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Symmetric Separating Equilibria in English Auctions[1]

Sushil Bikhchandani[2]

Anderson Graduate School of Management, University of California, Los Angeles, California 90095

Philip A. Haile[3]

Department of Economics, University of Wisconsin-Madison, Madison, Wisconsin 53706

and

John G. Riley[4]

Department of Economics, University of California, Los Angeles, California 90095

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We characterize the set of perfect Bayesian equilibria in symmetric separating strategies in the model of English auctions given by P. R. Milgrom and R. J. Weber (1982, *Econometrica* **50,** 1089–1122). There is a continuum of such equilibria. The equilibrium derived by Milgrom and Weber is that in which bids are maximal. Only in the case of pure private values does a restriction to weakly undominated strategies select a unique equilibrium. This has important implications for empirical studies of English auctions, particularly outside the pure private values paradigm. *Journal of Economic Literature* Classification Numbers: D44, D82.

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[2] E-mail: sbikhcha@anderson.ucla.edu.
[3] E-mail: pahaile@facstaff.wisc.edu. Financial support from National Science Foundation Grant SBR-9809082 is gratefully acknowledged.
[4] E-mail: riley@econ.ucla.edu.

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1. INTRODUCTION

Milgrom and Weber (1982) present the standard model of English auctions and propose a symmetric separating equilibrium in which, given a set of bidders who have not yet exited, each acts as if he believes that all other remaining bidders have the same private information he has. Milgrom and Weber's intuition for the optimality of this strategy is based on a bidder's asking at what price he would be unhappy if all remaining opponents simultaneously dropped out. However, under their natural distributional assumptions, a mass exit of this sort is a zero-probability event. As long as at least three bidders remain in the auction, each knows that he will have an opportunity to re-optimize once he sees one of his opponents exit. This suggests that there might be considerable flexibility in the strategies that could be used in the early rounds of the auction. While Milgrom and Weber consider only one equilibrium, they describe it as yielding a two-stage procedure in which the $n - 2$ bidders with the lowest signals first publicly reveal their types, and then the last two bidders engage in a second-price sealed bid auction. We show that this description characterizes all perfect Bayesian equilibria of the English auction in symmetric separating strategies, but that there are infinitely many such equilibria because there are infinitely many ways that the information revelation portion of the auction can take place.

While it is known that there is a continuum of *asymmetric* equilibria in a pure common values English auction (Bikhchandani and Riley, 1991), we focus on *symmetric* equilibria in the more general affiliated values framework. We derive necessary and sufficient conditions characterizing the equilibrium set. The equilibrium derived by Milgrom and Weber (1982) is that yielding the highest exit prices. All equilibria specify identical behavior in the final phase of the auction; i.e., the transaction price is unique. Hence, the multiplicity of equilibria does not affect well-known results ordering standard auctions by expected revenues. However, it has important implications for empirical and experimental research, since one cannot be certain of the interpretation of inframarginal bids.

2. MODEL AND RESULT

Our model is identical to Milgrom and Weber's model (1982) of symmetric affiliated values English auctions, up to one additional assumption imposed below. A seller holds an auction to sell a single indivisible object to one of n bidders. The value bidder i would receive from the object is $u(\mathbf{S}, X_i, \mathbf{X}_{-i})$, where $\mathbf{S}$ is a vector of random variables unobserved by bidders, $X_i \in [\underline{x}, \bar{x}]$ is bidder i's private information, and $\mathbf{X}_{-i}$ is that of his opponents. The random variables $\mathbf{S}, X_i, \mathbf{X}_{-i}$ are affiliated, with joint distribution F and associated joint density f. We make the following additional continuity assumption:[5]

CONT $E[u(\mathbf{S}, X_i, \mathbf{X}_{-i}) \mid X_i, \mathbf{X}_{-i}]$ is continuous in X_i and $\mathbf{X}_{-i}$.

