Answer Key to PS 8

1. Consider a second-price sealed bid auction. Let u_i denote the valuation of bidder i. Let $\underline{u}$ and $\overline{u}$ denote the minimum and maximum possible valuations. Suppose there are two bidders. Argue that the following strategies constitute Nash equilibrium: for bidder 1, $\beta_1(u_1) = \overline{u}$ for all u_1; for bidder 2, $\beta_2(u_2) = \underline{u}$ for all u_2. What is the seller's expected revenue? Can you offer an argument against these behavior in practice?

To show that the above strategies constitute a Nash equilibrium, we need to show that there is no unilateral profitable deviation for either player. Let us focus on bidder 1, since the case with bidder 2 is entirely symmetric. Given bidder 2's strategy, bidder 1 always wins the auction by bidding $\overline{u}$. He pays $\underline{u}$ for the object. His utility is: $u_1 - \underline{u} \geq 0$.

Let us consider other bidding strategy for bidder 1.

a) $\beta_1 > \underline{u}$: he wins the auction, pays $\underline{u}$, and gets the same utility as bidding $\beta_1(u_1) = \overline{u}$. $\Rightarrow$ He is no better off.

b) $\beta_1 < \underline{u}$: he loses the auction and gets 0. $\Rightarrow$ He is worse off.

c) $\beta_1 = \underline{u}$: Suppose the tie-breaking rule is to name bidder 1 the winner with probability $\alpha \in [0, 1]$ when both submit the same bid. The best possible case for bidder 1 is $\alpha = 1$. If that is the case, he is no better off. For any $\alpha < 1$, however, bidder 1 is worse off. So, for sure he is no better off.

Therefore, there is no unilateral profitable deviation for bidder 1. Similarly one can show that given bidder 1's strategy, there is no other more profitable bidding strategy for bidder 2.

The seller's revenue is always $\underline{u}$, so her expected revenue is also $\underline{u}$.

The above bidding strategy is unintuitive because it relies on one bidder bidding the lowest possible bid because he believes there is no hope of his winning, while another bidder makes the highest possible bid because he thinks there is no chance the other makes a bid above $\underline{u}$. Hence, this equilibrium relies heavily on the assumption that bidders perfectly understand the game and the equilibrium. Putting this a little differently, the bidding strategies in this equilibrium are not "robust": if one bidder thought there was even an arbitrarily small chance that the other bidder would make a mistake (and make some bid strictly between $\underline{u}$ and $\overline{u}$), the former would want to make a different bid. Either bidder could switch to bidding his valuation without any risk of lower his payoff, while getting a stricly higher expected payoff if the other bidder ever bid in some way other than that described in the equilibrium above. The equilibrium in which each follows the weakly dominant strategy may therefore be more compelling as a prediction of behavior.