

1. Suppose two players play the following prisoner's dilemma stage game in every period  $t = 1 \dots T$ . You can think of this as two firms choosing either a High price or Low price.

		Player 2	
		H	L
Player 1	L	(11,0)	(1,1)
	H	(10,10)	(0,15)

- a. If  $T$  is finite, what is the unique subgame perfect equilibrium? Why?
- b. If  $T$  is infinite and players discount the future using discount factor  $\delta$ , is the equilibrium in part a still a subgame perfect equilibrium? Explain.
- c. If  $T$  is infinite, what is the smallest value of  $\delta$  such that each player can obtain a payoff of 10 each period in a subgame perfect equilibrium?
- d. Now suppose that the stage game is played in period 1 and that, conditional on the game being played in a given period  $t$ , there is probability  $1 - p$  that the stage game is played in period  $t + 1$  and probability  $p$  that the repeated game ends after period  $t$ . For simplicity, assume now that firms do not discount future payments (i.e., that  $\rho = 0$ ). Show that for  $p$  sufficiently small, each player can obtain the payoff 10 in a subgame perfect equilibrium.

2. Now consider a richer specification of the same game in which firms can choose any quantity, not just "high" or "low." Suppose there are two identical firms with cost functions  $C(q) = 2q$  in an industry with inverse demand given by the equation  $P = 50 - 4Q$ , where  $Q$  is the market quantity.

- a. Suppose firms compete only once by choosing quantities simultaneously. What is the Nash equilibrium?
- b. If both firms produce the same quantity  $q_i$ , which value of  $q_i$  maximizes joint profits?
- c. Now suppose that firms choose quantities once a year and discount the future at an annual rate of 5%. Can joint profit-maximization (as derived above) be achieved in a subgame perfect equilibrium?

3. Suppose an industry consists of 2 firms that compete by choosing quantities in each period  $t = 1, 2, \dots$ . Inverse demand in the industry is given by the linear equation  $P = 50 - \frac{Q}{2}$ . Marginal cost is equal to zero.

- a. Suppose all firms discount future profit using the per-period discount factor  $\delta$ . How large must  $\delta$  be for there to be a subgame perfect equilibrium in which the industry makes the same profit a monopolist would?
- b. Suppose now that firms set quantities only in odd periods ( $t = 1, 3, 5, \dots$ ). How large must  $\delta$  be for industry to make the profit would if there were a monopolist? Provide some intuition for the differences from your answers in part a.