The auction is conducted according to the following rules. The price begins at zero with all bidders "in." Bidders may then exit observably and irreversibly as the price rises continuously and exogenously. The auction ends when only one bidder remains, with this bidder receiving the good and paying the price at which his final opponent quit. If at some point in the auction all remaining bidders quit simultaneously, one of these bidders is selected at random and named the winner at his exit price.

The auction can be viewed in a sequence of phases, beginning in phase zero and entering a new phase each time a bidder exits. Taking the perspective of an arbitrary bidder i, let $\mathbf{Y} = Y_1, \ldots, Y_{n-1}$ denote the ordered (highest to lowest) values of $\mathbf{X}_{-i}$. In a separating equilibrium, the prices at which bidders exit reveal their signals. For $k \geq 1$, let $\mathbf{Y}^k$ denote the signals $Y_{n-1}, \ldots, Y_{n-k}$ that have been revealed by exits in phases $0, \ldots, k-1$. To simplify the exposition, define $\mathbf{Y}^0 \equiv \varnothing$ and $Y_n \equiv \underline{x}$. Finally, let $F_k(\cdot; \mathbf{y}^k)$ denote the distribution of Y_{n-k-1} conditional on $\mathbf{Y}^k = \mathbf{y}^k$.

A bidder's strategy for phase k is given by a bid function $b^k(\cdot; \mathbf{y}^k)$, specifying his exit price as a function of his signal and the information revealed by prior exits. In the final phase (phase $n-2$), assume that all bidders use a strictly increasing bid function $b^{n-2}(\cdot; \mathbf{y}^{n-2})$.[6] Then when $X_i = x$, bidder i chooses $\tilde{x}$ to maximize

$$\pi^{n-2}(x, \tilde{x}; \mathbf{y}^{n-2}) = \int_{y_2}^{\max\{\tilde{x}, y_2\}} \{E[u(\mathbf{S}, X_i, \mathbf{X}_{-i}) \mid X_i = x,\, Y_1 = y,\, \mathbf{Y}^{n-2} = \mathbf{y}^{n-2}] - b^{n-2}(y; \mathbf{y}^{n-2})\} dF_{n-2}(y; \mathbf{y}^{n-2}).$$

The first-order condition implies the following result.

[5] We use this assumption only in showing necessity of the conditions we give to characterize the equilibrium set. Hence, even without this assumption we show that a continuum of symmetric separating equilibria exists.

[6] Note that we do not restrict the strategy space to bids above the prior exit price. Instead, we will prove that these are the only bids made in an equilibrium in which bidders' exit prices in each phase are weakly increasing in types everywhere and strictly increasing for types choosing bids of at least the prior exit price.

LEMMA. *In any symmetric Bayesian Nash equilibrium in strictly increasing strategies, in the final phase of the auction bidders use the bid function*

$$b_0^{n-2}(x;\mathbf{y}^{n-2}) = E\left[u(\mathbf{S}, X_i, \mathbf{X}_{-i}) \mid X_i = x,\ Y_1 = x,\ \mathbf{Y}^{n-2} = \mathbf{y}^{n-2}\right].$$

Note that affiliation and Milgrom and Weber's "nondegeneracy assumption"[7] imply that the bid function $b_0^{n-2}(\cdot;\mathbf{y}^{n-2})$ is strictly increasing and that $\frac{\partial}{\partial \tilde{x}}\pi^{n-2}(x,\tilde{x};\mathbf{y}^{n-2})$ is nondecreasing in x, ensuring that $b_0^{n-2}(\cdot;\mathbf{y}^{n-2})$ is a best response to itself.

