (w^*)^{1-\sigma}}. \tag{33}$$

Therefore, welfare equalization in (29) reads

[7] See Appendix for full derivation.

$$\left(\frac{w}{w^*}\right)^{1-\sigma} = \frac{(w)^{1-\sigma} + (w^*\tau^*)^{1-\sigma}}{(w\tau)^{1-\sigma} + (w^*)^{1-\sigma}}.$$

Rearrange to (finally) get

$$\begin{aligned}
\left(\frac{w}{w^*}\right)^{1-\sigma} &= \frac{(\frac{w}{w^*})^{1-\sigma} + (\tau^*)^{1-\sigma}}{(\frac{w}{w^*}\tau)^{1-\sigma} + 1} \Rightarrow \\
\left(\frac{w}{w^*}\right)^{2(1-\sigma)} \tau^{1-\sigma} + \left(\frac{w}{w^*}\right)^{1-\sigma} &= \left(\frac{w}{w^*}\right)^{1-\sigma} + (\tau^*)^{1-\sigma} \Rightarrow \\
\frac{w}{w^*} &= \sqrt{\frac{\tau^*}{\tau}}. \qquad (34)
\end{aligned}$$

Therefore, when exporting costs are relatively low, relative wage is high. Low export costs in one country mean low price of the exported good, and thus high demand. In order to produce more, the country has to attract a lot of labor force, implying higher relative wages. As we show in the appendix, extended -but straightforward- derivations reveal that the market clearing condition (13) and (34) imply that

$$\begin{aligned}
\left(\frac{p_c^{-\sigma}}{P^{1-\sigma}} wL + \tau \frac{p_c^{*-\sigma}}{(P^*)^{1-\sigma}} w^* L^*\right) &= L \\
\left(\left(\frac{w}{P}\right)^{1-\sigma} L + \tau \frac{w^{-\sigma} (\tau)^{-\sigma}}{(P^*)^{1-\sigma}} w^* L^*\right) &= L \\
\left(\frac{w}{P}\right)^{1-\sigma} \left(1 + \left(\frac{w}{w^*}\right)^{-\sigma} (\tau)^{1-\sigma} \frac{L^*}{L}\right) &= 1. \qquad (35)
\end{aligned}$$

where in the last line we used the expressions for prices in terms of wages and transport costs, and (29). By symmetry, we could rewrite (14) as

$$\left(\frac{w^*}{P^*}\right)^{1-\sigma} \left(1 + \left(\frac{w}{w^*}\right)^{\sigma} (\tau^*)^{1-\sigma} \frac{L}{L^*}\right) = 1. \qquad (36)$$

Dividing (35) by (36) and using (29) gives

$$\left(\frac{w}{w^*}\right)^{-\sigma} (\tau)^{1-\sigma} \frac{L^*}{L} = \left(\frac{w}{w^*}\right)^{\sigma} (\tau^*)^{1-\sigma} \frac{L}{L^*} \qquad (37)$$

Rearranging this expression and using (34), we finally obtain

$$\frac{L^*}{L} = \sqrt{\frac{\tau^*}{\tau}} \qquad (38)$$

This interesting result demonstrates that in a model with migration, people locate to places with better access - relatively lower importing costs. Finally, notice that we can solve for the welfare levels as a function of trade costs. In particular, substitute (38) in (35) and

$$\left(\frac{w}{P}\right)^{1-\sigma}\left[1+\left(\frac{\tau}{\tau^*}\right)^{\frac{\sigma}{2}}(\tau)^{1-\sigma}\left(\frac{\tau^*}{\tau}\right)^{\frac{1}{2}}\right]=1 \implies$$
$$\bar{W}=\frac{w}{P}=\left[1+(\tau\tau^*)^{\frac{1-\sigma}{2}}\right]^{1/(\sigma-1)}$$

Welfare is thus negatively related to trade barriers: as long as $\sigma > 1$, the higher the trade costs, the lower $\bar{W}$.

6 Winners and Losers from Relaxing Mobility Frictions

In the past few years, many countries have lifted the barriers of cross-country migration and have allowed people to freely move across borders. An excellent example is the institution of European Union, which has established an (almost) pan-European area, where citizens of the Union and their family members can move and reside freely. Because workers will choose to reallocate as a result, these types of agreements raise continuous debates about winners and losers from lifting mobility barriers. The following proposition proven in the Appendix describes under which conditions the union of a high and a low productivity country may benefit one or the other after the transition to a single integrated market.

