Problem Set 7
Answer Key

1 Problem 1

Question a: If the insurer knows each consumer's type, he will charge each one a price equal to his/her willingness to pay:

$$p = 1.5Kh \text{ for type } h$$

All consumers buy the insurance, since the price is no more than their willingness to pay.

This is efficient because it maximizes the social surplus. The social surplus is the sum of the insurer's profit and the consumer surplus:

$$\int_{all\ h\ with\ insurance} [(1.5Kh - p) + (p - Kh)]dh$$

where consumer surplus is $1.5Kh - p$ for type h and the social surplus sums over all types that buy the insurance. Notice that the integrand is positive for all h, so the integral achieves the maximum when all consumers buy the insurance.

Question b: When the insurer does not observe h, he charges price p. Type h consumer purchases the insurance iff $p \leq 1.5Kh$, that is:

$$h \geq \frac{p}{1.5K}$$

All consumers with their health levels between $\frac{1.5K}{p}$ and 1 will buy the insurance. Demand for the insurance is (which is simply $\Pr(h \geq \frac{p}{1.5K})$, because only consumers with $h \geq \frac{p}{1.5K}$ will buy the insurance):

$$q = 1 - \frac{p}{1.5K}$$

where I have used the formula that $\Pr(h \geq a) = 1 - \Pr(h < a)$.

The average health of the buyers is:

$$E(h|h \geq \frac{p}{1.5K}) = \frac{\frac{p}{1.5K} + 1}{2} = \frac{p}{3K} + \frac{1}{2}$$

(If s is uniformly distributed on $[0, 1]$, then $E(s|s > c) = \frac{1+c}{2}$, where $c \leq 1$.)

Question c: The insurer wants to sell at p if

$$\begin{aligned} p &\geq K(\frac{p}{3K} + \frac{1}{2}) \\ 4p &\geq 3K \\ p &\geq \frac{3K}{4} \end{aligned}$$

The insurer will choose to sell some insurance, but it will be so expensive that only unhealthy consumers buy the insurance.

Coming back to the question, the insurer chooses the price to maximize his expected profit (from part b, we know that the expected cost of providing the insurance given that $h \geq \frac{p}{1.5K}$ is $E(Kh|h \geq \frac{p}{1.5K}) = KE(h|h \geq \frac{p}{1.5K}) = (\frac{p}{3K} + \frac{1}{2})K$. Demand for the insurance is $q = 1 - \frac{p}{1.5K}$):

$$\begin{aligned} & \underset{p}{Max}(1 - \frac{p}{1.5K})(p - (\frac{p}{3K} + \frac{1}{2})K) \\ FOC \quad : \quad & 12K - 16p + 6K = 0 \\ p \quad = \quad & \frac{9}{8}K \end{aligned}$$

Who buys?

$$\begin{aligned} h &\geq \frac{p}{1.5K} \\ h &\geq \frac{9K}{12K} \\ h &\geq \frac{3}{4} \end{aligned}$$

The healthier consumers do not buy the insurance because adverse selection causes the price to be above the value they place on the insurance, even though the value of insurance is strictly higher than the cost ($1.5Kh > Kh$) for any type of consumers.

2 Problem 2

Question a: The downstream firm solves:

$$\underset{P_r}{Max}(P_r - \omega)(\frac{a - P_r}{b})$$

The f.o.c. is

$$a - 2P_r + \omega = 0$$

which implies:

$$P_r = \frac{a + \omega}{2}$$

Hence, the downstream firm's demand for wheels is

$$Q(\omega) = \frac{a - \omega}{2b}$$

Question b: The upstream firm solves

$$\underset{\omega}{Max}(\omega - c)(\frac{a - \omega}{2b})$$

The f.o.c. is:

$$a - 2\omega + c = 0$$

which implies:

$$\omega = \frac{a+c}{2}$$

and $Q = \frac{a-c}{4b}$.The upstream firm's profit is:

$$\Pi_\omega = \frac{a-c}{4b} * (\frac{a+c}{2} \ - c) = \frac{(a-c)^2}{8b}$$

and the downstream firm's profit is:

$$\Pi_r = (\frac{a-c}{4b}) * (a - b * \frac{a-c}{4b} - \frac{a+c}{2}) = \frac{(a-c)^2}{16b}$$

The total profit thus equals $\frac{3}{16b}(a-c)^2$. If they were vertically integrated they could obtain the monopoly profit Π_m of $\frac{1}{4b}(a-c)^2$. This is the 'double marginalization problem.' Both the downstream firm and the upstream firm restrict output to exert their market power, without taking into consideration the negative externality on the other firm. This leads to a smaller joint profit.

Question c: The upstream firm charges a two part tariff

$$\omega = A + BQ$$

The optimal choice is

$$\begin{aligned} B &= c \\ A &= \Pi_m = \frac{1}{4b}(a-c)^2 \end{aligned}$$

The downstream firm then maximizes

$$\underset{P_r}{Max}(P_r - c)(\frac{a-P_r}{b}) - A$$

which gives $P_r = \frac{a+c}{2}$ and $Q = \frac{a-c}{2b}$. The downstream firm makes zero profits and the upstream firm makes the monopoly profit equal to $A = \frac{1}{4b}(a-c)^2$. The two-part tariff reduces the upstream firm's incentive to integrate since it gives him the same profit if he were to integrate.