

Economics 200
Spring 2008
Prof. Haile
Problem Set 1
Due Tuesday, Jan. 29

1. Suppose a monopolist has no fixed costs and constant marginal cost of c . Demand is given by the equation $P = \alpha - \beta q$.
 - a. derive the profit maximizing price (for a single-price monopoly) and quantity.
 - b. show how the price can be expressed as a function of the demand elasticity at the profit maximizing price.
 - c. calculate the monopolist's profit.
 - d. calculate the consumer surplus.
 - e. calculate the deadweight loss.
 - f. Now suppose that there are 100 consumers with identical downward sloping demand curves $P = 10 - q$. What are the monopolist's profit and the deadweight loss?
 - g. In the case in part f, if the monopolist can use a two-part tariff, what will be the monopolist's profit and the deadweight loss?

2. Suppose there are two consumers in a market with utility functions $u_i(q, T) = \theta_i v(q) - T$, where q is the quantity consumed, T is expenditure on the good, and $\theta_2 > \theta_1$. Assume

$$v(q) = \frac{1 - (1 - q)^2}{2} \quad \text{for } 0 \leq q \leq 1.$$

The monopolist has no fixed cost and constant marginal cost c . Derive the optimal two-part tariff for the monopolist.

3. A nightclub manager realizes that the demand for drinks is more elastic among students, and is trying to determine the optimal pricing schedule. He estimates that patrons under 25 have demand curves $q = 18 - 5p$, while older patrons have demand curves $q = 10 - 2p$. The two groups visit the club in equal proportions. Producing each drink costs the nightclub \$2.
 - a. What is the uniform monopoly price?
 - b. If the nightclub can charge based on age but is limited to linear pricing, what price should be set for each group?
 - c. If the nightclub can set separate cover charges and drink prices for each group of consumers, what two-part pricing schemes should it choose to maximize profit?
 - d. Now suppose that it is impossible to discriminate based on age. If the nightclub owner lowers drink prices to \$2 and still wants to attract both types of patrons, what cover charge will it set?
 - e. Suppose again that the nightclub is restricted to linear pricing and that age discrimination is impossible. However, the manager believes that everyone remaining after midnight is under 25, while only $2/7$ of consumers who arrives before midnight is under 25. What are the optimal prices before and after midnight? How do the profits obtained here compare with those in parts c and d?

4. Consider a market for a product with two types of potential users: those in proportion λ have inverse demand curves $p = 5 - q/2$. The remainder have inverse demand curves $p = 10 - q$. Normalize the total number of consumers to be 1 and let $c = 2$ be the marginal cost.

a. What is the optimal (profit-maximizing) two-part tariff (as a function of λ) that induces both types of consumers to buy?

b. What is the optimal 2-part tariff when only high-demand consumers buy the good?

c. If $\lambda = 1/2$, which pricing scheme gives large profits? What if $\lambda = 3/4$?

5. Consider a monopolist in a market with two consumers. Consumer i has utility function $u_i(q, T) = \theta_i v(q) - T$, where q is the quantity consumed and T is the consumer's total expenditure on the good. Assume that $v(\cdot)$ is a strictly increasing and concave function, and that $\theta_2 > \theta_1$. The firm has marginal cost c and no fixed costs.

a. Suppose it is optimal for the monopolist to sell to both consumers and that the firm selects a fully nonlinear tariff in the following way: it first chooses (q_1, T_1) to maximize profit from consumer 1 without violating consumer 1's voluntary participation constraint. Then the monopolist chooses (q_2, T_2) to maximize profit from consumer 2 without violating the incentive compatibility constraint for consumer 2. Illustrate this in a diagram with q on the horizontal axis and T on the vertical axis. Show indifference curves for the level of utility each consumer receives, and show the profit the firm obtains from each type.

b. Show (mathematically) that the tariff in part a could not be the *optimal* fully nonlinear tariff. Hint 1: One way to do this is to show that there is some $(\hat{q}_1, \hat{T}_1, \hat{q}_2, \hat{T}_2)$ that gives the monopolist higher profit. Hint 2: one way to do this that simplifies things involves keeping $\hat{q}_2 = q_2$.

6. A cable TV monopolist has two types of programming to offer: sports and cooking. Consumers are also of two types and differ in their absolute and relative valuations of these two types of programs. Each type's maximum willingness to pay for each type of program is shown in the table below. For simplicity, assume zero marginal cost and that there is one consumer of each type.

	sports	cooking
Type 1	\$15	\$10
Type 2	\$8	\$12

a. Ignoring fixed costs (assuming they are zero), what profit could the firm obtain if it could perfectly price discriminate?

b. Suppose the firm is required by regulators to use uniform prices. If it charges a single price for each type of programming, what profit will the firm obtain?

c. Suppose the firm instead bundles the two types of programming. What profit can it obtain? Explain.