Proposition *Consider a 2-country asymmetric world with no trade costs, and assume that we move from an equilibrium with no mobility of labor (baseline trade model) to an equilibrium with full mobility (migration model). After the policy change, workers in the high productivity country are always worse off, whereas those in the low productivity country always better off, if the high productivity country labor force is smaller than the one in the low-productivity country or if the productivity advantage is large enough.*

Without trade costs, once the migration barriers are lifted, people will relocate themselves only if they would earn higher wages elsewhere. Real wages increase with productivity and decrease with the size of the labor force, as seen, for example, in (18).

Let us first assume that the two countries have the same population, i.e. the same labor force. When migration barriers are in place, workers in the high productivity country earn higher wages. Without barriers, workers from the low productivity country will move to the country paying higher wages, increasing the labor force there, until real wages will be equalized in the two countries. Since our measure of welfare is real wages, the high productivity country will thus be worse off after policy change, and vice versa.

Similarly, when the two populations have different sizes, the input compensation *per worker* in the high-productivity country is higher before the policy change, as long as its relative population is not too big. The intuition for what happens after the policy change is the same in this case.

Who have been the winners and losers after instituting the European Union? According to the Proposition above, the Union has benefited most the countries which were more densely populated but were less productive, like the Eastern European countries. Of course, our simple model ignores important gains from specialization due to agglomeration effects and increasing returns. In this case integration is likely to benefit all the participants in the union.

7 Extension: Wage Inequality within Countries

As a simple extension, we consider the baseline framework and assume now that within each country there are two types of workers: high-skilled and low-skilled workers. You can think of different skill types as different level of productivity of the labor input. High-skill labor force is denoted by H, and its productivity by A_H; low-skill labor and productivity are denoted by L and A_L, respectively. The final output in each country is a CES aggregator of these two types of inputs, with elasticity of substitution between inputs ς. The production function of good c (similarly for good f) can thus be written as

$$y_c = \left[(A_L L)^{\frac{\varsigma-1}{\varsigma}} + (A_H H)^{\frac{\varsigma-1}{\varsigma}}\right]^{\frac{\varsigma}{\varsigma-1}}. \tag{39}$$

Wages of each skill type are given by their marginal product. For the low-skill workers, this means

$$\begin{aligned} w_L &= \frac{\varsigma}{\varsigma-1}\left[(A_L L)^{\frac{\varsigma-1}{\varsigma}} + (A_H H)^{\frac{\varsigma-1}{\varsigma}}\right]^{\frac{1}{\varsigma-1}} \frac{\varsigma-1}{\varsigma} L^{-\frac{1}{\varsigma}} A_L^{\frac{\varsigma-1}{\varsigma}} \\ &= (L)^{\frac{1}{\varsigma}}\left[\left(A_L^{\frac{\varsigma-1}{\varsigma}} + \left(A_H \frac{H}{L}\right)^{\frac{\varsigma-1}{\varsigma}}\right)\right]^{\frac{1}{\varsigma-1}} L^{-\frac{1}{\varsigma}} A_L^{\frac{\varsigma-1}{\varsigma}} \\ &= \left[A_L^{\frac{\varsigma-1}{\varsigma}} + \left(A_H \frac{H}{L}\right)^{\frac{\varsigma-1}{\varsigma}}\right]^{\frac{1}{\varsigma-1}} A_L^{\frac{\varsigma-1}{\varsigma}}. \end{aligned} \tag{40}$$

Similarly, the wage of the high-skill workers is given by

$$w_H = \left[\left(A_L\frac{L}{H}\right)^{\frac{\varsigma-1}{\varsigma}} + A_H^{\frac{\varsigma-1}{\varsigma}}\right]^{\frac{1}{\varsigma-1}} A_H^{\frac{\varsigma-1}{\varsigma}}. \tag{41}$$

First thing to notice is that due to the imperfect substitutability of the two labor inputs, wages are increasing in the level of productivity of each type of workers. Moreover, w_L increases with $\frac{H}{L}$, while w_H increases with $\frac{L}{H}$. The scarcer an input, in relative terms, the higher its compensation. Dividing the two expressions, we can write the relative wages as[8]

$$\frac{w_H}{w_L} = \left(\frac{L}{H}\right)^{\frac{1}{\varsigma}} \left(\frac{A_H}{A_L}\right)^{\frac{\varsigma-1}{\varsigma}}. \tag{42}$$

In this simple "specific-factors" model the factor-specificity is directly linked to exogenous ratios of factor endowments and productivities. While in this model trade does not affect either of the ratios in the right-hand-side of this equation, it is easy to imagine cases in which it does: If workers were allowed to move across sectors and workers had different productivities, both the ratio of endowments and the relative productivities of the sectors would be tightly related to trade. A new literature in trade attempts to make this link between trade and inequality by using the modeling tools developed in Eaton and Kortum (2002). See, for example, Burstein and Vogel (2012), Burstein, Morales, and Vogel (2015), Lee (2015) (for models of worker assignment, endogenous skill allocation and inequality) and Simon Galle and Yi (2015) (for a model of worker assignment and inequality) and the review of Costinot and Vogel (2014).

