Answer Key, Problem Set 6

1 *Problem 1*

1.1 (a)

This is a 2-stage game and in order to solve for the SPE we use backwards induction.

So start with stage 2. Firm 1 takes the investment k decided last period as given and solves

$$\max_{q_1} (50 - 2q_1 - 2q_2 - 2 + \frac{k}{4}).$$

The first-order condition gives

$$q_1 = \frac{48 - 2q_2 + \frac{k}{4}}{4}.$$

Similarly, firm 2's first-order condition gives: $q2 = \frac{48-2q_1}{4}$.

Together, these two equations imply $q_1 = 8 + \frac{k}{12}$ and $q_2 = 8 - \frac{k}{24}$. This gives firm 1 profit of

$$\begin{aligned} & \left(8 + \frac{k}{12}\right)\left[50 - 2(16 + \frac{k}{24}) - 2 + \frac{k}{4}\right] - \frac{k^2}{18} \\ = & \left(8 + \frac{k}{12}\right)\left(16 + \frac{k}{6}\right) - \frac{k^2}{18}. \end{aligned}$$

Now let's move to the 1st stage and examine the choice of k: the firm takes as given the optimal choices of quantities as functions of k that will prevail in the 2nd stage.

Maximizing this with respect to k gives firm 1's optimal investment in stage 1. From the first order condition we have:

$$k(\frac{1}{9} - \frac{1}{36}) = (\frac{8}{6} + \frac{16}{12})$$

which has a solution of $k = 32$.

1.2 (b)

This is a top-dog strategy: the competition in strategic substitues leads firm 1 to be "tough" by "overinvesting" in capital. You can verify that investment makes firm 1 tough because firm 2's profit is decreasing in firm 1's investment. Indeed,

$$\frac{\partial \pi_2}{\partial q_1}\frac{\partial q_1}{\partial k} = -2q_2\frac{1}{12} < 0$$

and that the two firms' quantity choices are strategic substitutes – firm 2's quantity decreases in firm 1's quantity and vice versa (look at the slope of the reaction functions:

$$\begin{aligned} q_1 &= \frac{48 - 2q_2 + \frac{k}{4}}{4} \\ q2 &= \frac{48 - 2q_1}{4} \end{aligned}$$

2 Problem 2

2.1 (a)

This is Stackelberg duopoly. Again we use backwards induction to find the SPE.

For firm 2:

$$\max_{q_2} \left(10 - \frac{q_1}{100} - \frac{q_2}{100} - 0.25\right) q_2$$

The f.o.c. is

$$10 - \frac{q_1}{100} - \frac{2q_2}{100} - 0.25 = 0,$$

i.e.

$$q_2 = \frac{975 - q_1}{2}.$$

For firm 1:

$$\max_{q_1} \left(10 - \frac{q_1}{100} - \frac{975 - q_1}{200} - 0.25\right) q_1$$

The f.o.c. is

$$10 - \frac{2q_1}{100} - \frac{975}{200} + \frac{q_1}{100} - 0.25 = 0,$$

so $q_1 = \frac{975}{2}, q_2 = \frac{975}{4}$ and therefore

$$P = \left(10 - \frac{3 \times 975}{4} \frac{1}{100}\right) \approx 2.6875; \pi_1 = (2.6875–0.25)\frac{975}{2} = 1188;\ \pi_2 = \frac{\pi_1}{2} = 594.$$

2.2 (b)

Firm 1 will choose the "limit quantity,". q_1, which is the smallest quantity s.t. when firm 2 responds optimally (i.e., when $q_2 = \frac{975 - q_1}{2}$),

firm 2 makes no profit. So set

$$\pi_2 = \left(10 - \frac{q_1}{100} - \frac{975 - q_1}{200} - 0.25\right) \frac{975 - q_1}{2} - 2 = 0$$

and solve to get $q_1 \approx 946.7$. The monopoly quantity can be calculated by setting $MR = MC$, i.e.,

$$10 - \frac{2Q^m}{100} = 0.25,$$

from which we have $Q^m = \frac{975}{2}$. So firm 1 must produce more than the monopoly quantity to deter entry.

2.3 (c)

Profit for firm 1 if deterring entry is

$$\left(10 - \frac{946.7}{100} - 0.25\right) 946.7 - 2 = 265.92,$$

while profit with optimal accommodation is $1188 - 2 = 1186 > 265.92$. So it's better to accommodate. To find the smallest fixed cost f such that deterring entry would be profitable, note that we can write the quantity that makes firm 2 indifferent between entering the market and staying out as a function of $\dot{f}$:

$$\pi_2 = 0 \Leftrightarrow \left(10 - \frac{q_1}{100} - \frac{975 - q_1}{200} - 0.25\right) \frac{975 - q_1}{2} - f = 0$$

$$\Leftrightarrow \left(\frac{975 - q_1}{100} - \frac{975 - q_1}{200}\right) * \frac{975 - q_1}{2} = f$$

$$\Leftrightarrow q_1 = 975 - 20\sqrt{f}$$

Entry deterrence will be profitable whenever

$$\left(10 - \frac{975 - 20\sqrt{f}}{100} - .25\right)\left(975 - 20\sqrt{f}\right) > 1188$$

Which yields a fixed cost $f > 50.9401$.

3 Problem 3

If both firms locate at 0, Betrand competition implies zero profit for each of them. Firm 2 thus will choose to move to location 1 if profit at location 1 is bigger than r. Suppose now firm 1 is at 0 and firm 2 is at 1. The indifferent consumer is located at x, where x satisfies:

$$P_1 + tx^2 = P_2 + t(1 - x)^2$$

So

$$x = \frac{P_2 - P_1 + t}{2t}$$

$$Q_1(P_1, P_2) = \frac{P_2 - P_1 + t}{2t}$$

$$Q_2(P_1, P_2) = \frac{P_1 - P_2 + t}{2t}$$

Firm 1:

$$\max_{P_1}(P_1 - c)\frac{P_1 - P_2 + t}{2t}$$

The f.o.c. is:

$$P_1 = \frac{P_2 + t + c}{2} \tag{1}$$

Firm 2:

$$\max_{P_2}(P_2 - c)\frac{P_2 - P_1 + t}{2t}$$

The f.o.c. is:

$$P_2 = \frac{P_1 + t + c}{2} \tag{2}$$

The solution to equation 1 and 2 is:

$$P_1 = P_2 = t + c$$

So

$$\pi_1 = \pi_2 = \frac{t}{2}.$$

Therefore firm 2 will choose to move to location 1 if

$$r \leq \frac{t}{2}.$$

This investment strategy is "fat cat": investment is soft on opponent (because firm 1's profit increases from 0 to a positive number) and firms' prices are stragetic complements (firm 1's price increases in firm 2's price, and vice versa).