

Auctions with Resale

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Abstract

This paper studies auctions at which the good for sale may later be resold by the auction winner. Valuations at such auctions are endogenously determined, based in part on equilibrium expectations of resale outcomes. While these valuations sometimes reflect common value components introduced by the secondary market, the resale opportunity changes auctions in ways that cannot be captured by any model ignoring resale. The model predicts signaling through bids, multiple symmetric equilibria for some standard auctions, and potential gains to a seller from excluding bidders in order to create an active resale market. Key policy implications and empirical predictions can also overturn those based on models without resale.

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1 Introduction

After many auctions, the winner has an opportunity to resell the object in a secondary market. There are exceptions, where government restrictions, adverse selection, or the absence of property rights may preclude resale; however, active resale markets exist for many items sold at auction, including antiques, artwork, coins, emissions rights, livestock, operating licenses, real estate, stamps, treasury bills, and used cars. Furthermore, resale is possible in many other markets for which auctions may provide useful models of price formation. Since rational bidders must account for resale opportunities when choosing their bids, this suggests that the effects of a secondary market warrant careful consideration. Yet while resale has frequently been mentioned in both the theoretical and empirical auction literature, it has been given little careful attention. Almost all of the auction literature has either assumed that no resale markets exist or relied on conjectures regarding the effects of resale.

A resale opportunity is widely believed to add a common value component to bidders' valuations, providing one motivation for standard common or affiliated values models.¹ Investigating this conjecture is important since existing theory can have very different implications under the common and private value paradigms.² Erroneous policy prescriptions may result from incorrect assumptions regarding which model is more appropriate for particular applications. With common values, for example, revenue superiority of English auctions must be weighed against the protection against collusion offered by first-price auctions; with independent private values, revenue equivalence of these auctions (Myerson (1981), Riley and Samuelson (1981)) eliminates the need to consider such tradeoffs. In addition, assumptions regarding which paradigm is most

¹ Work suggesting this idea includes Ashenfelter and Genesove (1992), Chakravorti *et al.* (1995), Cramton (1995), Hausman and Wittman (1993), McAfee and McMillan (1987a, 1987b), McAfee, Quan, and Vincent (1995), McMillan (1994), Milgrom (1985, 1987, 1989), Milgrom and Weber (1982), Rosenthal and Wang (1996), or Rothkopf and Harstad (1994).

² For surveys of the auction literature see Klemperer (1999), McAfee and McMillan (1987a), Milgrom (1985, 1989), or Wilson (1992).

appropriate will often determine the specification of empirical models of auctions. Hence, knowing what circumstances imply common values is itself important to researchers applying theory to market design, testing auction theory empirically, or estimating policy relevant parameters.

The analysis below shows that a resale opportunity fundamentally changes the nature of the valuation upon which each bidder bases his bid, but that these valuations need not depend on other bidders' private information—i.e., resale does not always imply common or affiliated values. However, the question of whether and when resale implies common values turns out to be of relatively minor importance given the other effects of a resale opportunity. With a resale market, valuations are determined endogenously, in part by the expected option value the resale market provides in equilibrium. As a result, valuations vary with the rules governing resale trade and, in some cases, the rules of the auction in the primary market. The option value of the resale opportunity can be sufficiently large that a seller will benefit from excluding bidders from the auction in order to create an active resale market. In addition, informational linkages between the primary and secondary markets can cause a bidder's payoff in the resale market to depend on the bid he makes in the primary market auction. This can lead to signaling equilibria and to reversals of prior results regarding a seller's choice of auction, existence and uniqueness of equilibrium, and testable implications.

Prior work on auctions has paid surprisingly little attention to resale. Milgrom (1987) includes the first model of auctions with resale, while Kamien, Li, and Samet (1989) and Gale, Hausch, and Stegeman (1999) consider procurement bidding with subcontracting. Because players in these models have complete information, many of the most interesting issues raised by a resale opportunity cannot be addressed. Bikhchandani and Huang (1989) present a more closely related model with asymmetric information applicable to Treasury bill auctions, where pure common values, no reserve price and a perfectly competitive resale market are assumed. An important insight in their work is that a secondary market may create incentives for bidders to bid higher in hopes of convincing resale buyers that the object is more valuable. However,

their focus on Treasury bill auctions limits the scope of the analysis and its applicability to other environments, leaving important questions regarding auctions with resale unanswered.

A difficult but important choice in studying auctions with resale is how to model the secondary market. In some applications institutional features may suggest a particular model, but in most cases we have little information about secondary markets. Because important results turn out to be sensitive to the structure of the resale market, a single specification can be misleading. Regardless of the resale mechanism assumed, however, the effects of the resale opportunity on equilibrium bidding in the primary market arise only through bidders' expectations of payoffs conditional on the outcome of the auction. To focus on bidding in the primary market I employ a very general representation of these expectations, motivated by a few natural specifications. This approach makes it possible to incorporate a wide range of resale market structures while making clear which features of the resale market are essential to each result.

The paper is organized as follows. Section 2 presents a simple example in which the symmetric independent private values (IPV) model is extended to allow resale. This motivates the specification of the full model presented in section 3: a first-stage auction with affiliated values followed by a second-stage resale market in which new buyers arrive. Equilibrium bidding for the first-stage auction is derived in section 4. Implications for sellers are discussed in section 5 where, in addition to revenue comparisons across auction types, optimal policies toward bidder participation are discussed. This is followed by a discussion of implications for empirical tests for common values in section 6. Section 7 concludes and is followed by two appendices.

2 Resale in the Symmetric IPV Model

Consider the simplest model of a symmetric IPV auction with resale: a two-stage game in which n risk neutral bidders for a single indivisible object are also the only potential participants in

a post-auction resale market. Let X_i represent the component of bidder i 's private information corresponding to his "use value," i.e., the value he places on owning the object, ignoring any resale possibility.³ Each X_i is drawn independently from a continuously differentiable distribution $F(\cdot)$, which is strictly increasing on its support $[0, 1]$. Let Y_1 denote the highest use value among a given bidder's opponents. The first stage consists of an English, first-price sealed bid, or second-price sealed bid auction with reserve price $r \geq 0$. In the second stage, trade between the winner and the losers is permitted. I assume resale trade is voluntary and restrict attention to second-stage games that themselves have at least one perfect Bayesian equilibrium for any feasible beliefs. Otherwise, any specification of the resale market structure is permitted.

Theorem 1. *The unique symmetric Bayesian Nash equilibrium strategy for a first-price, second-price, or English auction without resale is also a perfect Bayesian equilibrium bidding strategy when the same auction is followed by a resale opportunity.*

Proof (second-price sealed bid or English auction): Suppose bidder i with use value x bids $\tilde{x} > x$ while all other bidders bid their use values. This would change i 's payoff only when $\tilde{x} \geq r$ and $x < y_1 < \tilde{x}$. When this occurs, i would pay $\max\{r, y_1\}$ for the object but could only resell for some price in the interval $[x, y_1]$. This leaves him nonpositive expected profit. By bidding x , i would have received zero profit with certainty when $x < y_1$. A similar argument shows that bidder i would not benefit from lowering his bid to something below x . \square

The proof for the first-price sealed bid auction, given in Appendix A, is slightly more subtle, but the intuition is the same: in expectation it cannot pay to outbid (underbid) an opponent in the hopes of selling to him (buying from him) in the resale market.⁴ However, the existence

³ This terminology is needed because "valuations" will be endogenously determined. A bidder's use value can be thought of as his present value of consuming the object himself, although this is more restrictive than necessary.

⁴ Bulow and Roberts (1989) and Porter (1995) anticipate this result, noting that because symmetric monotonic bidding equilibria yield efficient allocations, no resale trade should occur after an auction

of active resale market for many goods sold by auction indicates that something is missing in this simple model—something that allows gains from trade to arise in the resale market. The remainder of the paper focuses on one possibility originally suggested by Milgrom (1987): the presence buyers in the resale market who were not at the auction.⁵ In Treasury bill auctions, for example, competitive bidding is limited to a fairly small number of dealers and institutional investors (Bikhchandani and Huang (1989)). In procurement auctions, subcontracting often involves firms that did not pre-qualify and were, therefore, ineligible to bid. Bidders at an estate sale may anticipate resale from their own estates. Bidders for a house will anticipate the opportunity to sell to new potential buyers in the future, due to migration or changes in income. Of course, in many auction environments the arrival of new buyers is the natural result of entry, which often will be driven by factors outside the auction model. Firms entering the wireless telecommunications industry, for example, must purchase spectrum licenses, many of which were previously sold by auction.

3 The Model

Consider a two-stage game played by $n + m$ risk neutral buyers. In the first stage, n of these buyers compete for a single indivisible object at an auction of type $a \in \{f, s, e\}$ (first-price, second-price, English) with a reserve price r . Ties are resolved by uniform randomization. If no bid of at least r is submitted, the game ends and the seller keeps the object.⁶ Otherwise

even when it is permitted. Theorem 1 shows that this is correct, although even in this simple case one must show (as done here) that no profitable deviations are introduced when the auction game is extended to allow resale trade. An obvious question is whether there are other symmetric equilibria for the auction with resale. The results below provide conditions ensuring that there are not.

⁵ Other motivations for an active resale market are considered in Haile (1999a, 1999b, 1999c) (changes in information about use values), and Gupta and Lebrun (1997) (bidder asymmetry).

⁶ Horstman and LaCasse (1997) consider a complementary model, allowing the seller to reject all bids and re-auction the object but abstracting from resale by a winning bidder. McAfee and Vincent (1997) assume the absence of a resale market but allow the initial seller to hold another auction when there are no bids above the announced reserve price.

she awards it to the high bidder and announces all submitted bids. In the second stage, players participate in a resale game in which the first-stage winner may sell to one of the losing bidders or one of m new buyers.

