

1. Consider a Hotelling model of product differentiation in which there is a continuum of consumers uniformly distributed on the interval $[0, 1]$. Firms will also be located on this interval. Consumers have unit demands. A consumer who buys at price p from a firm located a distance Δ away obtains utility $v - p - t\Delta^2$. Assume all goods can be produced at zero marginal cost and that v is sufficiently large that all consumers will buy one good.

a. Suppose there are two firms and that prices are fixed at $p = 1$ for both firms. If firms choose their locations simultaneously, what locations do they choose in a pure strategy Nash equilibrium? Prove your answer.

b. Now suppose that the two firms move sequentially in choosing locations. What locations do the firms choose in a subgame perfect equilibrium?

c. Now suppose the locations of the firms are fixed at the endpoints of the interval, but prices are now chosen simultaneously by the firms. Derive the demand curves for the two firms. Solve for the Nash equilibrium prices.

d. What is the interpretation of the parameter t ? How do the prices in part c vary with this parameter? Provide some intuition.

2. Consider a version of the Hotelling model in which prices are endogenously determined. Two firms sell horizontally differentiated products located at opposite ends of the one-dimensional product space. Firm 0 is located at 0. Firm 1 is located at 1. M consumers are uniformly distributed between 0 and 1, with each consumer's location giving his most preferred type of product. Each consumer places value v on one unit of his most preferred product, but incurs a "transportation cost" λD^2 when purchasing a product which is located a distance D away. Assume v is sufficiently large that all consumers purchase one unit. Firms have no fixed costs but marginal costs of c per unit. Firms compete by choosing prices simultaneously.

a. Derive the demand curve for each firm as a function of the prices chosen. Explain the presence of firm j 's price p_j in the demand function for firm i ($i \neq j$). Is i 's demand increasing or decreasing in p_j ? Why?

b. Find the Nash equilibrium prices.

c. How do the Nash equilibrium prices vary with λ ? Provide some intuition for your answer.

d. Now suppose firm 0 can move to any location a between zero and 1 before the price-setting stage. Derive the demand curve for firm 0 as a function of a and the prices of each firm. You may assume that the indifferent consumer is located between the two firms. Using this demand function, explain intuitively the effects (there are two of them) of changes in a on firm 0's demand.

3. Consider a model of vertical differentiation. Consumer i is characterized by a taste parameter θ_i and would obtain utility $u_i(s, p) = \theta_i s - p$ from purchasing a good of quality s at price p . There is a continuum of consumers, each with their preference parameter drawn from a uniform distribution on the interval $[0, 2]$. Normalize the total number of consumers to be 1 (so if $0 < \alpha < 2$, the number of consumers with taste parameter between zero and α is $\frac{\alpha}{2}$). There are two firms producing goods of differing qualities s_2 and s_1 , with $s_2 > s_1 > 1$. The marginal cost of production for each firm is 1.

a. Construct the demand curves for each firm's product, taking s_1 and s_2 as given.

b. If the two firms compete by choosing prices simultaneously, what are the Nash equilibrium prices and profits? How do the profits vary with the qualities s_1 and s_2 and the difference between these?