8 Conclusions

This primer analyzes gains from openness, multinational production, and migration in a simple Armington model with two countries and perfect competition. Among other things, we showed that countries benefit from an increase in both their productivity and that of their trading partners. They also benefit when factors of production can and cannot move across space. This is prima-facie evidence that international trade can be a mutually beneficial arrangement between countries. We then showed that gains from trade are significantly

[8] Notice that because the workers face the same price index, their relative wage is also their relative welfare, since

$$\frac{w_L}{w_H} = \frac{w_L/P}{w_H/P}$$

which allows us to derive a measure of inequality in a model with multiple types of workers.

positive for all countries, and that they crucially depend on trade costs (negatively) and (positively) on the elasticity of substitution among different goods, i.e. on how much consumers value good variety. In addition to exporting, gains from trade are even larger in countries where foreign firms produce their goods.

Some concluding remarks follow. The main goal of these notes was to illustrate through the lens of a simple model how we can think of globalization and the global market as economists. We want to encourage the reader to understand that international trade is just another economic activity, and that open economies share the basic economic principles as closed ones. In particular, it is fundamentally wrong to think of trading partners as competitors. The benefits of international trade do not require a country to have an absolute advantage over its trading partners, but the benefits of economic integration can be accrued to all participants.

References

Allen, T., and C. Arkolakis (2014): "Trade and the topography of the spatial economy," *Quarterly Journal of Economics*, 129(6), 1085–1140.

Allen, T., C. Arkolakis, and Y. Takahashi (2014): "Universal Gravity," *mimeo*.

Anderson, J. E., and E. Van Wincoop (2004): "Trade Costs," *Journal of Economic Literature*, 42(3), 691–751.

Arkolakis, C., A. Costinot, and A. Rodríguez-Clare (2012): "New Trade Models, Same Old Gains?," *American Economic Review*, 102(1), 94–130.

Arkolakis, C., S. Demidova, P. J. Klenow, and A. Rodríguez-Clare (2008): "Endogenous Variety and the Gains from Trade," *American Economic Review, Papers and Proceedings*, 98(4), 444–450.

Arkolakis, C., N. Ramondo, A. Rodríguez-Clare, and S. Yeaple (2013): "Innovation and production in the global economy," *NBER*, 18792.

Armington, P. S. (1969): "A Theory of Demand for Products Distinguished by Place of Production," *International Monetary Fund Staff Papers*, 16, 159–178.

Burstein, A., E. Morales, and J. Vogel (2015): "Accounting for changes in between-group inequality," Discussion paper, Columbia, Princeton, and UCLA.

Burstein, A., and J. Vogel (2012): "International Trade, Technology, and the Skill Premium," Manuscript, Columbia University and UCLA.

Chaney, T. (2008): "Distorted Gravity: The Intensive and Extensive Margins of International Trade," *American Economic Review*, 98(4), 1707–1721.

Costinot, A., and J. Vogel (2014): "Beyond Ricardo: Assignment Models in International Trade," Discussion paper, National Bureau of Economic Research.

Eaton, J., and S. Kortum (2002): "Technology, Geography and Trade," *Econometrica*, 70(5), 1741–1779.

Helpman, E. (1998): "The Size of Regions," *Topics in Public Economics. Theoretical and Applied Analysis*, pp. 33–54.

Krugman, P. (1991): "Increasing Returns and Economic Geography," *The Journal of Political Economy*, 99(3), 483–499.

——— (1997): *Pop Internationalism*. MIT Press.

Lee, E. (2015): "Endogenous Allocation of Heterogeneous Workers and the Distributional Effect of Trade," Discussion paper, Yale University.

McGrattan, E., and E. C. Prescott (2010): "Technology Capital and the US Current Account," *American Economic Review*, 100(4), 1493–1522.

Melitz, M. J. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71(6), 1695–1725.

Ramondo, N. (2012): "A Quantitative Approach to Multinational Production," *Journal of International Economics*.

Ramondo, N., and A. Rodríguez-Clare (2013): "Trade, Multinational Production, and the Gains from Openness," *Journal of Political Economy*.