Each bidder i 's use value for the object is given by the nondecreasing function $u_b(X_i, \mathbf{S})$, where $X_i \in [0, 1]$ is i 's private information (type) and $\mathbf{S} \in \mathfrak{R}^q$ is unobserved at the time of the auction. Use values may be purely private, e.g., $u_b(X_i, \mathbf{S}) = X_i$; purely common, e.g., $u_b(X_i, \mathbf{S}) = V(\mathbf{S})$; or some combination of these. Similarly, the use value of each entrant buyer is given by the nondecreasing function $u_e(Z_j, \mathbf{S})$, $j = n + 1, \dots, n + m$, where $Z_j \in [0, 1]$ is buyer j 's private information.⁷ Note that the case $u_b(X_i, \mathbf{S}) < u_e(Z_i, \mathbf{S}) \forall X_i, Z_i$ is permitted (see example 3 in Appendix B).

Players know the numbers n and m as well as the joint distribution

$$\xi(X_1, \dots, X_n, \mathbf{S}, Z_{n+1}, \dots, Z_{n+m})$$

which is symmetric in its first n and last m arguments and affiliated.⁸ For simplicity I assume that continuous marginal densities exist for all subsets of $\{X_1, \dots, X_n, \mathbf{S}, Z_{n+1}, \dots, Z_{n+m}\}$. Let Y_j denote the j th highest type among a given bidder's $n - 1$ first-stage opponents and define $\mathbf{Z} = \{Z_{n+1}, \dots, Z_{n+m}\}$, $\mathbf{Y} \equiv \{Y_1, Y_2, \dots, Y_{n-1}\}$, and $\mathbf{Y}_{-1} \equiv \mathbf{Y} \setminus Y_1$. From the perspective of a representative bidder i let $F_1(\cdot | \Upsilon)$ give the distribution of Y_1 conditional on $\Upsilon \subset \{X_i, \mathbf{Y}_{-1}\}$ and assume that for all Υ , $F_1(\cdot | \Upsilon)$ has full support.

I focus on perfect Bayesian equilibria of the two-stage game in which first-stage bidding strategies are symmetric and strictly increasing. For any specification of the second-stage game,

⁷ In contrast to similar functions in Milgrom and Weber (1982), other players' types are not included as arguments in u_b or u_e . However, in the Milgrom and Weber (1982) framework, $u(X_i, \mathbf{X}_{-i}, \mathbf{S})$ represents bidder i 's expected utility from winning the auction, which may not be the same as his use value. It seems unlikely that a player's use value itself would depend on other players' private information in any well motivated example. Nonetheless, the results below remain valid in a more general model allowing such dependence.

⁸ See Milgrom and Weber (1982) for a discussion of affiliation. Loosely, affiliation implies that a higher realization of one of these random variables implies a (weakly) higher probability of higher realizations of the others.

we can represent the expected second-stage payoff to bidder i when he has won the first-stage auction (gross of the price paid to the initial seller) as

$$W(X_i, \mathbf{Y}, \mathbf{Z}, \mathcal{B})$$

where \mathcal{B} is a matrix of distribution functions \mathcal{B}_{ij} giving the beliefs of player i about the type of player j . In equilibrium these beliefs will be formed based on players' common priors and any information revealed during the auction or between stages. In particular, since a player's first-stage bid reveals his type in equilibrium, \mathcal{B}_{ij} will be degenerate for all i when bidder j submits a bid that is revealed during or after the first-stage auction.⁹ Similarly, we can let

$$L(X_i, \mathbf{Y}, \mathbf{Z}, \mathcal{B})$$

represent the expected payoff to bidder i when he loses the first-stage auction.

At the time of the auction, bidders typically will know only their own signals and must take expectations over \mathbf{Y}, \mathbf{Z} , and \mathcal{B} . However, the outcome of the auction will itself reveal whether the highest opposing bid was higher or lower than a one's own bid. When all other bidders use equilibrium strategies this is a revelation about Y_1 . Furthermore, bidder i 's own bid will typically determine the beliefs \mathcal{B}_{jj} of his opponents. If $b(\cdot)$ is the equilibrium bid function for a first-stage sealed bid auction, we can construct the conditional expectations¹⁰

$$w(x, y, \tilde{x}) = E [W(X_i, \mathbf{Y}, \mathbf{Z}, \mathcal{B}) | X_i = x, Y_1 = y, b_i = b(\tilde{x})]$$

and

$$\ell(x, y, \tilde{x}) = E [L(X_i, \mathbf{Y}, \mathbf{Z}, \mathcal{B}) | X_i = x, Y_1 = y, b_i = b(\tilde{x})].$$

⁹ Given the assumed bid announcement policy of the seller, in equilibrium this will be all bids in the case of a sealed bid auction. An English auction, however, will end before the winner's bid is reached, leading to nondegenerate beliefs. This is discussed further below. For simplicity I consider only one bid announcement policy—one consistent with disclosure requirements for most government auctions. However, the question of an optimal bid announcement policy is an important one for future research.

¹⁰ I implicitly restrict attention to bids in the support of $b(\cdot)$. One can easily construct beliefs off the equilibrium path ensuring that deviation to any other bid is suboptimal.

A bidder’s *valuation*—the expected value of winning the auction—is the difference between the second-stage payoff conditional on winning and that conditional on losing, i.e.,

$$v(x, y, \tilde{x}) = w(x, y, \tilde{x}) - \ell(x, y, \tilde{x}).$$

The analysis below allows many models of the resale market by working directly with these expectations.¹¹ Appendix B shows the derivation of these expectations for several specifications of the resale market, each of which is shown to possess the following properties.

SYM. (symmetry) For all \tilde{x} , $\ell(x, y, \tilde{x}) = 0 \forall y \geq x$.

MON. (monotonicity) (i) $w(x, y, \tilde{x})$ and $v(x, y, \tilde{x})$ are nondecreasing in x and y ; (ii) $w(x, y, \tilde{x})$ strictly increases in x when $y \leq x$; (iii) if \mathcal{B} and $\hat{\mathcal{B}}$ are identical except that $\hat{\mathcal{B}}_{j_i}$ first-order stochastically dominates \mathcal{B}_{j_i} for all $j \neq i$, then $W(X_i, \mathbf{Y}, \mathbf{Z}, \hat{\mathcal{B}}) \geq W(X_i, \mathbf{Y}, \mathbf{Z}, \mathcal{B})$.

DIF. (differentiability) $w(x, y, \tilde{x})$ is differentiable (i) with respect to x , (ii) with respect to y at $y = x$, and (iii) with respect to \tilde{x} at $\tilde{x} = x$

SM. (supermodularity) $\frac{\partial}{\partial \tilde{x}} w(x, y, \tilde{x})$ is nondecreasing in x and y .

BIL. (belief irrelevance for losers) $\frac{\partial}{\partial \tilde{x}} \ell(x, y, \tilde{x}) = 0 \forall x, y, \tilde{x}$.

These properties are assumed throughout the remainder of the paper.¹² SYM rules out gains to trade between a winning bidder with a high type and a loser with a low type. If only one resale trade is permitted, this is implied by the symmetry of bidder’s use value functions. When multiple resale transactions are possible, SYM rules out asymmetries in bidders’ abilities to extract surplus from entrant buyers. Parts (i) and (ii) of MON reflect natural monotonicity properties, including the necessary condition (for a separating equilibrium) that valuations increase in types. Part (iii) arises because in many bargaining games with asymmetric information, a player’s surplus extraction depends on his opponents’ beliefs. Parts (i) and (ii) of DIF

¹¹ A slight modification of this notation will be necessary when studying English auctions, due to differences in the information revealed during the auction. This is introduced below.

¹² Alternatively, one can restrict attention to the resale market structures considered in Appendix B and view the statement of these properties as a lemma.

generally follow from the assumption of continuous marginal densities. Part (iii), however, is an important restriction: Haile (1996) shows that this condition fails for some simple specifications of the resale market (e.g., an ultimatum offer from a randomly chosen resale buyer), causing a “ratchet effect” that precludes existence of a separating equilibrium.¹³ Because such ratchet effects are not robust to minor changes in the information structure, restricting attention to resale mechanisms satisfying DIF avoids pathological results. SM and BIL are more restrictive properties assumed to ensure existence of a separating bidding equilibrium. SM requires that each bidder i 's return from shifting the beliefs $\mathcal{B}_{ji}, j \neq i$ increase (weakly) in i 's own type and in the type of his top bidding opponent.¹⁴ BIL requires that a loser's expected payoff be independent of his own first-stage bid, even when he submits a deviant first-stage bid. This is less restrictive than it may seem, since all opponents place probability zero on there being gains to trade with a first-stage loser.

Appendix B verifies each of these properties for several natural specifications of the resale market. Dividing gains to trade according to Shapley values is a simple trading rule capturing features of efficient resale markets. With independent private values, an auction with an optimal reserve price is the optimal resale mechanism and, therefore, a natural choice. With correlated types the bargaining literature is less well developed; however, the competitive resale market modeled by Bikhchandani and Huang (1989) can be shown to satisfy the above properties.¹⁵ The results that follow hold for these examples as well as any other specifications satisfying these five properties.

¹³ Similar phenomena are found in Laffont and Tirole (1988) and Waehrer (1998).

¹⁴ This is a generalization of Bikhchandani and Huang's (1989) “information complementarity” assumption, which plays the same role in their analysis.

¹⁵ Bikhchandani and Huang (1989) allow auctions of multiple units. However, restricting attention to the single-unit case as done here does not significantly affect their analysis.

