Suggested Answers: 2004 Final Exam

(see old problem set solutions for problems 1-3)
4. The firm solves the following maximization problem:

$$\max_{q_1,q_2}(P(q_1)-c_1)q_1+\beta(P(q_2)-c(q_1))q_2$$

The first order conditions yield:

$$\begin{aligned} P(q_1)+P'(q_1)q_1-c_1-\beta c'(q_1)q_2 &= 0 \\ P(q_2)+P'(q_2)q_2-c(q_1) &= 0 \end{aligned}$$

While the second equations is the standard requirement that marginal revenue
equals marginal cost, in the first equation there is an extra term that captures the effect of first period output on second period cost. Since that term is positive, we have that marginal revenue is less than marginal cost in the first period, so there will more output than if there were no learning by doing. The monopolist produces extra output to be able to produce at a lower cost in the second period.
5.a) The downstream firm solves:

$$\underset{Q_s}{Max} Q_s(100-5Q_s-\omega) \tag{1}$$

The f.o.c. is

$$Q_s: \qquad (100-10Q_s-\omega)=0$$

and 1

$$Q_s=\frac{100-\omega}{10}$$

Hence, the downstream firm's demand is

$$Q_\omega(\omega)=\frac{100-\omega}{10}$$

or

$$\omega=100-10Q_\omega$$

b) The upstream firm solves

$$\underset{Q\omega}{Max} Q_\omega(100-10Q_\omega-c)$$

which yields

$$Q_\omega=\frac{100-c}{20}$$

and

$$\omega = \frac{100+c}{2}$$

The upstream firm's profit becomes

$$\Pi_\omega = \frac{100-c}{20} * (\frac{100+c}{2} - c) = \frac{(100-c)^2}{40}$$

and the downstream firm's profit is:

$$\Pi_s = (\frac{100-c}{20}) * (100 - 5 * \frac{100-c}{20} - \frac{100+c}{2}) = \frac{(100-c)^2}{80}$$

The total profits thus equals $\frac{3}{80}(a-c)^2$.
If they were vertically integrated they could obtain the monopoly profit Π_m of
$\frac{1}{20}(100-c)^2$. This is the 'double marginalization problem.' The downstream firm wants to restrict output to increase the retail price above the wholesale price.
c)The upstream firm charges a two part tariff

$$\omega = A + BQ$$

The optimal choice is

$$\begin{aligned} B &= c \\ A &= \Pi_m = \frac{1}{80}(100-c)^2 \end{aligned}$$

The downstream firm then maximizes

$$\underset{Q_s}{Max} Q_s(100 - 5Q_s - c) - A$$

which yields the monopoly quantity of $\frac{100-c}{10}$ and price of $\frac{100+c}{2}$. The downstream firm makes zero profits and the upstream firm
makes the monopoly profit equal to $A = \frac{1}{20}(100-c)^2$. The two-part tariff reduces the upstream firm's incentive to integrate. It eliminates the distortion caused by the above marginal cost pricing by the upstream firm.
6. a) Firm i maximizes

$$(20 - q_i - q_j - 5)q_i \tag{2}$$

The first order condition yields:

$$q_i = \frac{15 - q_j}{2}$$

And $q_i = 5$, total quantity $Q = 10$ and price $P = 10$. Since each firm's market share equalt 50 percent, the Herfindahl index is

$$H = 50^2 + 50^2 = 5,000$$

b) Total surplus equals Consumer surplus $CS = \frac{(20-10)10}{2} = 50$ plus total profits $\Pi = (10-5)10 = 50$. Total surplus thus is

$$TS = CS + \Pi = 100$$

c) The firm would have a market share of 100 percent. The Herfindahl index would increase to $H = 100^2 = 10,000$. This would concern the anti-trust authorities in the U.S. because $H > 1,800$ and the change in the index $\Delta H > 100$.

d) If the two firms merged and face marginal cost c, they would maximize

$$(20 - q - c)q$$

which yields: monopoly quantity $q_m = (20-c)/2$ and price $P_m = (20+c)/2$. Consumer surplus would be $CS = (20 - \frac{20+c}{2})(\frac{20-c}{2})/2$ and profits would be $\Pi = \frac{20-c}{2}(\frac{20+c}{2} - c)$ and total surplus would be in the case of $c = 4$:

$$TS = \Pi + CS = 64 + 32 = 96$$

Hence, total welfare would be 4 units less. To make consumers better off post-merger, the post-merger surplus has to larger than the pre-merger surplus:

$$CS = (20 - \frac{20+c}{2})(\frac{20-c}{2})/2 = \frac{(20-c)^2}{8} > 50$$

which requires $c < 0$.

7. a) Suppose share p is non-rBGH. Consumers' expected value (willingness to pay) is:

$$3p + 2(1-p) = 2 + p$$

while the cost is: \$ 0.25

So we need $p = .25$ to get a price of \$2.25. All the farmers will receive the same price. The existence of the 'high types' will raise the profits of the others.

b) None. It will be more profitable to produce with rGBH. So all will produce "low quality".

c) No. Advertising would not be credible because consumers cannot tell whether milk is high or low quality just by buying it, and it is just as easy for the 'low types' to send the signal as for the 'high types.'