THEOREM. *Under assumption CONT, the strategies* $\mathbf{b}_0 = (b_0^0, b_0^1, \ldots, b_0^{n-2})$ *constitute a perfect Bayesian equilibrium in symmetric strictly increasing strategies if and only if each* $b_0^k(x;\mathbf{y}^k)$ *is strictly increasing in* x *for all* $x \geq y_{n-k}$ *and*

(i) $b_0^{n-2}(x;\mathbf{y}^{n-2}) = E[u(\mathbf{S}, X_i, \mathbf{X}_{-i}) \mid X_i = Y_1 = x,\ \mathbf{Y}^{n-2} = \mathbf{y}^{n-2}]$;

(ii) $b_0^{k+1}(y;\{x,\mathbf{y}^k\}) > b_0^k(x;\mathbf{y}^k)$ *for all* $y > x \geq y_{n-k}$;

(iii) $b_0^k(x;\mathbf{y}^k) \leq E[u(\mathbf{S}, X_i, \mathbf{X}_{-i}) \mid X_i = Y_1 = \cdots = Y_{n-1-k} = x,$ $\mathbf{Y}^k = \mathbf{y}^k]$.

Proof. (Sufficiency) Suppose all bidders besides i use a strategy $\mathbf{b}_0$ satisfying (i), (ii), and (iii) and consider a phase $k < n-2$. Letting $\pi^k(x,\tilde{x};\mathbf{y}^k)$ denote the expected payoff of a bidder with signal x who bids $b_0^k(\tilde{x};\mathbf{y}^k)$ in phase k, suppose for the moment that for all x, y, and $\mathbf{y}^k$

$$\frac{\partial}{\partial \tilde{x}}\pi^{k+1}\left(x,\tilde{x};\{y,\mathbf{y}^k\}\right) \begin{matrix} \geq 0 & \quad \text{for } \tilde{x} \leq x \\ \leq 0 & \quad \text{for } \tilde{x} \geq x, \end{matrix} \tag{1}$$

where $\{y,\mathbf{y}^k\}$ denotes a feasible realization of $\mathbf{Y}^{k+1}$ given $\mathbf{Y}^k = \mathbf{y}^k$. If bidder i exits in phase k he gets nothing; otherwise he continues to the next phase, where (1) implies that his payoff is $\pi^{k+1}(x_i, x_i, \mathbf{y}^{k+1})$. Hence, when $X_i = x$, i chooses $\tilde{x}$ (i.e., an exit price $b_0^k(\tilde{x};\mathbf{y}^k)$) to maximize

$$\pi^k\left(x,\tilde{x};\mathbf{y}^k\right) = \int_{y_{n-k}}^{\max\{\tilde{x},\, y_{n-k}\}} \pi^{k+1}\left(x,x,\{y,\mathbf{y}^k\}\right) dF_k\left(y;\mathbf{y}^k\right).$$

Deviating to a bid $b_0^k(\tilde{x};\mathbf{y}^k) < b_0^k(x;\mathbf{y}^k)$ affects i only when he actually exits in phase k, in which case he receives a payoff of zero. Condition (ii) ensures that this is a lower bound on the payoff he would receive by remaining to the next phase. The same argument implies that all reductions in i's bid can only reduce his expected payoff. Deviating to a bid above $b_0^k(x;\mathbf{y}^k)$ has no effect on his payoff, since if he reaches phase $k+1$

[7] This is the assumption that $E[u(\mathbf{S}, X_i, \mathbf{X}_{-i}) \mid X_i = x,\ \mathbf{X}_{-i} = \mathbf{x}_{-i}]$ strictly increases in x.

by remaining in the auction past the price $b_0^k(x;\mathbf{y}^k)$, (1) ensures that it will be optimal for him to exit immediately when phase $k+1$ begins, giving him the payoff of zero that he would have obtained by bidding $b_0^k(x;\mathbf{y}^k)$. Hence

$$\frac{\partial}{\partial \tilde{x}} \pi^k\left(x, \tilde{x}; \mathbf{y}^k\right) \begin{matrix} \geq 0 & \text{for all } \tilde{x} \leq x \\ = 0 & \text{for all } \tilde{x} \geq x. \end{matrix}$$

This ensures optimality of the bid $b_0^k(x;\mathbf{y}^k)$ for phase k, under the assumption that (1) holds. Optimality of the entire bidding strategy $\mathbf{b}_0$ then follows by induction, recalling that $\frac{\partial}{\partial \tilde{x}} \pi^{n-2}(x, \tilde{x}; \mathbf{y}^{n-2})$ is nondecreasing in x.