With this notation in place, one final assumption is made regarding the reserve price:

$$r \in \left[w(0, 0, 0), \int_0^1 w(1, y, 1) dF_1(y|1) \right). \quad (1)$$

Reserve prices above this range will preclude existence of a symmetric equilibrium in which any player bids. Reserve prices below this range do not bind and are, therefore, equivalent to a reserve price of $w(0, 0, 0)$.

4 Equilibrium Bidding

4.1 Second-Price Sealed-Bid Auction

Suppose $b(\cdot)$ is an equilibrium bid function when a second price sealed bid auction is held in the first stage and define

$$x_0^s = \inf\{x \in [0, 1] : b(x) \geq r\}.$$

Define \underline{x} by the equation (DIF and (1) ensure that \underline{x} is well defined)

$$E[w(X_i, Y_1, X_i) | X_i = \underline{x}, Y_1 \leq \underline{x}] = r.$$

Lemma 1. *In any perfect Bayesian equilibrium in symmetric, strictly increasing bidding strategies, $x_0^s = \underline{x}$.*

Proof: See Appendix A.

If a bidder with signal x bids $b(\tilde{x}) \geq r$, his expected payoff is

$$\pi(x, \tilde{x}) = \int_0^{\tilde{x}} [w(x, y, \tilde{x}) - \max\{r, b(y)\}] dF_1(y|x) + \int_{\tilde{x}}^1 \ell(x, y, \tilde{x}) dF_1(y|x). \quad (2)$$

For $b(\cdot)$ to be an equilibrium bid function, (2) must be maximized at $\tilde{x} = x$ for all $x > x_0^s$. Differentiating with respect to \tilde{x} and setting $\tilde{x} = x$ gives (recalling SYM) the unique candidate equilibrium bid function

$$b^s(x) = \theta(x) \equiv w(x, x, x) + \psi(x)$$

where

$$\psi(x) \equiv \frac{\int_0^x w_3(x, y, x) dF_1(y|x)}{f_1(x|x)}$$

and $w_3(x, y, \tilde{x})$ is the partial derivative of $w(x, y, \tilde{x})$ with respect to \tilde{x} . This proves the following result.

Theorem 2. *If a second-price sealed bid auction is held in the first stage, in any perfect Bayesian equilibrium in symmetric strictly increasing bidding strategies, all bidders with types above x_0^s bid $w(x, x, x) + \psi(x)$ while bidders with types below x_0^s do not bid.*

Theorem 2 gives necessary conditions for a perfect Bayesian equilibrium in symmetric separating bidding strategies, implying that at most one equilibrium bidding strategy exists. However, this equilibrium need not exist if the signaling incentive is too strong.¹⁶ The reason is that in a second-price auction, an increase in i 's bid to something higher than $b(X_i)$ affects the price he pays only when he “jumps over” another bidder with a higher type. If the cost induced by this possibility is smaller than the signaling benefit of raising his bid, i would deviate from any proposed separating equilibrium. The following theorem provides a strong sufficient condition (the absence of signaling incentives) ensuring that this is not the case.

Theorem 3. *If $w_3(x, y, x) = 0 \forall x, y$, $b^s(\cdot)$ gives a perfect Bayesian equilibrium bidding strategy for the second-price sealed bid auction with resale.*

Proof: See Appendix A.

Despite the possible nonexistence of a separating equilibrium—which we will see is unique to the second-price auction—the analysis for this auction illustrates several key features that carry through to the other auction types. First, in equilibrium a bidder does not expect to buy in the resale market if he loses the auction. Hence, the expected payoff to a losing bidder does not appear in the equilibrium bid function. Second, if one ignores the term $\psi(x)$, the

¹⁶ Bikhchandani and Huang (1989) obtain a similar result.

equilibrium bid function resembles that in an affiliated values auction without resale, with the endogenous valuation $v(x, x, x) = w(x, x, x)$ replacing the primitive valuation $v(x, x)$ of Milgrom and Weber's (1982) affiliated values model. Third, however, is the presence of the term $\psi(x)$, which is nonnegative (by MON) and reflects bidders' incentives to raise their bids in hopes of signaling a higher type to resale buyers. The presence of this term implies that bidding differs from that in standard models of auctions without resale even when one accounts for the endogeneity of valuations.

4.2 First-Price Sealed Bid Auction

In a first-price sealed bid auction, the analysis is similar. Here, however, not only is an equilibrium sure to exist, there will often be a continuum of equilibria. Suppose $b(\cdot)$ is a symmetric strictly increasing and differentiable equilibrium bid function for a first-price sealed bid auction and define

$$x_0^f = \inf\{x \in [0, 1] : b(x) \geq r\}.$$

Let $b(x_0^f) = \lim_{x \downarrow x_0^f} b(x)$.

Lemma 2. $b(x_0^f) = \int_0^{x_0^f} w(x_0^f, y, x_0^f) \frac{f_1(y|x_0^f)}{F_1(x_0^f|x_0^f)} dy.$

Proof: See Appendix A.

Consider a bidder i with type x . If he bids $b(\tilde{x}) \geq r$ while all other bidders follow $b(\cdot)$, his expected payoff will be

$$\pi(x, \tilde{x}) \equiv \int_0^{\tilde{x}} [w(x, y, \tilde{x}) - b(\tilde{x})] dF_1(y|x) + \int_{\tilde{x}}^1 \ell(x, y, \tilde{x}) dF_1(y|x). \quad (3)$$

The first-order condition is a differential equation similar to that characterizing bidding in a first-price auction without resale; the difference is that the exogenous valuation $v(x, x)$ from Milgrom and Weber's (1982) model is replaced with $\theta(x) \equiv v(x, x, x) + \psi(x)$, just as in the

second-price auction. Making this substitution suggests the equilibrium bid function

$$b^f(x) = \begin{cases} b(x_0^f) \eta(x_0^f|x) + \int_{x_0^f}^x \theta(t) \eta'(t|x) dt & x \geq x_0^f \\ \text{"no bid"} & x < x_0^f \end{cases} \quad (4)$$

where

$$\eta(t|x) \equiv \exp \left\{ - \int_t^x \frac{f_1(s|s)}{F_1(s|s)} ds \right\}.$$

Standard arguments show that this is indeed the unique solution, proving the following result.

Theorem 4. *If a first-price auction is held in the first stage, in any perfect Bayesian equilibrium in symmetric strictly increasing differentiable bidding strategies, all players bid according to $b^f(\cdot)$.*

While Theorem 4 characterizes a unique equilibrium bid function given expected outcomes in the resale continuation game and a value of x_0^f , if beliefs affect the winner's payoff there generally will be continuum of equilibrium values for x_0^f . Define

$$\bar{x}(t) = \begin{cases} 1 & E[w(1, Y_1, 0) | Y_1 \leq t] < r \\ \inf \left\{ x \in [0, 1] : E[w(X_i, Y_1, 0) | X_i = x, Y_1 \leq t] \geq r \right\} & \text{otherwise.} \end{cases}$$

Letting \bar{x} denote the unique fixed point of $\bar{x}(\cdot)$,¹⁷ \underline{x} and \bar{x} define participation margins conditional on two extreme specifications of the beliefs induced by a bid equal to the reserve price—the optimal deviant bid for a type below x_0^f . When the reserve price is an equilibrium bid, bidding r results in beliefs that are degenerate at the marginal type. With those beliefs \underline{x} will be the marginal type, as in the second-price auction. If all equilibrium bids lie above r , however, equilibrium beliefs induced by a bid of r (or anything else below $b^f(x_0^f)$) are arbitrary; \bar{x} is determined using the least favorable beliefs possible. If $w_3(x, y, \tilde{x}) = 0$ for all x, y, \tilde{x} (so that these beliefs do not matter), then $\underline{x} = \bar{x}$ and x_0^f must equal x_0^s . When $w(x, y, \tilde{x})$ varies with \tilde{x} , however, there will be a continuum of equilibrium values for x_0^f —all but one of them larger than x_0^s .

¹⁷ MON, DIF and (1) ensure that $\bar{x}(t)$ is well defined for all t , with $\bar{x}(0) > 0$ and $\bar{x}(1) \leq 1$. Since MON and DIF ensure that $\bar{x}(t)$ is nonincreasing and continuous, there must be a unique fixed point.

Lemma 3. $w^f(x, x, x) - b^f(x) \geq 0 \quad \forall x \geq x_0^f$, with the inequality strict for $x > x_0^f$.

Proof: See Appendix A.

Theorem 5. For any $x_0^f \in [\underline{x}, \bar{x}]$, $b^f(\cdot)$ gives a perfect Bayesian equilibrium bidding strategy for the first-price sealed bid auction with resale.

Proof: See Appendix A.

To understand the reason for the multiplicity of equilibria, first note that there is one equilibrium in which participation in the first-price auction is exactly as in the second-price auction; i.e., $x_0^f = \underline{x} = x_0^s$. However, types just above the participation margin in that equilibrium could be forced out of the bidding if opponents form sufficiently unfavorable beliefs in response to the bids these types make in that equilibrium. If these beliefs do force such types out, a new equilibrium is obtained, since the unfavorable beliefs concern bids that are now off the equilibrium path. Figure 1 illustrates. Here

$$\phi(x_0^f, \hat{x}) \equiv \int_0^{x_0^f} w(x_0^f, y, \hat{x}) \frac{f_1(y|x_0^f)}{F_1(x_0^f|x_0^f)} dy.$$

For every set of degenerate beliefs represented by $\hat{x} \in [0, \underline{x}]$, there is a unique solution for x_0^f defined by $\phi(x_0^f, \hat{x}) = r$. To see why this multiplicity arises only in the first-price auction, note that the new equilibrium with a higher participation threshold can be obtained because the marginal type in the new equilibrium makes zero profit (by Lemma 2), implying that lower types could not profit by deviating to an equilibrium bid. This contrasts with a second-price auction, where if $x_0^s > \underline{x}$, a bidder with type in (\underline{x}, x_0^s) could deviate by bidding $b^s(x_0^s)$. While this bid itself is above r , a bidder who wins with this bid pays only r , giving the deviant bidder a profit (by MON and the definition of \underline{x}).

