Answer Key: Problem Set 3

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Problem 1

Consider a market with inverse demand $P = 11 - Q$, where Q is the sum of output produced by all firms in the market. Firms have identical cost functions $c(q) = 2 + q$.

Question a

Suppose there is a single firm in this market. What price and quantity will the firm choose? What will be the consumer surplus? What will be the firm's profit?

ANSWER: The firm solves

$$\max_q \{(11 - q)q - (2 + q)\}$$

It's easy to see the f.o.c. is

$$11 - 2q - 1 = 0$$

which gives

$$\begin{aligned} q^m &= 5, p^m = 6 \\ \pi^m &= (10 - 5)5 - 2 = 23 \\ CS^m &= 5 \cdot 5 \cdot \frac{1}{2} = 12.5 \end{aligned}$$

Question b

Suppose there are n firms in the market that compete by simultaneously choosing quantities. What quantities, $q^c(n)$, will they choose in equilibrium? How do these quantities change with the number of firms? What is the total quantity $Q^c(n)$ produced in equilibrium and how does it change with the number of firms in the market? What is the profit $\pi^c(n)$ and how does it change with n ? What is the consumer surplus and how does it change with n ?

ANSWER: Firm i solves

$$\max_{q_i}\{(11 - q_i - Q_{-i})q_i - (2 + q_i)\}$$

The f.o.c.(w.r.t. q_i) is

$$11 - 2q_i - Q_{-i} - 1 = 0$$

Symmetry implies $Q_{-i} = (n-1)q_i$,which reduces the f.o.c. to $10-(n+1)q_i = 0$. (Note that if you make this substitution *before* deriving the FOC, you will *not* get the right soultion; remember that firm i does not get to choose the quantities of its competitors!) Therefore:

$$\begin{aligned}
q_i^c(n) &= \frac{10}{n+1} \\
Q^c(n) &= \frac{10n}{n+1} \\
P^c(n) &= \frac{11+n}{n+1} \\
\pi^c(n) &= n\pi_i = \frac{100n}{(n+1)^2} - 2n \\
CS^c(n) &= \frac{1}{2}\left(\frac{10n}{n+1}\right)^2.
\end{aligned}$$

To see how these quantities change with the number of firms, we need to take the derivatives with respect to n:

$$\begin{aligned}
\frac{dq_i^c}{dn} &= -\frac{10}{(n+1)^2} < 0 \text{ , decreasing in } n \\
\frac{dQ^c(n)}{dn} &= \frac{10}{(n+1)^2} > 0 \text{ , increasing in } n \\
\frac{d\pi^c(n)}{dn} &= \frac{100}{(n+1)^3}(1-n) - 2 < 0\text{,decreasingin } n \\
\frac{d(CS^c)}{dn} &= \frac{100n}{(n+1)^3} > 0\text{,increasing in } n
\end{aligned}$$

Question c

Suppose now the number of firms is determined endogenously. What will be the free-entry equilibrium number of firms? What is the sum of consumer and industry profits in this equilibrium?

ANSWER: If there is free-entry:

$$\begin{aligned} \pi_i &= 0 \\ \frac{100}{(n+1)^2} - 2 &= 0 \end{aligned}$$

This gives $n \approx 6.07$. Therefore π is 0; CS is 36.86. The sum $CS + \pi = 36.86$ (plug $n = 6.07$ into the formula in part (b)).

Question d

What is the efficient number of firms in the market? What is the sum of consumer surplus and industry profits?

ANSWER: To find the efficient number of firms, we need to solve

$$\max_n (CS + \pi)$$

i.e.

$$\max_n \left\{ \frac{1}{2} \left(\frac{10n}{n+1} \right)^2 + \frac{100n}{(n+1)^2} - 2n \right\}$$

The f.o.c. is

$$\frac{100n}{(n+1)^3} + \frac{100(1-n)}{(n+1)^3} - 2 = 0,$$

from which we can solve $n \approx 2.68$. Therefore $CS + \pi = 40.94$.

Question e

Provide some intuition for the failure of free entry to lead to an efficient equilibrium.