Redding, S. J. (2012): "Goods Trade, Factor Mobility and Welfare," *mimeo*.

Simon Galle, A. R.-C., and M. Yi (2015): "Slicing the Pie: Quantifying the Aggregate and Distributional Effects of Trade," Discussion paper, Berkeley University.

Appendix

8.1 Formal derivation of consumer's optimal consumption

We want to solve the problem

$$\begin{aligned} & \max_{c_c, c_f} \left(c_c^{\frac{\sigma-1}{\sigma}} + c_f^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \\ s.t \qquad & p_c c_c + p_f c_f = wL \end{aligned}$$

where prices are taken as given.
The Lagrangian of the problem is

$$\mathcal{L}(\lambda; c_c, c_f) = \left(c_c^{\frac{\sigma-1}{\sigma}} + c_f^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} + \mu(wL - p_c c_c + p_f c_f) \tag{43}$$

To derive consumer's demand we look at the first order conditions of the problem. They are

$$\{c_c\}: \quad \frac{\sigma}{\sigma-1} \left(c_c^{\frac{\sigma-1}{\sigma}} + c_f^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \frac{\sigma-1}{\sigma} c_c^{-\frac{1}{\sigma}} = \mu p_c \tag{44}$$

$$\{c_f\}: \quad \frac{\sigma}{\sigma-1} \left(c_c^{\frac{\sigma-1}{\sigma}} + c_f^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \frac{\sigma-1}{\sigma} c_f^{-\frac{1}{\sigma}} = \mu p_f \tag{45}$$

$$\{\mu\}: \quad p_c c_c + p_f c_f = wL \tag{46}$$

Divide (44) by (45) to get (4), namely

$$\frac{c_h}{c_f} = \left(\frac{p_h}{p_f} \right)^{-\sigma}.$$

Note that these results do not depend on the fact that the consumer can choose only between two goods. In the case with MNEs, for example, the Lagrangean of the consumer's problem reads

$$\mathcal{L}(\lambda; c_c, c_f, c_{c^M}, c_{f^M}) = \left(c_c^{\frac{\sigma-1}{\sigma}} + c_f^{\frac{\sigma-1}{\sigma}} + c_{c^M}^{\frac{\sigma-1}{\sigma}} + c_{f^M}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} + \mu(wL - p_c c_c - p_f c_f - p_{c^M} c_{c^M} - p_{f^M} c_{f^M})$$

The FOC for c_c and c_f will differ from (44) and (45) only in the term within brackets, which is common across goods and will cancel out when dividing the two conditions. Therefore, the equilibrium condition for c_c will be the same as in the model with only two goods. This result can be generalized to the case of N goods.

8.2 Extension: Nested CES

Consider the model with Multinational Enterprises where the consumer can choose among four different goods, c, c^M, f and f^M. In this section, we will show that the problem we studied in section 4 is similar in terms of solution to a situation where the consumer chooses how much to spend on good produced by the US and Greece first, and then allocate those amounts to the different goods.

Let c_c, c_{c^M}, c_f and c_{f^M} be the consumption of each of the four goods, as before. Define a "composite" of goods produced by the US as C_C, such that

$$C_C = c_c^{\frac{\nu-1}{\nu}} + c_{c^M}^{\frac{\nu-1}{\nu}}.$$

Similarly, let C_F be defined as

$$C_F = c_f^{\frac{\nu-1}{\nu}} + c_{f^M}^{\frac{\nu-1}{\nu}}.$$

The elasticity of substitution ν measures the degree of substitability of the "varieties" of goods produced by one country. In our case, a variety is defined by the country of production.[9]

We can define price indices for these composite goods as weighted mean over prices, namely

$$\begin{aligned} P_C &= \left(p_c^{\nu} + p_{c^M}^{\nu}\right)^{\frac{1}{\nu}} \\ P_F &= \left(p_f^{\nu} + p_{f^M}^{\nu}\right)^{\frac{1}{\nu}}. \end{aligned}$$

The consumer problem can be written as a two-tier problem as follows. First, she solves the following utility maximization problem over composites:

$$\begin{aligned} \max_{C_C, C_F} U(C_C, C_F) &\equiv \left(C_C^{\frac{\theta-1}{\theta}} + C_F^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \\ s.t. \quad & P_C C_C + P_F C_F = wL \end{aligned}$$