4.3 English Auction

As usual, the analysis of the English auction is similar to that for the second-price sealed bid auction but must account for the fact that bidders observe opponents exiting during the auction. We will see that here a second distinction also arises, due to the informational linkages between the first and second stages. I follow the convention of modeling an English auction as a “button” auction, where the price is continuously raised by the auctioneer as bidders observably and irreversibly drop out. Following Milgrom and Weber (1982), I model the auction in a sequence of phases, starting in phase 0 and beginning a new phase each time a bidder exits. Phase k then begins when the k th bidder drops out at some price d_k . The auction ends when only one bidder remains,¹⁸ implying that in contrast to the sealed bid auctions, marginal changes in the winner’s bid cannot affect second-stage beliefs.

Let

$$D_k = \begin{cases} \emptyset & k = 0 \\ \{d_1, \dots, d_k\} & k = 1, \dots, n - 1. \end{cases}$$

represent the dropout prices observed prior to phase k of the auction. A bidding strategy for phase k depends on the observed information D_k as well as a bidder’s own type. Suppose $b_k(\cdot, D_k)$ is the symmetric strictly increasing equilibrium bid function for phase k given D_k .

Let

$$\Omega(y, D_k) \equiv \begin{cases} \{Y_1 = \dots = Y_{n-k-1} = y, Y_{n-k} = b_{k-1}^{-1}(d_k; D_{k-1}), \dots, Y_{n-1} = b_0^{-1}(d_1; D_0)\} & k > 0 \\ \{Y_1 = \dots = Y_{n-1} = y\} & k = 0. \end{cases}$$

As usual, this represents the information regarding \mathbf{Y} that a bidder would have if he were to win the auction in phase k at price $b_k(y, D_k)$.¹⁹ Conditioning on this information, let

$$w^e(x, \Omega(y, D_k)) = E[W(X_i, \mathbf{Y}, \mathbf{Z}, \mathcal{B}) | X_i = x, \Omega(y, D_k)]$$

¹⁸ In contrast to auctions without resale, this assumption has bite here, since MON implies that the winning bidder might actually like to offer a price above that of the last exit in order to signal a high type. Such bidding can be ruled out with (self-confirming) beliefs that ignore all signals. However, in some circumstances one can also construct equilibria in which such signaling occurs. DasVarma (1997) explores this possibility in a related model.

¹⁹ When \hat{n} of the n bidders choose not to participate, the auction would actually begin in phase \hat{n} , requiring a straightforward modification of the conditioning in the information set $\Omega(\cdot)$.

and

$$\ell^e(x, \Omega(y, D_k), \tilde{x}) = E[L(X_i, \mathbf{Y}, \mathbf{Z}, \mathcal{B}) | X_i = x, \Omega(y, D_k), b_i = b(\tilde{x})].$$

Note that the argument \tilde{x} does not appear in $w^e(\cdot)$ since the winner's bid will not be revealed and, therefore, will have no effect on his expected payoff conditional on $\Omega(y, D_k)$. The expectations $w^e(x, \Omega(y, D_k))$, $\ell^e(x, \Omega(y, D_k), \tilde{x})$ and

$$v^e(x, \Omega(y, D_k), \tilde{x}) \equiv w^e(x, \Omega(y, D_k)) - \ell^e(x, \Omega(y, D_k), \tilde{x})$$

are assumed to satisfy the properties assumed above regarding $w(x, y, \tilde{x})$, $\ell(x, y, \tilde{x})$ and $v(x, y, \tilde{x})$.²⁰

Besides eliminating the incentive to signal, the fact that the winner's type is not revealed by the first-stage auction implies that he expects to extract more surplus (greater information rents) in the resale market than he does after winning a sealed bid auction.

Lemma 4. $w(x, x, x) \leq E[w^e(X_i, \Omega(X_i, D_{n-2})) | X_i = x] \quad \forall x, D_{n-2}$.

Proof: See Appendix A.

Define

$$x_0^e = \inf\{x \in [0, 1] : b_0^e(x, \emptyset) \geq r\}.$$

Following the proof of Lemma 1, it is straightforward to show the following:

Lemma 5. *In any perfect Bayesian equilibrium in symmetric, strictly increasing bidding strategies, x_0^e equals the unique fixed point of $\underline{x}^e(t) \equiv \inf\{x \in [0, 1] : E[w^e(X_i, \Omega(Y_1, D_0)) | X_i = x, Y_1 \leq t] \geq r\}$.*

Proof: Omitted.

The following results show that in equilibrium, players bid their expected valuations in each phase k , conditional on $\Omega(y, D_k)$.

²⁰ This is easily verified for the examples in Appendix B.

Theorem 6. *If an English auction is held in the first stage, in any perfect Bayesian equilibrium in symmetric strictly increasing bidding strategies, types below x_0^e do not participate while in phase $k \in \{0, \dots, n-2\}$ each type $x > x_0^e$ drops out when the price reaches $w^e(x, \Omega(x, D_k))$.*

Proof: See Appendix A.

Theorem 7. *$b_k^e(\cdot, D_k) = w^e(x, \Omega(x, D_k))$ gives a perfect Bayesian equilibrium bidding strategy for each phase $k \in \{0, \dots, n-2\}$ of an English auction with resale.*

Proof: See Appendix A.

4.4 Discussion

The defining characteristic of a common value auction is that each bidder's expectation of the value of winning depends on his opponents' private information. This characteristic—which accounts for the winner's curse, for example—is present in standard common value models (e.g., Wilson (1977)) and, in fact, in any affiliated values auction other than the special case of pure private values. This characteristic is also a feature of the present model when types are not independent, since opponents' types are then signals of the use values of entrant buyers, which affect the option value of the resale market. More formally, if

$$W(X_i, \mathbf{Y}, \mathbf{Z}, \mathcal{B}) - L(X_i, \mathbf{Y}, \mathbf{Z}, \mathcal{B})$$

varies with \mathbf{Z} and if \mathbf{Y} contains informative signals of \mathbf{Z} , then the valuations $v(x, y, \tilde{x})$ and $v^e(x, \Omega(y, D_k), \tilde{x})$ will depend on y and D_k . This is true even if use values themselves do not vary with \mathbf{Y}, \mathbf{Z} , or \mathbf{S} . Hence the resale market can add a common value component to what otherwise would be a private value auction.

Remark 1. *When bidders' types are correlated with entrant buyers' types, in equilibrium the value a bidder places on winning the auction depends on the realizations of losing bidders'*

types even if use values are purely private; i.e., resale creates common values.

Suppose, however, that bidders' types are independent of those of entrant buyers—a natural assumption if one considers extending the standard IPV model to allow resale, for example. In this case, it is still true that the value of winning the auction increases in Y_1 , for example, as long as $Y_1 > X_i$. However, in equilibrium this is never the case when i wins. Once a bidder conditions on the information implied by his winning the auction in equilibrium, no opponent's private information affects the expected value of winning since no bidder has private information about \mathbf{Z} . For this reason, the auction is one with private values.²¹ In fact, we will see below that in some cases equilibrium bidding in a given auction is exactly as in an appropriately respecified private values auction without resale.

Remark 2. *When bidders' types are independent of those of resale buyers, the value a bidder places on winning the auction in equilibrium does not depend on the realization of other bidders' types; i.e., resale does not create common values.*

However, the results above should already suggest that the question of whether resale creates common values is actually of minor importance given the other effects of the resale opportunity. The following section shows a number of ways in which policy implications and testable implications differ due to the resale opportunity. An exception, however, is the special case in which $m = 0$, where

$$w(x, x, x) = E[u_b(X_i, \mathbf{S}) | X_i = x, Y_1 = x]$$

²¹ The distinction between dependence of $W(X_i, \mathbf{Y}, \mathbf{Z}, \mathcal{B}) - L(X_i, \mathbf{Y}, \mathbf{Z}, \mathcal{B})$ on \mathbf{Y} unconditionally and conditional on $Y_1 < X_i$ may appear arbitrary, since we are used to thinking about valuations (and therefore the distinction between common and private values) as primitives. Here, however, valuations are not primitives; they are defined only based on equilibrium expectations of outcomes in the resale continuation game—outcomes that depend on beliefs formed on the assumption of separating equilibrium bidding behavior in the first stage. It, therefore, seems inconsistent not to condition on all restrictions implied by equilibrium when examining the nature of valuations. From a more practical perspective, expectations of $W(X_i, \mathbf{Y}, \mathbf{Z}, \mathcal{B})$ or $L(X_i, \mathbf{Y}, \mathbf{Z}, \mathcal{B})$ when $Y_1 > X_i$ do not appear in the equilibrium bid functions and therefore have no relevance to questions of seller policies or interpretation of bidding data.

while $w(x, \Omega(x, D_k)) =$

$$E [u_b(X_i, \mathbf{S}) | X_i = Y_1 = \dots = Y_{n-k-1} = x, Y_{n-k} = b_{k-1}^{-1}(d_k; D_{k-1}), \dots, Y_{n-1} = b_0^{-1}(d_1; D_0)].$$

Comparing the equilibrium bid functions above to those in Milgrom and Weber (1982), we see that the unique bidding equilibrium is then identical to that in the same auction without resale, generalizing and strengthening the result in Theorem 1.

Corollary 1. *When $m = 0$, bidding in any perfect Bayesian equilibrium in symmetric strictly increasing bidding strategies is identical to that for the same auction when there is no resale market.*

5 Implications for Sellers

5.1 The Effects of Resale on Seller Revenues

The fact that the resale opportunity cannot hurt an auction winner implies that the seller always benefits from the existence of the resale market in this model.