ANSWER: The inefficiency of free entry is due to business-stealing effect: part of an additional entrant's profit comes at the expense of existing firms. When firms make entry decisions, they do not take into consideration the decrease in their opponents' quantities when they enter. In other words, the private benefit for an additional firm exceeds the social benefit.

Question f

Would eliminating the fixed cost associated with entry eliminate the discrepancy between the free entry equilibrium and the social optimum?

ANSWER: Yes. When the fixed cost associated with entry is eliminated, the efficient number of firms will be ∞. Since entry always ensures a positive profit, the number of firms in the free-entry equilibrium also tends to ∞. Therefore discrepancy disappears when there is no fixed cost.

Problem 2

Consider a market with inverse demand $P = a - 2Q$. Firms have no fixed cost and constant marginal cost c.

Question a

Derive expressions for industry price, quantity , profit, and the Lerner index if this market is served by a monopoly.

ANSWER: $P = a - 2Q, MR = a - 4Q$.

By setting $MR = MC$, we can easily solve

$$Q^m = \frac{a-c}{4}, \qquad P^m = \frac{a+c}{2}, \qquad \pi^m = \frac{(a-c)^2}{8}, \qquad L^m = \frac{a-c}{a+c}.$$

Question b

Derive expressions for the Nash equilibrium industry price, quantity, profit, and the Lerner index if the market is served by Cournot duopolists. Compare these to your answers in part 1.

ANSWER: Solving this symmetric game by

$$\max_{q_i}(a - 2q_i - 2q_{-i} - c)q_i.$$

The f.o.c is $a - 4q_i - 2q_{-i} - c = 0$. Imposing $q_i = q_{-i}$, we can solve

$$q_i^c = \frac{a-c}{6}, \qquad P^c = \frac{a+2c}{3}, \qquad \pi^c = \frac{(a-c)^2}{9}, \qquad L^c = \frac{a-c}{a+2c}.$$

Comparing (a) and (b), we can see

$$Q^c = 2q_i^c > q^m, \qquad P^c < P^m, \qquad \pi^c < \pi^m, \qquad L^c < L^m.$$

Question c

Do the same thing for the case in which the market is served by Bertrand duopolists.

ANSWER: For the Bertrand game,

$$Q^B = \frac{a-c}{2}, \qquad P^B = c, \qquad \pi^B = 0, \qquad L^B = 0.$$

Question d

If the duopolists could choose whether to compete by choosing prices simultaneously or by choosing quantities simultaneously, which would they prefer? Which would consumers prefer? Which is more efficient?

ANSWER: Firms prefer cournot competition since they can get positive profits by choosing quantities simultaneously. Consumers prefer Bertrand competition since they can get more surplus. Bertrand is more efficient since it yields the perfect competitive outcome.

Question e

What will the firm profits be in Nash equilibrium if there are n Cournot competitors? Show how these profits vary with n. As n goes to infinity what happens to the market price? What happens to industry profits?

ANSWER: In the n-firm cournot game,

$$\max_{q_i}(a - 2q_i - 2Q_{-i} - c)q_i$$

The f.o.c. is

$$a - 4q_i - 2Q_{-i} - c = 0,$$

where $Q_{-i} = (n-1)q_i$ by symmetry. So

$$q_i^c = \frac{a-c}{2(n+1)}, \qquad P^c = \frac{a+nc}{n+1}, \qquad \pi_i^c = \frac{1}{2}\left(\frac{a-c}{n+1}\right)^2, \ \pi^c = \frac{n}{2}\left(\frac{a-c}{n+1}\right)^2.$$

Clearly, both firm profits and industry profits are decreasing in n: $\frac{d\pi_i^c}{dn} = -\frac{(a-c)^2}{(n+1)^3} < 0$, $\frac{d\pi^c}{dn} = \frac{(a-c)^2(1-n)}{2(n+1)^3} < 0$. As n goes to infinity, we have:

$$\lim_{n\to\infty} P^c = c \quad and \quad \lim_{n\to\infty} \pi^c = 0.$$