Here, θ is the elasticity of substitution between composite goods. It measure how much the individual is willing to shift consumption among goods produced by each country. Define the total expenditure on goods produced by US and Greece as $y_C \equiv P_C C_C$ and $y_F \equiv p_F c_F$, respectively. The consumer chooses how to allocate income optimally between composite goods C_H and C_F, i.e. she chooses y_H and y_F. Note that this problem has the same structure as the standard Armington model with two countries we discussed in section 2. Hence, we can use the results derived there, in particular equation (9), and write:

[9]Note that we are assuming that it is constant for different producing countries.

$$y_C = \left(\frac{P_C}{P}\right)^{1-\theta} wL$$

$$y_F = \left(\frac{P_F}{P}\right)^{1-\theta} wL$$

In a second stage, the consumer takes y_C and y_F as given, and decides how to allocate those amounts to each particular good. We assume that the second stage problem maintains the CES structure, and thus write it as

$$\begin{aligned} \max_{c_c, c_{c^M}} u(c_c, c_{c^M}) &\equiv \left(c_c^{\frac{\nu-1}{\nu}} + c_{c^M}^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}} \\ s.t. \qquad & p_c c_c + p_{c^M} c_{c^M} = y_C \end{aligned}$$

and

$$\begin{aligned} \max_{c_f, c_{f^M}} u(c_f, c_{f^M}) &\equiv \left(c_f^{\frac{\nu-1}{\nu}} + c_{f^M}^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}} \\ s.t. \qquad & p_f c_f + p_{f^M} c_{f^M} = y_F \end{aligned}$$

Again, we can use the results we derived in section 3 and write the solutions to these 2 "second-stage" problems as

$$\begin{aligned} c_c &= \frac{p_c^{-\nu}}{P_C^{1-\nu}} y_C \\ c_{c^M} &= \frac{p_{c^M}^{-\nu}}{P_C^{1-\nu}} y_C \end{aligned}$$

and similarly for c_f and c_f^M. We can substitute for the optimal y_H and write

$$\begin{aligned} c_c &= \frac{p_c^{-\nu}}{P_C^{1-\nu}} y_C \\ &= \frac{p_c^{-\nu}}{P_C^{1-\nu}} \left(\frac{P_C}{P}\right)^{1-\theta} wL \end{aligned}$$

and similarly for the other goods. Note that if we assume that the elasticity of substitution between composites is the same as the elasticity of substitution between varieties, i.e if we assume that $\theta = \nu$, we can further simplify the expression and write

$$c_c = \left(\frac{p_c^{-\nu}}{P^{1-\nu}} \right) wL$$

which is the same as (7). Therefore, if the elasticity of substitution between and within composite goods is the same, choosing among different goods is equivalent to a two-stage problem where the consumer choose how to allocate income across composite goods first, and then solve the "within" problem and choose varieties of each composite. Now consider welfare. Again, notice that in the model with MNE the home shares of production and consumption do not coincide. In particular, the home share of production is given by

$$\begin{aligned} \lambda &= \frac{y_C}{wL} \\ &= \left(\frac{P_C}{P} \right)^{1-\theta}. \end{aligned}$$

On the other hand, the home share of consumption is given by

$$\begin{aligned} \widetilde{\lambda} &= \frac{p_c c_c}{wL} \\ &= \frac{p_c^{1-\nu}}{P_C^{1-\nu}} \left(\frac{P_C}{P} \right)^{1-\theta} \\ &= \left(\frac{w}{P_C} \right)^{1-\nu} \left(\frac{P_C}{P} \right)^{1-\theta} \\ &= \left(\frac{w}{P} \right)^{1-\nu} \left(\frac{P_C}{P} \right)^{\nu-\theta} \\ &= \left(\frac{w}{P} \right)^{1-\nu} (\lambda)^{\frac{\nu-\theta}{1-\theta}} \end{aligned}$$

Finally, we can write welfare as

$$\left(\frac{w}{P} \right) = \left(\widetilde{\lambda} \right)^{\frac{1}{1-\nu}} (\lambda)^{\frac{\theta-\nu}{(1-\theta)(1-\nu)}}.$$

Obviously, if $\theta = \nu$, we go back to the previous version where

$$\left(\frac{w}{P} \right) = \left(\widetilde{\lambda} \right)^{\frac{1}{1-\nu}}$$

as discussed in the main text.