Theorem 8. *The seller's expected revenue in any symmetric separating equilibrium of a first-price, second-price, or English auction with resale is at least as large as that from the same auction without a resale opportunity.*

Proof: For a second-price sealed bid auction, let $b_R^s(\cdot)$ and $b_{NR}^s(\cdot)$ denote, respectively, the equilibrium bid function with and without a resale opportunity. Then

$$\begin{aligned} b_R^s(x) &= w(x, x, x) + \psi(x) \\ &\geq w(x, x, x) \\ &\geq E [u_b(X_i, \mathbf{S}) | X_i = Y_1 = x] \\ &= b_{NR}^s(x). \end{aligned}$$

Hence the bid of any player when a resale market exists is at least as high as that when there

is no resale opportunity. Furthermore, the fact that

$$w(x, y, \tilde{x}) \geq E[u_b(X_i, \mathbf{S}) | X_i = x, Y_1 = y]$$

for any x, y, \tilde{x} ensures that any bidder willing to participate in the auction without resale will also participate when resale is allowed. A similar argument applies to the other auctions types.

□

5.2 Excluding Bidders to Create a Resale Market

Given Theorem 8, it is natural to ask whether the seller could benefit from excluding some buyers from the auction to create a resale market; i.e., whether the entry of new buyers in the resale market could arise from profit-maximizing behavior by the initial seller. If the initial seller can use any mechanism she chooses in the first stage and no information is exogenously revealed between stages, the Revelation Principle ensures that the answer is no—some one-shot mechanism including all potential buyers will be optimal. However, a seller may be restricted (by law or by complexity costs, for example) to standard auction formats. Such a restriction will not always bind: with independent types and a regularity condition, Myerson (1981) shows that a one-shot standard auction is optimal among all selling mechanisms. In other cases, however, this may not be true. For example, the first-stage winner may have access to information or commitment technologies enabling him to extract more surplus from other buyers than the initial seller can. In such cases the seller may benefit from letting buyers compete for the right to sell in the second stage, thereby passing through at least some of the added surplus extraction realized in the second stage.

This can be seen in a simple example in which there are n potential buyers with use values $u_b(X_i, \mathbf{S}) = X_i$ drawn independently from the standard uniform distribution. Suppose that the first-stage winner extracts a share $\alpha \in (0, 1)$ of any surplus in the resale market. For simplicity, consider a second-price sealed bid auction with reserve price $\alpha/2$ in the first stage. There are

no entrant buyers arriving exogenously; however, the initial seller may exclude bidders from the initial auction, forcing them into the resale market.

Proposition 1. *For any $n \geq 3$, the seller strictly prefers to exclude a bidder from the auction iff $\alpha > \frac{2}{3}$.*

Proof: If all n buyers participate in the first-stage auction Corollary 1 implies that each bidder bids his use value, so the seller's expected revenue R_n is the expected value of the second highest use value, $\frac{n-1}{n+1}$. If one of the bidders is excluded at random, this creates an opportunity for the winner from among the remaining $n - 1$ bidders to sell in the second stage. In this case

$$\begin{aligned} w(x, x, x) &= xF(x) + \int_x^1 [x + \alpha(z - x)]f(z) dz \\ &= x(1 - \alpha) + \frac{\alpha}{2}(1 + x^2). \end{aligned}$$

Since $n \geq 3$ and the reserve price $\frac{\alpha}{2}$ equals $w(0, 0, 0)$, the seller's expected revenue is

$$\begin{aligned} R_{n-1} &= \int_0^1 \left[x(1 - \alpha) + \frac{\alpha}{2}(1 + x^2) \right] (n - 1)(n - 2)f(y)(1 - F(y))F(y)^{n-3} dy \\ &= \frac{n^2 - n - 2 + 3\alpha}{n(n + 1)} \end{aligned}$$

which exceeds $\frac{n-1}{n+1}$ iff $\alpha > \frac{2}{3}$. □

5.3 Choice of Auction

For auctions without resale, there are well known results ranking expected revenues from different auction forms (e.g., Milgrom and Weber (1982)). The forces behind those results are present in the current model but can be confounded by the multiplicity of equilibria for the first-price auction and informational linkages between the auction and the resale market. To focus on these issues, I address only the case of independent types, where revenue equivalence holds without resale. Extensions to the case of affiliated values then follow standard arguments (Haile (1996)), although often with ambiguous predictions.

Theorem 9. *If all types are independent, the seller's expected revenue in a given equilibrium of a first-price auction with reserve price r^f , defined by the selection of x_0^f , is identical to that obtained in equilibrium from a second-price auction with a reserve price r^s such that $x_0^s = x_0^f$.*

Proof: See Appendix A.

It is useful to abstract from cases in which the beliefs B_{ji} affect i 's payoff from winning for some realizations of types and not others, since such possibilities complicate the exposition without adding insight. Consider instead the following two cases:

belief dependence. *If \mathcal{B} and $\hat{\mathcal{B}}$ are identical except that $\hat{\mathcal{B}}_{ji}$ strictly first-order stochastically dominates \mathcal{B}_{ji} for all $j \neq i$, then $W(X_i, \mathbf{Y}, \mathbf{Z}, \hat{\mathcal{B}}) > W(X_i, \mathbf{Y}, \mathbf{Z}, \mathcal{B})$.*

belief independence. *$W(X_i, \mathbf{Y}, \mathbf{Z}, \mathcal{B})$ does not vary with \mathcal{B}_{ji} for any $j \neq i$.*

Viewing x_0^s and x_0^f as functions of the reserve price, belief independence implies $x_0^f(r) = x_0^s(r)$, so that revenue equivalence holds for a fixed reserve price. With belief dependence, however, $x_0^f(r) > x_0^s(r)$ generically, so that the revenue equivalence obtained in Theorem 9 is not the standard sort. Indeed, Theorem 9 implies that with independent types and a given reserve price, the first-price auction may yield higher or lower expected revenues. To see this, let r^* be an optimal reserve price for the second-price auction, defined as

$$\arg \max_r \quad r n F(x_0^s(r))^{n-1} (1 - F(x_0^s)) + \int_{x_0}^1 \theta(y) n (n-1) f(y) (1 - F(y)) F(y)^{n-2} dy.$$

Corollary 2. *Suppose types are independent and that for reserve prices in a neighborhood of r^* an equilibrium in symmetric separating bidding strategies exists for the second-price auction. Under belief dependence there exists a reserve price r^f (r^s) such that there is an equilibrium of the first-price auction yielding revenues at least as high as (no higher than) those from the second-price auction.*

Proof: Theorem 5 implies that for r^f below but close to r^* , there is an equilibrium of the first-price sealed bid auction in which $x_0^f(r^f) = x_0^s(r^*)$. Theorem 9 and optimality of r^* imply that with a reserve price of r^f , revenues in this equilibrium of the first-price auction are higher than those from a second-price auction with reserve price r^f —strictly higher in the “regular” case in which revenues are strictly concave in the reserve price. Similarly, when $r^s = r^*$, revenues from a first-price sealed bid auction with reserve price r^s will generically be lower (strictly lower in the when r^* is a unique optimum) than those from the second-price auction. \square

For different reasons, revenue equivalence between English and second-price sealed bid auctions can also break down. However, with independent types and belief independence, all differences between the English and second-price sealed bid auctions disappear. In particular, $\psi(x) = 0$ for any x and

$$w(x, y, \tilde{x}) = E[w^e(X_i, \Omega(Y_1, D_{n-2})) | X_i = x, Y_1 = y] \quad \forall x, y, \tilde{x}.$$

In this case, the bidding strategies derived above for all three auctions are exactly as if bidders had independent private values given by

$$\Theta_i = w(X_i, X_i, X_i)$$

in an auction without resale.²² Hence, revenue equivalence follows from standard results (Myerson (1981), Riley and Samuelson (1981)).

Theorem 10. *With belief independence and independent types, the seller’s expected revenue is the same at any of the three auctions.*

²² Even with belief dependence, each type above x_0^s in a second-price auction with independent types bids as if he were in an IPV auction without resale, with private values $\Theta_i = \theta(X_i)$. In a first-price auction with independent types this equivalence fails only due to the boundary condition, which is not $b(x_0^f) = \theta(x_0^f)$ but $b(x_0^f) = E[w(x_0^f, y, x_0^f) | y \leq x_0^f]$. However, the results below will demonstrate important differences across auctions in the case of independence.

Under belief dependence, however, there are two important differences between an English auction and a second-price sealed bid auction, even with independent types. First, $\psi(x) > 0$ for all x , raising bids in the sealed bid auction, whereas there is no signaling in an English auction. Second, belief dependence implies that the inequality in Lemma 4 is strict: bidders place a strictly higher value on winning an English auction than winning a sealed bid auction, due to the greater information rents extracted by the resale seller when the first-stage auction does not fully reveal his type. Hence, with belief dependence, bidders have higher endogenous valuations at an English auction but raise their bids above their valuations in a second-price auction. This makes the revenue ranking of the English and second-price auctions ambiguous. Let R^e denote the expected revenue from an English auction and define

$$f_1(y) = n(n-1)f(y)F(y)^{n-2}(1-F(y)).$$

Then

$$\begin{aligned} R^e &= r n F(x_0^e)^{n-1}(1-F(x_0^e)) + \int_{x_0^e}^1 E[w(y, \Omega(y, D_{n-2}), y)|y]f_1(y) dy \\ &> r n F(x_0^s)^{n-1}(1-F(x_0^s)) + \int_{x_0^s}^1 E[w(y, \Omega(y, D_{n-2}), y)|y]f_1(y) dy \\ &> r n F(x_0^s)^{n-1}(1-F(x_0^s)) + \int_{x_0^s}^1 w(y, y, y)f_1(y) dy \\ &< r n F(x_0^s)^{n-1}(1-F(x_0^s)) + \int_{x_0^s}^1 [w(y, y, y) + \psi(y)]f_1(y) dy \\ &= R^s. \end{aligned}$$

The first and second inequalities are implied by Lemma 4 and the last inequality by the fact that $\psi(x) > 0$ under belief dependence. Haile (1996) provides examples showing that the revenue ranking can go either way.

