Useful Fact for Geometric Series

Fact. *For* $\delta \in (0,1)$ *and any constant* K, $\sum_{t=0}^{\infty} K\delta^t = \frac{K}{1-\delta}$.
Now let's show this. Let

$$S_T = \sum_{t=0}^{T} \delta^t.$$

Then

$$\begin{aligned}
(1-\delta)\, S_T &= (1-\delta) \sum_{t=0}^{T} \delta^t \\
&= \sum_{t=0}^{T} \delta^t - \sum_{t=0}^{T} \delta^{t+1} \\
&= \sum_{t=0}^{T} \delta^t - \sum_{t=1}^{T+1} \delta^t \\
&= \delta^0 - \delta^{T+1} \\
&= 1 - \delta^{T+1}.
\end{aligned}$$

So

$$S_T = \frac{1-\delta^{T+1}}{(1-\delta)}.$$

Now look at the limit at $T \to \infty$.

$$\begin{aligned}
\lim_{T\to\infty} S_T &= \lim_{T\to\infty} \frac{1-\delta^{T+1}}{(1-\delta)} \\
&= \frac{1}{1-\delta}
\end{aligned}$$

since $\lim_{T\to\infty} \delta^{T+1}$ is zero.
So, since $\sum_{t=0}^{\infty} \delta^t = \lim_{T\to\infty} S_T, \sum_{t=0}^{\infty} K\delta^t = \frac{K}{1-\delta}$.