8.3 Extension: Symmetric Trade Costs, Labor Fixed, Same Productivities

Consider the baseline model, and assume that there productivies are the same, i.e. $A = A^* = 1$. To make progress analytically we follow the approach of Allen, Arkolakis, and Takahashi (2014) and assume that countries have symmetric trade costs, i.e. $\tau = \tau^* > 1$. Use the market clearing condition (13) to write

$$\begin{aligned}\left(\frac{p_c^{-\sigma}}{P^{1-\sigma}}wL + \tau\frac{p_c^{*-\sigma}}{(P^*)^{1-\sigma}}w^*L^*\right) &= L \\ \left(\frac{w}{P}\right)^{1-\sigma}\left(1+\tau^{1-\sigma}\left(\frac{P}{P^*}\right)^{1-\sigma}\left(\frac{w}{w^*}\right)^{-1}\frac{L^*}{L}\right) &= 1 \qquad (47)\end{aligned}$$

By symmetry

$$\left(\frac{w^*}{P^*}\right)^{1-\sigma}\left(1+\tau^{1-\sigma}\left(\frac{P^*}{P}\right)^{1-\sigma}\left(\frac{w^*}{w}\right)^{-1}\frac{L}{L^*}\right) = 1$$

Now the price index can be written as

$$P^{1-\sigma} = \left(w^{1-\sigma} + (w^*\tau)^{1-\sigma}\right)$$

and

$$(P^*)^{1-\sigma} = \left((w^*)^{1-\sigma} + (w\tau)^{1-\sigma}\right)$$

so that

$$\begin{aligned}\left(\frac{P^*}{P}\right)^{1-\sigma} &= \frac{(w^*)^{1-\sigma} + (w\tau)^{1-\sigma}}{w^{1-\sigma} + (w^*\tau)^{1-\sigma}} \\ \left(\frac{P^*}{P}\right)^{1-\sigma} &= \frac{(w)^{1-\sigma}}{(w^*)^{1-\sigma}}\cdot\frac{\left(\frac{w^*}{w}\right)^{1-\sigma} + (\tau)^{1-\sigma}}{\left(\frac{w}{w^*}\right)^{1-\sigma} + (\tau)^{1-\sigma}} \implies \\ \left(\frac{\frac{w}{P}}{\frac{w^*}{P^*}}\right)^{1-\sigma} &= \frac{\left(\frac{w^*}{w}\right)^{1-\sigma} + (\tau)^{1-\sigma}}{\left(\frac{w}{w^*}\right)^{1-\sigma} + (\tau)^{1-\sigma}}\end{aligned}$$

Rewrite (47) as

$$\begin{aligned}\left(\frac{w}{P}\right)^{1-\sigma}\left(1+\tau^{1-\sigma}\left(\frac{w^*/P^*}{w/P}\right)^{1-\sigma}\left(\frac{w}{w^*}\right)^{-\sigma}\frac{L^*}{L}\right) &= 1 \\ \left(\frac{w}{P}\right)^{1-\sigma}\left(1+\tau^{1-\sigma}\frac{\left(\frac{w}{w^*}\right)^{1-\sigma} + (\tau)^{1-\sigma}}{\left(\frac{w^*}{w}\right)^{1-\sigma} + (\tau)^{1-\sigma}}\left(\frac{w}{w^*}\right)^{-\sigma}\frac{L^*}{L}\right) &= 1 \qquad (48)\end{aligned}$$

and

$$\left(\frac{w^*}{P^*}\right)^{1-\sigma}\left(1+\tau^{1-\sigma}\frac{\left(\frac{w^*}{w}\right)^{1-\sigma}+(\tau)^{1-\sigma}}{\left(\frac{w}{w^*}\right)^{1-\sigma}+(\tau)^{1-\sigma}}\left(\frac{w^*}{w}\right)^{-\sigma}\frac{L}{L^*}\right)=1. \tag{49}$$

Dividing (48) by (49), get

$$\frac{\left(\frac{w^*}{w}\right)^{1-\sigma}+(\tau)^{1-\sigma}}{\left(\frac{w}{w^*}\right)^{1-\sigma}+(\tau)^{1-\sigma}}\cdot\frac{\left(1+\tau^{1-\sigma}\frac{\left(\frac{w}{w^*}\right)^{1-\sigma}+(\tau)^{1-\sigma}}{\left(\frac{w^*}{w}\right)^{1-\sigma}+(\tau)^{1-\sigma}}\left(\frac{w}{w^*}\right)^{-\sigma}\frac{L^*}{L}\right)}{\left(1+\tau^{1-\sigma}\frac{\left(\frac{w^*}{w}\right)^{1-\sigma}+(\tau)^{1-\sigma}}{\left(\frac{w}{w^*}\right)^{1-\sigma}+(\tau)^{1-\sigma}}\left(\frac{w^*}{w}\right)^{-\sigma}\frac{L}{L^*}\right)}=1$$