6 Empirical Implications: Tests for Common Values

The differences between bidding with and without a resale market have other important empirical implications as well. Two of these concern tests proposed to distinguish auctions with

private values from those with common values. There are well known results for auctions without resale regarding the effect of the number of bidders on valuations. Recall that in the Milgrom-Weber (1982) model of auctions without resale, the function $v(X_i, Y_1)$ represents the expected value of winning the auction. This expectation is unaffected by n in a private values auction ($v(X_i, Y_1) = X_i$ w.l.o.g.) but decreases in n in a common value auction, due to a more severe winner's curse when n is large. Such predictions are often suggested as empirical tests of the theory or as a means of distinguishing common and private values (see, for example, Hendricks and Porter (1999)). By recovering the underlying valuations $v(X_i, X_i)$ from a sample of observed bids in sealed bid auctions,²³ one can examine whether and how $v(x, x)$ varies with n .²⁴ When there are no incentives for bidders to signal through their bids, similar results hold in the present model.²⁵ As noted previously, however, in the present model the analog of $v(x, x)$ is $\theta(x)$. Hence applying the same empirical techniques leads to tests of variation in $\theta(x)$ with n . With belief dependence, $\theta(x)$ will decline in n even with independent types.²⁶

Theorem 11. *Suppose types are independent and that $w(x, y, \tilde{x})$ does not increase with n when $y \leq x$. Under belief dependence, $\theta(x)$ decreases in n .*

²³ This is trivial in a second-price auction, where $b(x) = v(x, x)$, but can be for first-price auctions done using techniques developed by Guerre, Perrigne, and Vuong (1995) and Li, Perrigne, and Vuong (1996).

²⁴ See also Hendricks, Porter, and Pinske (1999).

²⁵ See Haile (1996). However, Haile (1999b) shows that even without signaling these results can be overturned if bidders observe private values with noise at the time of the auction.

²⁶ The theorem assumes $w(x, y, \tilde{x})$ does not increase in n when $y \leq x$. An assumption that $w(x, y, \tilde{x})$ not vary at all with n when $y \leq x$ is natural with independent types, since in equilibrium it is common knowledge that no losing bidder will buy in the resale market. In a model of resale bargaining based on costly random matching, adding losing bidders (i.e., increasing n) could *reduce* the expected payoff $w(x, y, x)$ when $y \leq x$.

Proof:

$$\begin{aligned}
\frac{\partial \theta(x)}{\partial n} &= \frac{\partial w(x, x, x)}{\partial n} + \frac{\partial \psi(x)}{\partial n} \\
&= \frac{\partial}{\partial n} \int_0^x w_3(x, y, x) \frac{f_1(y|x)}{f_1(x|x)} dy \\
&= \int_0^x w_3(x, y, x) \frac{f(y)}{f(x)} \frac{\partial}{\partial n} \left\{ \left(\frac{F(y)}{F(x)} \right)^{n-2} \right\} dy \\
&< 0.
\end{aligned}$$

□

This should not be taken as support for the conjecture that resale introduces elements of common values to an auction. The dependence of $\theta(x)$ on n has nothing to do with a winner's curse, but results entirely from the signaling component $\psi(x)$. In fact, as already noted, bidding in a second-price auction with independent types, a fixed number of bidders, and a resale opportunity is identical to that in an auction without resale where bidders have independent private values $\Theta_i = \theta(X_i)$. Hence Theorem 11 suggests that this test can incorrectly suggest the presence of a winner's curse and, therefore, common values if applied to auctions with resale opportunities.

Hendricks, Pinkse, and Porter (1999) have recently proposed an alternative test for common values based on the observation that with a binding reserve price, the valuation of the lowest type to participate in an affiliated values auction without resale lies strictly above the reserve price except in a private values auction. This implies that a test for common values can be constructed by examining the lower end of the distribution of the valuations $v(x, x)$ inferred from bids at a sealed bid auction. In a second-price sealed bid auction, for example, this implies that the lowest equilibrium bid lies strictly above the reserve price in a common value auction but equals the reserve in a private auction.²⁷ If there is no signaling incentive, such a test will be valid in the model here as well. However, when equilibrium bidding involves signaling, a gap between $\theta(x)$ (the object interpreted as a bidder's valuation $v(x, x)$ if one incorrectly

²⁷ This observation was first made in Milgrom and Weber (1982). Hendricks, Pinske, and Porter's (1999) test is developed for offshore mineral lease auctions, where resale is rare (Porter (1995)).

assumes an affiliated values auction without resale) and the reserve price will exist even with independent private values, since for $a \in \{f, s\}$

$$w(x_0^a, x_0^a, x_0^a) + \psi(x_0^a) \geq r + \psi(x_0^a) > r.$$

This proves the following result.

Theorem 12. *With independent types and belief dependence, $\theta(x_0^a) > r$ for $a \in \{f, s\}$.*

7 Conclusion

This paper has examined a model of affiliated values auctions with resale opportunities. The results show that standard (static) auction models, regardless of their information structures, will often miss important aspects of bidding strategies and resulting implications for policy. The endogeneity of bidder valuations and the information linkages between primary and secondary markets can preclude existence of a symmetric separating equilibrium in a second-price auction, introduce a continuum of equilibria in a first-price auction, reverse standard predictions regarding the seller's revenue, and invalidate empirical implications based on models excluding a resale opportunity. Hence this work has important implications for our understanding of price formation in auctions and other markets with asymmetric information, for the implementation of auction theory in the design of markets, and for the growing volume of empirical work on auctions.

Appendix A Proofs Omitted from the Text

Proof of Theorem 1 (first-price sealed bid auction): Without resale, bidders use the equilibrium bid function $b(\cdot)$, where

$$b(x) = \begin{cases} \frac{rF_1(r) + \int_r^x y dF_1(y)}{F_1(x)} & x \geq r \\ \text{"no bid"} & x < r \end{cases} \quad (5)$$

with $F_1(\cdot)$ giving the distribution of Y_1 . If all bidders follow $b(\cdot)$ in the extended game, the bidder with the highest use value will win and there will be no resale trade. Thus, a bidder with use value $X_i = x \geq r$ who bids according to (5) receives expected payoff

$$\pi(x, x) = (x - b(x)) F_1(x).$$

Any bid above $b(1)$ is dominated by $b(1)$. So suppose bidder i with use value $x \geq r$ deviates to a bid $b(\tilde{x})$, $\tilde{x} \in [r, 1]$. Let $\pi(x, \tilde{x})$ give his payoff. The best i could do is extract all gains from trade in the resale market; thus

$$\begin{aligned} \pi(x, \tilde{x}) &\leq (x - b(\tilde{x})) F_1(\tilde{x}) + \mathbf{1}\{\tilde{x} \geq x\} \int_x^{\tilde{x}} (y - x) dF_1(y) + \mathbf{1}\{\tilde{x} < x\} \int_{\tilde{x}}^x (x - y) dF_1(y) \\ &= (x - b(\tilde{x})) F_1(\tilde{x}) + \int_x^{\tilde{x}} (y - x) dF_1(y) \\ &= xF_1(x) - b(\tilde{x}) F_1(\tilde{x}) + \int_r^{\tilde{x}} y dF_1(y) - \int_r^x y dF_1(y) \\ &= (x - b(x)) F_1(x) \\ &= \pi(x, x) \end{aligned}$$

where $\mathbf{1}\{\cdot\}$ is an indicator function. Hence the deviation is not profitable. Suppose instead that i deviates to a bid below r . Since the object will go unsold if no opponent has a use value greater than r , the most i could obtain is

$$\begin{aligned} \int_r^x (x - y) dF_1(y) &\leq xF_1(x) - rF_1(r) - \int_r^x y dF_1(y) \\ &= \pi(x, x). \end{aligned}$$

Finally, suppose a bidder with use value $x < r$ bids $b(\tilde{x}) \geq r$. If this bid wins, the greatest payoff he could obtain is

$$(x - b(\tilde{x})) F_1(x) + \int_x^{\tilde{x}} (y - b(\tilde{x})) dF_1(y).$$

Integrating by (recalling (5)) leaves the expected payoff

$$- \int_x^r F_1(y) dy \leq 0. \quad \square$$

Proof of Lemma 1: Suppose $x_0^s > \underline{x}$. Then types $\hat{x} \in (\underline{x}, x_0^s)$ do not bid in equilibrium and receive payoff zero. For small $\epsilon > 0$, deviation to $b^s(x_0 + \epsilon)$ would give such a type a payoff arbitrarily close (by DIF) to

$$\begin{aligned} \int_0^{x_0^s} [w(\hat{x}, y, x_0^s) - r] dF_1(y|\hat{x}) &= F_1(x_0^s|\hat{x}) \int_0^{x_0^s} [w(\hat{x}, y, x_0^s) - r] \frac{f_1(y|\hat{x})}{F_1(x_0^s|\hat{x})} dy \\ &> F_1(x_0^s|\hat{x}) \int_0^{x_0^s} [w(\underline{x}, y, \underline{x}) - r] \frac{f_1(y|\hat{x})}{F_1(x_0^s|\hat{x})} dy \\ &\geq F_1(x_0^s|\hat{x}) \int_0^{x_0^s} [w(\underline{x}, y, \underline{x}) - r] \frac{f_1(y|\underline{x})}{F_1(x_0^s|\underline{x})} dy \\ &\geq 0. \end{aligned}$$

where the first inequality follows from MON, the second from MON and affiliation, and the final inequality from MON and the definition of \underline{x} . So suppose instead that $x_0^s < \underline{x}$. Then a bidder with type $\hat{x} \in (x_0^s, \underline{x})$ bids at least r and receives payoff

$$\int_0^{x_0^s} [w(\hat{x}, y, \hat{x}) - r] dF_1(y|\hat{x}) + \int_{x_0^s}^{\hat{x}} [w(\hat{x}, y, \hat{x}) - b(y)] dF_1(y|\hat{x}).$$