(Necessity) Necessity of (i) is immediate from the lemma. To show necessity of (ii), suppose to the contrary that for some $k < n-2$ and some $\mathbf{y}^k$ we have $b_0^{k+1}(y;\{x,\mathbf{y}^k\}) \leq b_0^k(x,\mathbf{y}^k)$ for some $y > x \geq y_{n-k}$. Fix $\mathbf{Y}^{k+1} = \{x, \mathbf{y}^k\}$ and let $\bar{y} = \sup\{y : b_0^{k+1}(y; \{x, \mathbf{y}^k\}) \leq b_0^k(x, \mathbf{y}^k)\}$. Since $b_0^{k+1}(\cdot;\{x,\mathbf{y}^k\})$ must be strictly increasing, in equilibrium all types $z \in [x, \bar{y})$ exit immediately when phase $k+1$ begins. Hence, with positive probability, such a bidder wins the auction at price $b_0^k(x,\mathbf{y}^k)$ and obtains payoff

$$E\left[u(\mathbf{X}, X_i, \mathbf{X}_{-i}) \mid X_i = z,\ Y_j \in [x, \bar{y}] \text{ for } 1 \leq j < n-k-1,\right.$$
$$\left. Y_{n-k-1} = x,\ \mathbf{Y}^k = \mathbf{y}^k\right] - b_0^k\left(x, \mathbf{y}^k\right).$$

The nondegeneracy assumption and affiliation imply that this payoff equals zero for at most one type z. If this payoff is positive for any z, such a type would strictly prefer to raise his bid by a small amount in phase $k+1$, since this yields a discrete increase in the probability of winning. If this payoff is negative for all $z \in [x, \bar{y})$, then a bidder with type sufficiently close to x would have been better off exiting at a price below $b_0^k(x,\mathbf{y}^k)$ in phase k, since this would reduce his chance of a profitable win continuously while avoiding a discrete loss. Hence (ii) must hold. Finally, to show necessity of (iii), first note that this holds with equality in phase $n-2$ by (i). Now suppose that (iii) holds for all $k \in \{1, \ldots, n-2\}$ greater than some $\bar{k}$. Then for all such k

$$\begin{aligned} b_0^{k-1}&\left(x;\mathbf{y}^{k-1}\right) \\ &\leq \lim_{z \downarrow x} b_0^k\left(z;\{x,\mathbf{y}^{k-1}\}\right) \\ &\leq \lim_{z \downarrow x} E\left[u(\mathbf{S}, X_i, \mathbf{X}_{-i}) \mid X_i = Y_1 = \cdots = Y_{n-1-k} = z,\right. \\ &\qquad\qquad \left. Y_{n-k} = x,\ \mathbf{Y}^{k-1} = \mathbf{y}^{k-1}\right] \\ &= E\left[u(\mathbf{S}, X_i, \mathbf{X}_{-i}) \mid X_i = Y_1 = \cdots = Y_{n-k} = x,\ \mathbf{Y}^{k-1} = \mathbf{y}^{k-1}\right], \end{aligned}$$

where the first inequality follows from (ii), the second from the induction hypothesis, and the final equality from assumption CONT. Thus (iii) is true for all k. ■

When $n = 2$, the lemma implies that the equilibrium identified by Milgrom and Weber (1982) is the unique symmetric separating equilibrium. However, when $n > 2$, there is a continuum of such equilibria. The equilibrium derived by Milgrom and Weber is that in which bidding in each phase is maximal, since condition (iii) of the theorem then holds with equality.

