

1. Two firms producing a homogeneous good compete in a two-stage game. In stage 1, firm 1 can purchase cost-reducing capital equipment k . In stage 2, firms compete by simultaneously choosing quantities. Market (inverse) demand is given by the equation $P = 50 - 2Q$. Firm 1's total cost (including the cost of the capital equipment) is $q_1(2 - \frac{k}{4}) + \frac{k^2}{18}$, where q_1 is firm 1's output. Firm 2's cost is $2q_2$.

- a. Find the subgame perfect equilibrium quantities. How much investment does firm 1 make?
- b. Is this a "top dog" or "puppy dog" investment strategy? Explain.

2. Inverse demand in the market for frozen waffles is given by the equation $P = 10 - \frac{Q}{100}$ where Q is the market quantity of waffles. Waffles cost .25 each to produce. Firms compete by choosing quantities.

a. Suppose there is no fixed cost of production and that there are two firms in this industry. Firm 1 is able to commit to its output level before firm 2 can. What will be the subgame perfect equilibrium quantities and profits of each firm?

b. Now suppose that there is a fixed cost of 2 to produce waffles. What is the smallest quantity firm 1 could produce and still deter entry by firm 2? How does this compare to the monopoly quantity?

c. Does firm 1 want to deter entry? What is the smallest fixed cost f for which deterring entry would be profitable?

3. Consider a version of the Hotelling model of duopoly with product differentiation with both firms initially located at 0 in the product space. Consumers are uniformly distributed between 0 and 1, with each consumer's location giving his most preferred type of product. For simplicity, normalize the total measure of consumers to be 1. Each consumer places value s on one unit of his most preferred product, but encounters a "transportation cost" $t\Delta^2$ when purchasing a product which is located a distance Δ away. Assume s is sufficiently large that all consumers purchase one unit. Firms have no fixed costs but marginal costs of c per unit. Firms compete by choosing prices simultaneously. However, before the firms compete in prices, firm 1 has an opportunity to "move" to location 1 (i.e., to invest in product differentiation). Changing its product characteristics to make this move costs firm 1 r . Under what conditions will firm 1 choose to make this move? When these conditions are satisfied, what name would be given to firm 1's investment strategy in the "dog and cat" taxonomy presented in class. Explain briefly.