For \hat{x} close to x_0^s this is strictly negative by MON, affiliation, and the definition of \underline{x} . \square

Proof of Theorem 3: Since

$$\pi_2(x, \tilde{x}) = [v(x, \tilde{x}, \tilde{x}) - b(\tilde{x})] f_1(\tilde{x}|x)$$

MON and the first-order condition $\pi_2(\tilde{x}, \tilde{x}) = 0$ imply that $\pi_2(x, \tilde{x}) > 0$ for $\tilde{x} < x$ and $\pi_2(x, \tilde{x}) \leq 0$ for $\tilde{x} > x$. \square

Proof of Lemma 2: Suppose that at $x = x_0^f$

$$b(x) > \int_0^x w(x, y, x) \frac{f_1(y|x)}{F_1(x|x)} dy. \quad (6)$$

Since both sides of the inequality are continuous in x , this implies that for types \hat{x} just above x_0^f ,

$$b(\hat{x}) > \int_0^{\hat{x}} w(\hat{x}, y, \hat{x}) \frac{f_1(y|\hat{x})}{F_1(\hat{x}|\hat{x})} dy.$$

implying that a type \hat{x} bidder receives a negative equilibrium payoff. If instead the inequality (6) is reversed at $x = x_0^f$, DIF implies that for \hat{x} just above x_0^f

$$b(\hat{x}) < \int_0^{\hat{x}} w(x_0^f, y, \hat{x}) \frac{f_1(y|x_0^f)}{F_1(\hat{x}|x_0^f)} dy. \quad (7)$$

A bidder with type $x_0^f - \epsilon$ could then deviate to a bid of $b(\hat{x})$ and obtain payoff

$$\int_0^{\hat{x}} \left[w(x_0^f - \epsilon, y, \hat{x}) - b(\hat{x}) \right] dF_1(y|x_0^f - \epsilon)$$

which, by DIF and (7), is strictly positive for ϵ sufficiently small. \square

Proof of Lemma 3: Splitting $\theta(t)$ into its components and integrating (4) by parts (noting that $\eta'(t|x) = \frac{f_1(t|t)}{F_1(t|t)}\eta(t|x)$) shows that for $x > x_0^f$

$$\begin{aligned} w(x, x, x) - b^f(x) &= \eta(x_0^f|x) \left[w(x_0^f, x_0^f, x_0^f) - b^f(x_0) \right] \\ &\quad + \int_{x_0^f}^x \eta(t|x) \left\{ \frac{\partial}{\partial t} w(t, t, t) - \int_0^t w_3(t, y, t) \frac{f_1(y|t)}{F_1(t|t)} dy \right\} dt \\ &\geq \int_{x_0^f}^x \eta(t|x) \left[w_1(t, t, t) + w_2(t, t, t) + w_3(t, t, t) - \frac{\int_0^t w_3(t, y, t) dF_1(y|t)}{F_1(t|t)} \right] dt \\ &> 0. \end{aligned}$$

Here the first inequality follows from Lemma 3 while the strict inequality follows from MON and SM. \square

Proof of Theorem 5: Differentiation of (3) give the first-order condition

$$b'(x) = [v(x, x, x) + \psi(x) - b(x)] \frac{f_1(x|x)}{F_1(x|x)}. \quad (8)$$

MON implies $\psi(x) \geq 0 \forall x$; hence SYM and Lemma 3 ensure that the right side of (8) is strictly positive for all $x > x_0^f$. This ensures that the solution to this differential equation, $b^f(\cdot)$, is

strictly increasing. To show that $b^f(\cdot)$ is an optimal bidding strategy for types $x > x_0^f$, let $b(\cdot)$ denote $b^f(\cdot)$ and note that for $\tilde{x} \in [x_0^f, x)$, optimization by type \tilde{x} requires

$$\begin{aligned}
0 &= \pi_2(\tilde{x}, \tilde{x}) \\
&= F_1(\tilde{x}|\tilde{x}) \left\{ [v(\tilde{x}, \tilde{x}, \tilde{x}) - b(\tilde{x})] \frac{f_1(\tilde{x}|\tilde{x})}{F_1(\tilde{x}|\tilde{x})} + \int_0^{\tilde{x}} w_3(\tilde{x}, y, \tilde{x}) \frac{f_1(y|\tilde{x})}{F_1(\tilde{x}|\tilde{x})} dy - b'(\tilde{x}) \right\} \\
&< F_1(\tilde{x}|\tilde{x}) \left\{ [v(x, \tilde{x}, \tilde{x}) - b(\tilde{x})] \frac{f_1(\tilde{x}|\tilde{x})}{F_1(\tilde{x}|\tilde{x})} + \int_0^{\tilde{x}} w_3(x, y, \tilde{x}) \frac{f_1(y|\tilde{x})}{F_1(\tilde{x}|\tilde{x})} dy - b'(\tilde{x}) \right\} \\
&\leq F_1(\tilde{x}|\tilde{x}) \left\{ [v(x, \tilde{x}, \tilde{x}) - b(\tilde{x})] \frac{f_1(\tilde{x}|x)}{F_1(\tilde{x}|x)} + \int_0^{\tilde{x}} w_3(x, y, \tilde{x}) \frac{f_1(y|x)}{F_1(\tilde{x}|x)} dy - b'(\tilde{x}) \right\} \\
&= \frac{F_1(\tilde{x}|\tilde{x})}{F_1(\tilde{x}|x)} \pi_2(x, \tilde{x}).
\end{aligned}$$

The first inequality follows from MON and SM, the second from affiliation and SM, and the final equality from BIL. Hence $\pi_2(x, \tilde{x}) > 0$ for any $\tilde{x} < x$. A similar argument shows that $\pi_2(x, \tilde{x}) \leq 0$ for $\tilde{x} > x$. Finally, note that Lemma 3 ensures that submitting no bid is strictly worse than following $b^f(\cdot)$ for types above x_0^f . Hence $b^f(\cdot)$ gives an optimal strategy for all types $x \geq x_0^f$.

The identity of the marginal type x_0^f must satisfy three additional constraints. First, x_0^f must be sufficiently large that $b(x_0^f)$ (given by Lemma 2) is at least the reserve price:

$$b(x_0^f) = \int_0^{x_0^f} w^f(x_0^f, y, x_0^f) \frac{f_1(y|x_0^f)}{F_1(x_0^f|x_0^f)} dy \geq r. \quad (9)$$

Hence the smallest possible x_0^f is \underline{x} . Second, if $x_0^f > \underline{x}$, so that $b(x_0^f) > r$, then there must be no types below x_0^f willing to submit a bid $\hat{b} \in [r, b(x_0^f))$ given the beliefs that such a bid would induce. Suppose that when a bidder submits any bid \hat{b} in this range, buyers in the second stage form beliefs regarding this bidder's type which are degenerate at some value \hat{x} . Given these beliefs, a bid of r will dominate any bid in $(r, b(x_0^f))$. Since

$$\int_0^{x_0^f} w(x, y, \hat{x}) \frac{f_1(y|x)}{F_1(x_0^f|x)} dy \quad (10)$$

is strictly increasing in x , this second constraint requires that for all $x < x_0^f$ (10) be no larger than r . Given MON and DIF, this implies that for $\hat{x} \in [0, \underline{x}]$

$$\phi(x_0^f; \hat{x}) \equiv \int_0^{x_0^f} w(x_0^f, y, \hat{x}) \frac{f_1(y|x_0^f)}{F_1(x_0^f|x_0^f)} dy = r \quad (11)$$

unless $\phi(1, \hat{x}) < r$, in which case $x_0^f = 1$. Otherwise since MON and affiliation imply that $\phi(x; \hat{x})$ strictly increases in x , (11) defines a unique solution for x_0^f given any $\hat{x} \in [0, \underline{x}]$. Since the solution x_0^f must decrease in \hat{x} , the constraint (9) will be slack except at $\hat{x} = x_0^f$. Finally, a bidder with type $x < x_0^f$ must not want to bid $b^f(\tilde{x}) \geq b^f(x_0^f)$. If he did, his expected payoff would be

$$\begin{aligned} \pi(x, \tilde{x}) &= F_1(\tilde{x}|x) \int_0^{\tilde{x}} [w(x, y, \tilde{x}) - b^f(\tilde{x})] \frac{f_1(y|x)}{F_1(\tilde{x}|x)} dy \\ &< F_1(\tilde{x}|x) \int_0^{\tilde{x}} [w(x_0^f, y, \tilde{x}) - b^f(\tilde{x})] \frac{f_1(y|x_0^f)}{F_1(\tilde{x}|x_0^f)} dy \\ &= \frac{F_1(\tilde{x}|x)}{F_1(\tilde{x}|x_0^f)} \pi(x_0^f, \tilde{x}) \\ &\leq \frac{F_1(\tilde{x}|x)}{F_1(\tilde{x}|x_0^f)} \pi(x_0^f, x_0^f) \\ &= 0 \end{aligned}$$

where the strict inequality follows from MON and affiliation, the weak inequality from optimization by type x_0^f , and the final equality from Lemma 2. Hence there are no profitable deviations. \square

Proof of Lemma 4: Suppose the next-to-last bidder in the English auction bids $b_{n-2}^e(x, D_{n-2})$.

