Answer Key: Problem Set 2

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Problem 1

See also chapter 4 in the textbook.

(a) Under leasing, the monopolist chooses monopoly pricing each period. The profit of the monopolist in each period is:

$$\pi_{t=}P_tQ_t$$

Demand in each period is:

$$P_t = 1000 - Q_t$$

where $t = 1, 2$. The first order conditions w.r.t Q_t is:

$$1000 - 2Q_t = 0$$

where $t = 1, 2$. Solving them gives us $Q_1 = Q_2 = 500$, the profit-maximizing rental rate $P_1 = P_2 = 500$, profit per period \$250,000, and total profits \$ $250,000(1 + \delta)$.

(b) A way to think about this problem is start with the second period. In the second period everything that happened in the first period (whatever that was) can be taken as given. So we can infer the optimal behavior of the monopolist as a function of the first period choices: solve for q_2 as a function of q_1. Once we know this optimal behavior we can go back to the first period and look at the total profits $(\pi_1 + \delta\pi_2)$ as a function of q_1 for which we solve. We follow this procedure to solve the problem:

The monopolist knows that her first-period sales are irreversible, so in the second-period, she faces a residual demand:

$$Q_2(P_2) = Q(P_2) - Q_1 = 1000 - P_2 - Q_1$$

So the inverse demand function for the second period is:

$$P_2 = 1000 - Q_1 - Q_2$$

The monopolist's profit in the second period (given Q_1 units sold in the first period) is:

$$\pi_2 = Q_2(1000 - Q_1 - Q_2)$$

The first order condition is: $\frac{1000-Q_1}{2} = Q_2$. Price in the second period is: $P_2 = \frac{1000-Q_1}{2}$, and the profit in the second period is $\pi_2 = \frac{(1000-Q_1)^2}{4}$.

In the first period, consumers who buy the durable good this period can consume it for two periods. Therefore, their valuation should incorporate the benefit they get in the second period (what if they could sell the durable good in the second period at price p_2). They are willing to pay:

$$P_1 = 1000 - Q_1 + \delta P_2 = 1000 - Q_1 + \delta(\frac{1000 - Q_1}{2}) = (1 + \frac{\delta}{2})(1000 - Q_1)$$

The monopoly chooses the optimal quantity Q_1 to maximize her total profit in the two periods (remember, her profit in the second period is a function of the quantity she sells in the first period, since she can not commit to a second-period price (or quantity) beforehand. In other words, the quantity she sells in the second period is contingent on the quantity she sells in the first period.):

$$\pi = (1 + \frac{\delta}{2})(1000 - Q_1)Q_1 + \delta\frac{(1000 - Q_1)^2}{4}$$

This is a function of Q_1 only. From the F.O.C, we have $Q_1 = \frac{2000}{4+\delta}$, and $Q_2 = \frac{500(2+\delta)}{4+\delta}$ and $P_1 = \frac{500(2+\delta)^2}{4+\delta}$ and $P_2 = \frac{500(2+\delta)}{4+\delta}$. Notice here there is some intertemporal price discrimination. Aggregate profits are:

$$\pi = (1,000,000 + \delta 250,000)(\frac{2+\delta}{4+\delta})^2$$

(c) Suppose the monopolist can commit to P_1,P_2 ($P_1 > P_2$). The residual demand in the second period is: $Q_2 = 1000 - Q_1 - P_2$, the same as before. In period one, consumers are willing to pay: $P_1 = 1000 - Q_1 + \delta P_2$. So

$Q_1 = 1000 - P_1 + \delta P_2$. The monopolist chooses P_1, P_2 to maximize her profit in both periods:

$$\max_{P_1, P_2} P_1(1000 - P1 + \delta P_2) + \delta P_2(1000 - Q_1 - P_2)$$

Plug in $Q_1 = 1000 - P1 + \delta P_2$ in the second term, we have:

$$\max_{P_1, P_2} P_1(1000 - P1 + \delta P_2) + \delta P_2(P_1 - P_2(1 + \delta))$$

$>$From the first order conditions with respect to P_1 and P_2, we get: $P_1 = 500 + \delta P_2$, and $P_2 = \frac{P_1}{(1+\delta)}$. Solving these two equations gives us the profit maximizing prices in both periods: $P_1 = 500(1 + \delta)$, and $P_2 = 500$. All sales occur in period one. The quantity sold in period one is $Q_1 = 500$. (You can check that quantity sold in period 2 is zero.) The monopolist's profit is: $\pi = 250,000(1 + \delta)$. It is the same as the profit she gets under leasing.

(d) We know from part (b) that if the monopolist produces the durable good, she gets $\pi = (\frac{3}{5})^2 * 1,250,000 = 450,000$. If she produces the non-durable good, her profit in each period is:

$$\pi_t = (1000 - Q_t - c)Q_t$$

where $t = 1, 2$. From the first order condition, we know the optimal quantity is $Q_t = \frac{1000-c}{2}$ and price is $P_t = \frac{1000+c}{2}$ for both periods. When $\delta = 1$, the total profit is $\pi = 2 * (\frac{1000-c}{2})^2 = \frac{(1000-c)^2}{2}$. The monopolist will choose to produce the nondurable good only when: $\frac{(1000-c)^2}{2} > 450,000$, or $c < 51$.

(e) Under leasing, the monopolist's profit is 500,000. Since the marginal cost c can't be negative, the profit she gets by producing a nondurable good at cost c will not be bigger than her leasing profit. So she will not choose to produce the nondurable good if she can produce the durable good and lease it (when $c = 0$, she is indifferent).

(f) First, leasing does not involve intertemporal price discrimination, so for that purpose the policy (forbidding leasing) is poorly designed.

When $c < 51$, the monopolist will produce and sell the nondurable good. Here there is no intertemporal price discrimination, but it involves efficiency loss: less quantity is provided for both periods ($Q_t = \frac{1000-c}{2} < 500$) at a higher cost ($c$ instead of zero).

If $c \geq 51$, the monopolist will produce the durable good. In period one, she sells 400 units at $P_1 = 900$. Her profit is 360,000, and the total consumer surplus from two periods' consumption of these 400 units is: 280,000 (the area under the demand curve times 2 minus the total payment). In the second period, the monopolists sells 300 units at $P_2 = 300$. Her profit is 90,000, and consumer surplus is 45,000. The total profit is 450,000, and the total consumer surplus is 325,000. The social surplus is 775,000. Under leasing, the total social surplus is 750,000. Here forbidding leasing improves efficiency, but it involves intertemporal price discrimination.