or

$$\frac{x^{\sigma-1}+(\tau)^{1-\sigma}}{x^{1-\sigma}+(\tau)^{1-\sigma}}\cdot\left(1+\tau^{1-\sigma}\frac{x^{1-\sigma}+(\tau)^{1-\sigma}}{x^{\sigma-1}+(\tau)^{1-\sigma}}x^{-\sigma}\frac{L^*}{L}\right)=\left(1+\tau^{1-\sigma}\frac{x^{\sigma-1}+(\tau)^{1-\sigma}}{x^{1-\sigma}+(\tau)^{1-\sigma}}x^{\sigma}\frac{L}{L^*}\right)$$

where we defined $x\equiv\frac{w}{w^*}$. Rewrite as

$$\begin{aligned}(x^{\sigma-1}+(\tau)^{1-\sigma})\cdot\left(1+\tau^{1-\sigma}\frac{x^{1-\sigma}+(\tau)^{1-\sigma}}{x^{\sigma-1}+(\tau)^{1-\sigma}}x^{-\sigma}\frac{L^*}{L}\right) &= x^{1-\sigma}+(\tau)^{1-\sigma}+\tau^{1-\sigma}(x^{\sigma-1}+(\tau)^{1-\sigma})x^{\sigma}\frac{L}{L^*}\\ x^{\sigma-1}+\tau^{1-\sigma}(x^{1-\sigma}+(\tau)^{1-\sigma})x^{-\sigma}\frac{L^*}{L} &= x^{1-\sigma}+\tau^{1-\sigma}(x^{\sigma-1}+(\tau)^{1-\sigma})x^{\sigma}\frac{L}{L^*}\end{aligned}$$

$$\Rightarrow\quad a_0x^{2\sigma-1}+a_1x^{\sigma}-a_2x^{\sigma-1}+a_3x^{1-\sigma}-a_4x^{-\sigma}-a_5x^{1-2\sigma}=0 \tag{50}$$

where $a_0=\tau^{1-\sigma}l,\ a_1=\tau^{2(1-\sigma)}l,\ a_2=a_3=1,\ a_4=\tau^{2(1-\sigma)}l^{-1},\ a_5=\tau^{1-\sigma}l^{-1}$ and $l\equiv\frac{L}{L^*}$. Equation (50) represents a non-linear 'polynomial equation' in x. Itcan be used to derive the wage ratio x as a function of trade costs τ and the labor ratio. Substituting x in (48) and (49) finally gives a relationship between welfare and trade costs.

8.4 Extension: No Trade Costs, Different Productivities

Consider the model with migration studied in section 5, but now assume that there are no costs of trade, i.e. $\tau=\tau^*=1$, Clearly, since they can freely move across countries, equation (29) holds in this model. Because there are no trade costs prices equalize across countries, $P=P^*$, which implies

$$w=w^*. \tag{51}$$

Use the market clearing condition (13) to write

$$\begin{aligned} \left(\frac{p_c^{-\sigma}}{P^{1-\sigma}} wL + \frac{p_c^{*-\sigma}}{(P^*)^{1-\sigma}} w^* L^* \right) &= L \\ \left(\left(\frac{w}{P}\right)^{1-\sigma} \left(\frac{1}{A}\right)^{-\sigma} L + \frac{w^{1-\sigma}}{(P^*)^{1-\sigma}} \left(\frac{1}{A}\right)^{-\sigma} \left(\frac{w^*}{w}\right) L^* \right) &= L \\ \left(\frac{w}{P}\right)^{1-\sigma} \left(\frac{1}{A}\right)^{-\sigma} \left(1 + \frac{L^*}{L}\right) &= 1. \end{aligned} \tag{52}$$

By symmetry, starting from (14) we could write

$$\left(\frac{w^*}{P^*}\right)^{1-\sigma} \left(\frac{1}{A^*}\right)^{-\sigma} \left(1 + \frac{L}{L^*}\right) = 1 \tag{53}$$

Divide (52) by (53), and use (29) to get

$$\begin{aligned} \left(\frac{1}{A}\right)^{-\sigma} \left(1 + \frac{L^*}{L}\right) &= \left(\frac{1}{A^*}\right)^{-\sigma} \left(1 + \frac{L}{L^*}\right) \\ \left(\frac{A^*}{A}\right)^{-\sigma} &= \frac{L}{L^*}. \end{aligned} \tag{54}$$

Hence, everything else equal, labor is more abundant in places where productivity is higher. Note that using (52), we can write welfare as

$$\left(\frac{w}{P}\right) = \frac{1}{(A^\sigma + A^{*\sigma})^{\frac{1}{1-\sigma}}}$$

which says that welfare is increasing in the productivity of both countries.

