Econ 200b: Firms Markets and Competition
Midterm Solutions

Spring 2008

1 Write your Name

2 $n-$Firm Cournot Oligopoly

2.1 Total Profit

Total profit in the industry is

$$\pi^{total} = (10 - p)(p - 2) - n$$

since the inverse demand function is $p = 10 - Q$.
We wish to solve

$$\max_p (10 - p)(p - 2) - n$$

The FOC is

$$-(p - 2) + (10 - p) = 0$$

or

$$2p = 12$$

or

$$p = 6$$

2.2 Cournot Equilibrium Price and Profit

Each firm i solves:

$$\max_{q_i} (10 - q_i - Q_{-i}) q_i - 2q_i - 1$$

where

$$Q_{-i} = \sum_{j \neq i} q_j$$

The FOC is

$$8 - Q_{-i} = 2q_i$$

Since the firms are symmetric, in equilibrium

$$q_i^C = q^C, \text{ all } i$$

and therefore

$$Q_{-i} = (n-1)\, q^C$$

Substituting in the FOC we get

$$q^C(n) = \frac{8}{n+1}$$

Substitute the quantity in the demand function to get the price p^C:

$$p^C(n) = 10 - nq^C = \frac{2n+10}{n+1}$$

Finally, firm i's profits equal:

$$\pi_i^C(n) = \left(\frac{8}{n+1}\right)^2 - 1$$

2.3 Free Entry

The free entry condition state that

$$\pi_i^C(n) = 0$$

or

$$\left(\frac{8}{n+1}\right)^2 = 1$$

or

$$n = 7$$

2.4 Compare to Efficiency

There are too many firms in this equilibrium compared to the efficient number of firms (i.e. the one that maximizes total surplus). This is because of the business stealing effect that entrants ignore when making their entry decision. Indeed, the quantity produced decreases with the number of firms.

3 Durable Good Monopolist

3.1 Rent

When the monopolist rents the good in both periods, it acts like a monopolist of a non-durable good and solves (in both periods) the following problem:

$$\max_p \,(10-p)\, p$$

The FOC gives us

$$p = 5$$

and therefore

$$\pi = 25$$

Therefore, price is $p = 5$ in each period and total profit is 50.

3.2 Durable

In the second period Q_1 units have already been sold. Therefore, (the "residual") demand in the second period is:

$$p_2 = (10 - Q_1) - Q_2$$

The monopolist then solves

$$\max_{Q_2} \left((10 - Q_1) - Q_2\right) Q_2$$

The FOC is

$$Q_2 = \frac{10 - Q_1}{2}$$

Profit in the second period, therefore is

$$\pi_2 = \left(\frac{10 - Q_1}{2}\right)^2$$

In the first period, consumers' willingness to pay is

$$\begin{aligned} p_1 &= 10 - Q_1 + p_2 = \\ &= 10 - Q_1 + (10 - Q_1) - \frac{10 - Q_1}{2} \end{aligned}$$

or

$$p_1 = \frac{3}{2}(10 - Q_1)$$

The monopolist solves

$$\max_{Q_1} \frac{3}{2}(10 - Q_1) Q_1 + \left(\frac{10 - Q_1}{2}\right)^2$$

The FOC is

$$\frac{3}{2}(10 - Q_1) - \frac{3}{2}Q_1 - (10 - Q_1) = 0$$

or

$$Q_1 = 4$$

and total profits

$$\pi = 45$$

4 Pencils and Erasers, Nerds and Jocks

4.1 Two Part Tariff for Both Types

The monopolist will set a two-part tariff $T = a + bq$ by choosing a and b optimally to maximize profits from selling to both consumer types. a will equal the lower type's (jock) surplus:

$$a = \frac{(10-b)^2}{2}$$

The monopolist solves:

$$\max_b 2\frac{(10-b)^2}{2} + q_N(b) + q_J(b)$$

where $q_N(b)$ $(q_J(b))$ is the demand of nerds (jocks) so that

$$\max_b 2\frac{(10-b)^2}{2} + (b-c)(30-3b)$$

The FOC is

$$-2(10-b) + 30 - 3b - 3(b-c) = 0$$

or

$$b = \frac{5}{2} = 2.5$$

and therefore

$$a = \frac{1}{2}\left(\frac{15}{2}\right)$$

4.2 Two Part Tariff Only for Nerds

In this case,

$$b = c = 0$$

and therefore

$$a = \frac{1}{2}10 \times 20 = 100$$

4.3 Bundling

The demand for the bundle is

$$p = 10 - q + 10 - \frac{q}{2} = 20 - \frac{3}{2}q$$

and it is the same for both nerds and jocks. Therefore, it is as if the monopolist is facing only one type. The monopolist can achieve the maximum payoff by extracting all the surplus from this type. This is done by setting:

$$b = 0$$

and

$$a = \frac{1}{2} 20 \times \frac{40}{3} = \frac{400}{3}$$

We know without doing any calculations that the profit achieved from bundling is higher than two independent monopolists, since in this case the monopolist can extract the entire surplus that is generated at the efficient quantity.

5 A Cartel

5.1 Monopoly Pricing

Let π^C be the profit each firm gets when both charge the monopoly price (profits from "cooperating"). The best profitable deviation of firm i when the other firm is charging the monopoly price (price just below the monopoly price) yields profit $\pi^D = 2\pi^C$ (since instead of half the monopoly profits, the firm can get the entire monopoly profits). Moreover, the profit in the punishment phase is equal to zero (NE). To sustain an equilibrium it must be

$$\frac{\pi^C}{1-\delta} \geq 2\pi^C$$

or

$$\delta \geq \frac{1}{2}$$

5.2 Cournot Pricing

Same as above. Let π^C be the profit each firm gets when both charge the Cournot price (profits from "cooperating"). The best profitable deviation of firm i when the other firm is charging the Cournot price (price just below the Cournot price) yields profit $\pi^D = 2\pi^C$ (since instead of half the Cournot profits, the firm can get the entire Cournot profits). Moreover, the profit in the punishment phase is equal to zero (NE). To sustain an equilibrium it must be

$$\frac{\pi^C}{1-\delta} \geq 2\pi^C$$

or

$$\delta \geq \frac{1}{2}$$

5.3 Price "doesn't matter"

There is no difference in the level of patience required to sustain either the monopoly or the Cournot price. If the punishment consists of marginal cost pricing, all that matters is the ratio of π^C/π^D- here this equals $1/2$, which is the same regardless of what the collusive price is (in fact, the two terms cancel out completely!).