3. REFINEMENTS

We briefly explore other equilibrium concepts that might shrink the equilibrium set. In Milgrom and Weber's equilibrium buyers do not regret their bids even after learning those of their opponents (Milgrom, 1981). An equilibrium concept that captures this notion is that of *posterior implementability* (Green and Laffont, 1987; Lopomo, 2001). It requires that, with probability one, each player's equilibrium action (bid) is a best response to the equilibrium actions (bids) of his opponents.[8] Formally, let B_i and $\mathbf{B}_{-i}$ denote, respectively, the random variables corresponding to the equilibrium bid of player i and those of his opponents. Let μ denote the equilibrium joint distribution of $(B_i, \mathbf{B}_{-i}, X_i, \mathbf{X}_{-i})$. Then the equilibrium strategies are *posterior optimal* (i.e., the equilibrium outcome is posterior implementable) if, for all i and μ-almost every $(b_i, \mathbf{b}_{-i}, x_i, \mathbf{x}_{-i})$, b_i is a best response (given x_i) to $\mathbf{b}_{-i}$.

We focus on the following example in which there are three bidders, with $[\underline{x}, \bar{x}] = [0, 1]$. The value bidder i places on the object is

$$u(x_i, x_j, x_h) = ax_i + (1 - a)\frac{x_j + x_h}{2}, \qquad i \neq j \neq h,$$

where $a \in [\frac{1}{3}, 1]$. Thus, the example ranges from a pure common values model to a pure private values model as the parameter a goes from $\frac{1}{3}$ to 1. Observe that if $x_i > x_j$ then $u(x_i, x_j, x_h) \geq u(x_j, x_i, x_h)$ with strict inequality when $a > \frac{1}{3}$. By the theorem, the following strategies yield a symmetric

[8] A closely related concept, due to Perry and Reny (1999), is that of *ex post equilibrium*, which requires that for each realization of types, the equilibrium actions are Nash equilibrium strategies for the game in which these types are common knowledge. The equilibria of the theorem satisfy this requirement for almost all realizations of types.

perfect Bayesian equilibrium for any $\beta \in (0, 1]$:

$$b_0^0(x) = \beta u(x, x, x) = \beta x,$$
$$b_0^1(x; y_2) = u(x, x, y_2) = \frac{1+a}{2}x + \frac{1-a}{2}y_2. \quad (2)$$

To verify that the strategies in (2) are posterior optimal, we can ignore (zero-probability) ties in types. So without loss of generality assume $x_1 > x_2 > x_3$. In equilibrium, bidder 3 is the first to drop out at a price of $b_3 = \beta x_3$, and later, after bidder 2's subsequent exit at price $b_2 = u(x_2, x_2, x_3)$, bidder 1 is declared the winner. Suppose the bids of bidders 2 and 3 are known and that bidder 1's intention of staying in the bidding until the price reaches $b_1 = u(x_1, x_2, x_3)$ is also known.[9] We verify that players' actions remain best responses even with this additional information, noting that knowledge of (b_1, b_2, b_3) implies knowledge of (x_1, x_2, x_3).

Bidder 1's bid remains a best response, as his valuation $u(x_1, x_2, x_3)$ exceeds b_2. Further, bidder 2's valuation $u(x_2, x_1, x_3)$ is less than the price b_1 at which he could have won the object, so his bid is a best response. Similarly, the fact that $b_1 > u(x_3, x_1, x_2)$ implies that 3 could not profit from a deviation. Thus all equilibria satisfying (2) are posterior implementable.

A stronger restriction is to weakly undominated strategies. Consider first a pure private values version of our example ($a = 1$). When $\beta < 1$, the symmetric equilibrium in (2) involves use of a (weakly) dominated strategy, since $b_0^0(x) < x$. With pure private values, the unique symmetric equilibrium in undominated strategies is that in which $b^k(x) = x$, $\forall k$. More generally, with $\beta < a \leq 1$, the strategy for the initial phase in (2) is dominated. To see this, suppose that bidders j and h follow the (nonequilibrium) strategies of dropping out at prices p_j, p_h regardless of their signals. If $p_j, p_h \in (\beta X_i, aX_i)$, the strategy $b_0^0(X_i) = \beta X_i$ yields a strictly lower payoff than the strategy $b_a^0(X_i) = aX_i$. Furthermore, because aX_i is the smallest possible valuation of bidder i, there are no possible bids by j and h to which $b_0^0(X_i)$ would be a strictly better response than $b_a^0(X_i)$.