8.4.1 Gains from Migration in a 2-countries asymmetric world with no trade costs

What are the gains from moving from a trade equilibrium with no mobility to an equilibrium with migration, in a 2-country asymmetric world with no trade costs? Notice that we already derived closed form expressions for welfare in the two cases, which makes the comparison easy. Let the relative productivity and size in the US and Greece be denoted by $a \equiv \frac{A^*}{A}$, and $l \equiv \frac{L^*}{L}$.

Welfare in the high productivity country

Let us first assume that the US has a productivity advantage over Greece, i.e $A > A^*$, or $a \in [0,1)$.

In the trade equilibrium with no mobility, welfare in the high-productivity country is given by (17), which we can rewrite as

$$\begin{aligned} W^{H,TRADE} \equiv \frac{w}{P} &= \left(\frac{A^{1-\sigma}}{A + A^* l^{\frac{\sigma-1}{\sigma}}}\right)^{\frac{1}{1-\sigma}} \\ &= \left(\frac{1}{A^\sigma + A^{\sigma-1}A^* l^{\frac{\sigma-1}{\sigma}}}\right)^{\frac{1}{1-\sigma}} \\ &= \left(\frac{1}{A^\sigma(1 + a l^{\frac{\sigma-1}{\sigma}})}\right)^{\frac{1}{1-\sigma}} \end{aligned} \tag{55}$$

In the equilibrium with geography, welfare in the high-productivity country is given by (30), which we can rewrite as

$$\begin{aligned} W^{H,GEO} \equiv \frac{w}{P} &= \left(\frac{1}{A^\sigma + (A^*)^\sigma}\right)^{\frac{1}{1-\sigma}} \\ &= \left(\frac{1}{A^\sigma(1 + a^\sigma)}\right)^{\frac{1}{1-\sigma}} \end{aligned} \tag{56}$$

The gains from moving to the equilibrium with migration in the high productivity country are thus given by

$$\hat{w}^H \equiv \frac{W^{H,GEO}}{W^{H,TRADE}} = \left(\frac{1 + a l^{\frac{\sigma-1}{\sigma}}}{1 + a^\sigma}\right)^{\frac{1}{1-\sigma}} \tag{57}$$

Notice that the high productivity countries gains from opening to migration only if

$$\begin{aligned} \hat{w}^H &> 1 \\ \Longleftrightarrow \quad \left(\hat{w}^H\right)^{1-\sigma} &< 1 \\ \Longleftrightarrow \quad \left(\tfrac{1+a l^{\frac{\sigma-1}{\sigma}}}{1+a^\sigma}\right) &< 1 \\ \Longleftrightarrow \quad l^{\frac{\sigma-1}{\sigma}} &< a^{\sigma-1} \\ \Longleftrightarrow \quad l &< a^\sigma. \end{aligned} \tag{58}$$

Notice that $l \in [0, \infty]$, whereas $a \in [0,1)$. Therefore, (58) cannot be satisfied if $l > 1$, namely $L^* > L$. The same thing is true in the limit case when the productivity advantage is very big, i.e. $a \longrightarrow 0$. In both cases, the high productivity country is always worse off when it moves from a trade to a migration equilibrium.

Welfare in the low productivity country

How about welfare in the low productivity country? Let us now assume that the US is the low productivity country, i.e. $a > 1$. Notice that nothing in the derivation above relied on the assumption $a \in [0, 1)$, so that we can just use (57) and rewrite it as:

$$\hat{w}^L \equiv \frac{W^{L,GEO}}{W^{L,TRADE}} = \left(\frac{1 + al^{\frac{\sigma-1}{\sigma}}}{1 + a^\sigma} \right)^{\frac{1}{1-\sigma}} \tag{59}$$

As in (58), the low productivity country benefits from moving to a migration equilibrium if $l < a^\sigma$. However, since now $a > 1$, this inequality is satisfied more often than in the previous case. First notice that if the low productivity country is bigger than the high productivity country, i.e. $l < 1$, (59) is always satisfied, i.e. the low productivity country is always better off when it moves to an equilibrium with migration. The same thing is true in the limit case when the productivity advantage is very big, i.e. $a \longrightarrow \infty$, under the (mild) assumption that the difference in size of the two countries is not too big, i.e. $l < \infty$.

We thus, have proved the Proposition in paragraph 6.