It is easily verified, however, that under the restriction $\beta \in [a, 1]$ all symmetric equilibria in (2) involve weakly undominated strategies. Indeed,

[9] The latter may suggest more ex post knowledge than that in the original motivations for posterior implementability, since in practice even after the auction is over, the planned exit price of the winner will not be known to his opponents. Requiring bids to be optimal against the actual planned exit of the winner (as we do) rather than against a nondegenerate posterior on this planned exit can only strengthen the requirements of the solution concept.

in the general model the inequality

$$b_0^k(x;\mathbf{y}^k) \geq E\left[u(\mathbf{S}, X_i, \mathbf{X}_{-i}) \mid X_i = x,\ Y_1 = \cdots = Y_{n-1-k} = \underline{x},\ \mathbf{Y}^k = \mathbf{y}^k\right] \quad (3)$$

together with (i), (ii), and (iii) of the theorem characterizes the set of perfect Bayesian equilibrium in symmetric, strictly increasing, and weakly undominated strategies. In a pure private values setting (i.e., when $E[u(\mathbf{S}, X_i, \mathbf{X}_{-i}) \mid X_i, \mathbf{X}_{-i}] = E[u(\mathbf{S}, X_i, \mathbf{X}_{-i}) \mid X_i]$), this leaves only the Milgrom–Weber equilibrium. Outside the pure private values setting, the case for the Milgrom–Weber equilibrium is less clear, since one can construct multiple equilibria satisfying (3). To see this, for $x \geq y_{n-k}$ and any $\alpha \in [0,1]$ define

$$\hat{b}^k(x;\mathbf{y}^k) = E\left[u(\mathbf{S}, X_i, \mathbf{X}_{-i}) \mid X_i = x,\ Y_j = \alpha x + (1-\alpha) y_{n-k} \text{ for } 1 \leq j < n-k,\ \mathbf{Y}^k = \mathbf{Y}^k\right].$$

Each $\hat{b}^k(\cdot;\mathbf{y}^k)$ satisfies the conditions of the theorem as well as (3). Of course, bidders need not condition on an assumption of equal types among remaining opponents. For example, the following bid functions also yield an equilibrium in weakly undominated strategies:

$$\bar{b}^k(x;\mathbf{y}^k) = E\left[u(\mathbf{S}, X_i, \mathbf{X}_{-i}) \mid X_i = Y_1 = x,\ Y_j \in [y_{n-k}, x] \text{ for } 1 < j < n-k,\ \mathbf{Y}^k = \mathbf{y}^k\right].$$

Hence, except in the case of pure private values, the restriction to weakly undominated strategies leaves a continuum of equilibria.

4. DISCUSSION

We have shown that Milgrom and Weber's model (1982) of English auctions has a continuum of equilibria in symmetric separating strategies. For a seller's choice of auction, this multiplicity of equilibria is inconsequential since the transaction price is identical in all equilibria. However, our results have important implications for empirical analysis of English auctions, which often relies on an exact interpretation of bids below the second highest (e.g., Donald and Paarsch, 1996; Paarsch, 1997; Hong and Shum, 1999; Athey and Haile, 2000). The multiplicity of equilibrium exit patterns consistent with a single distribution of signals suggests one cannot make such an interpretation without a compelling equilibrium selection rule. Experimental studies of English auctions (e.g., Levin *et al*. 1996) have likewise implicitly relied on an assumption that the Milgrom–Weber equilibrium provides a unique prediction regarding observed bids. In the case of pure private values auctions, a restriction to weakly undominated strategies provides a justification for focusing on the Milgrom–Weber equilibrium. Outside the pure private values paradigm, however, a continuum of equilibria remains even with this restriction.

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