If i is the winner, \mathcal{B}_i is given by

$$\mathcal{B}_i^e(t) = \begin{cases} \frac{F(t)}{1-F(x)} & t \geq x \\ 0 & t < x. \end{cases}$$

After a sealed bid auction that bidder i wins with bid $b^s(x)$ \mathcal{B}_i is given by

$$\mathcal{B}_i^s(t) = \begin{cases} 1 & t \geq x \\ 0 & t < x. \end{cases}$$

The result then follows from MON. \square

Proof of Theorem 6: Suppose the sequence of bid functions $b_k(\cdot, \cdot), k = 0, \dots, n-2$ gives the equilibrium bidding strategies for each phase k . Given D_k , a bidder with type $x > x_0^e$ who plans to stay in the auction up to price $b_k(\tilde{x}, D_k)$ in phase k has expected payoff

$$\begin{aligned} \pi^k(x, \tilde{x}) &\equiv \int_{b_{k-1}^{-1}(d_k; D_{k-1})}^{\tilde{x}} [w^e(x, \Omega(y, D_k)) - b_k(y, D_k)] d\hat{F}_1(y|x, D_k) \\ &\quad + \int_{\tilde{x}}^1 \ell^e(x, \Omega(y, D_k), \tilde{x}) d\hat{F}_1(y|x, D_k) \end{aligned}$$

where

$$\hat{F}_1(y|x, D_k) \equiv \begin{cases} \frac{F_1\left(y \mid x, b^{-1}(d_k; D_{k-1}), \dots, b^{-1}(d_1; D_0)\right)}{1 - F_1\left(b_k^{-1}(d_k; D_{k-1}) \mid x, b^{-1}(d_k; D_{k-1}), \dots, b^{-1}(d_1; D_0)\right)} & k > 0 \\ F_1(y|x) & k = 0 \end{cases}$$

and

$$b_k^{-1}(d_k; D_{k-1}) = \inf\{x \in [0, 1] : b_{k-1}(x; D_{k-1}) \geq d_k\}.$$

Differentiating with respect to \tilde{x} , setting $\tilde{x} = x$, and recalling SYM gives the first-order condition

$$b_k^e(x, D_k) = w^e(x, \Omega(x, D_k))$$

as the unique candidate equilibrium bid function. □

Proof of Theorem 7: MON and BIL imply that

$$\pi_2^k(x, \tilde{x}) = f_1(\tilde{x}|x) [v^e(x, \Omega(\tilde{x}, D_k), \tilde{x}) - b^k(\tilde{x}, D_k)]$$

is positive for $\tilde{x} < x$ and nonpositive for $\tilde{x} > x$, ensuring that the bid function $b_k(\cdot, \cdot)$ gives a best response for all types above x_0^e in each phase. MON also implies that $v^e(x, \Omega(x, D_k), x)$ is strictly increasing in x and that for any equilibrium realization of $\{d_1, \dots, d_k\}$

$$d_k \leq b_{k+1}^e(x, D_k) \quad \forall x, k;$$

i.e., no bidder will remaining in the auction up to the beginning of any phase $k + 1$ using this strategy. Hence, equilibrium bidding within each phase and over the entire auction is strictly increasing, as assumed. □

Proof of Theorem 9: Let R^f and R^s denote the expected revenues from the first- and

second-price auctions, respectively, and let $x_0 = x_0^s = x_0^f$. Then, letting $\rho = 1 - F(x_0)^n$

$$\begin{aligned}
\frac{R^f}{\rho} &= E[b^f(X_1)|Y_1 \leq X_1, X_1 > x_0] \\
&= E\left[b^f(x_0) \Pr(Y_1 \leq x_0|Y_1 \leq X_1) \right. \\
&\quad \left. + \Pr(Y_1 > x_0|Y_1 \leq X_1)E[\theta(Y_1)|Y_1 \in (x_0, X_1)] \mid X_1 > x_0, Y_1 \leq X_1\right] \\
&= b^f(x_0) \Pr(Y_1 \leq x_0|X_1 > x_0, Y_1 \leq X_1) \\
&\quad + \Pr(Y_1 > x_0|X_1 > x_0, Y_1 \leq X_1)E[\theta(Y_1)|X_1 > x_0, Y_1 \in (x_0, X_1)] \\
&= E[w(X_i, Y_1, X_i)|X_i = x_0, Y_1 \leq x_0] \Pr(Y_1 \leq x_0|X_1 > x_0, Y_1 \leq X_1) \\
&\quad + \Pr(Y_1 > x_0|X_1 > x_0, Y_1 \leq X_1)E[b^s(Y_1)|X_1 > x_0, Y_1 \in (x_0, X_1)] \\
&= \frac{R^s}{\rho}.
\end{aligned}$$

□

Appendix B. Examples of Resale Market Structures

Example 1. Gains to Trade Divided According to Shapley Values

Suppose gains to trade in the resale market are divided according to Shapley (1953) values. Let U_1, \dots, U_{n+m} represent the realizations of all use values and \mathbf{U}_{-j} those other than player j 's. Assume that $U_i \equiv u_b(X_i, \mathbf{S})$ is strictly increasing in X_i .²⁸ Let N denote the set of all $n+m$ players and N_{-j} the subset that excludes player j . Conditional on the realization of all use values, a bidder i who wins the first-stage auction has expected payoff

$$\phi^w(U_i, \mathbf{U}_{-i}) = U_i + \sum_{C \subseteq N_{-i}} \frac{|C|!(|N| - |C| - 1)!}{|N|!} \max\{0, \max\{U_j, j \in C\} - U_i\}$$

where the second term is i 's Shapley value. If i is a losing bidder, let w denote the index of the first-stage winner. Then i 's Shapley value (his payoff in the resale market) is

$$\phi^\ell(U_i, \mathbf{U}_{-i}) = \sum_{C \subseteq N_{-i}} \frac{|C|!(|N| - |C| - 1)!}{|N|!} \mathbf{1}\{w \in C\} \max\{0, U_i - \max\{U_j, j \in C\}\}.$$

Hence,

$$w(x, y, \tilde{x}) = E[\phi^w(U_i, \mathbf{U}_{-i}) | X_i = x, Y_1 = y] \quad (12)$$

and

$$\ell(x, y, \tilde{x}) = E[\phi^\ell(U_i, \mathbf{U}_{-i}) | X_i = x, Y_1 = y]. \quad (13)$$

Any buyer in the resale market with use value below that of the first-stage winner has a Shapley value of zero. Hence, SYM is satisfied. Furthermore, because beliefs have no role in determining resale outcomes, SM, BIL, and part (iii) of MON and DIF are all trivially satisfied. Parts (i) and (ii) of DIF follow from differentiability of $\phi^w(U_i, \mathbf{U}_{-i})$ and the assumption of continuous marginal densities, which implies that the conditional density underlying the expectation (12) is continuous in the conditioning variables. Affiliation of Y_1, \mathbf{Y}_{-1} and \mathbf{Z} ensures that $w(x, y, \tilde{x})$ is nondecreasing in y while $\ell(x, y, \tilde{x})$ is nonincreasing in y , so that $v(x, y, \tilde{x})$ is nondecreasing in y . Note that

$$\frac{\partial}{\partial U_i} \phi^w(U_i, \mathbf{U}_{-i}) = 1 - \sum_{C \subseteq N_{-i}} \frac{|C|!(m+n-|C|-1)!}{(m+n)!} \mathbf{1}\{\max\{U_j, j \in C\} > U_i\}$$

which is positive, giving part (ii) of MON. Since

$$\frac{\partial}{\partial U_i} \phi^\ell(U_i, \mathbf{U}_{-i}) = \sum_{C \subseteq N_{-i}} \frac{|C|!(m+n-|C|-1)!}{(m+n)!} \mathbf{1}\{\max\{U_j, j \in C\} \leq U_i, w \in C\}$$

we know

$$\frac{\partial}{\partial U_i} [\phi^w(U_i, \mathbf{U}_{-i}) - \phi^\ell(U_i, \mathbf{U}_{-i})] > 0.$$

Since

$$v(x, y, \tilde{x}) = E[\phi^w(U_i, \mathbf{U}_{-i}) - \phi^\ell(U_i, \mathbf{U}_{-i}) | X_i = x, Y_1 = y]$$

this implies that $v(x, y, \tilde{x})$ strictly increases in x , ensuring that part (i) of MON holds.

²⁸ Any assumption that avoids the trivial case in which bidders' signals do not affect their willingness to pay at the auction will do.

Example 2. English Auction with an Optimal Reserve Price

Suppose use values are private ($u_b(X_i, \mathbf{S}) = X_i$ and $u_e(Z_i, \mathbf{S}) = Z_i$ for simplicity) and independently distributed. The second stage consists of an English auction with a reserve price chosen by the first-stage winner. After this second auction, no further transactions are possible. For simplicity, assume the “regular” case (Myerson (1981)), in which the distribution, $G(\cdot)$, of each entrant buyer’s type is such that

$$t - \frac{1 - G(t)}{g(t)}$$

increases in t . Let $r(x)$ denote the second-stage reserve price set by a winner with type x . Since the first-stage winner believes that only the entrant buyers could have use values above his own, this optimal reserve price must satisfy (Riley and Samuelson (1981), Myerson (1981))

$$r(x) = x + \frac{1 - G(r(x))}{g(r(x))}. \quad (14)$$

This reserve price will be strictly greater than x except at $x = 1$ and will increase in x . This ensures that SYM and part (ii) of DIF are satisfied.

Let $Z_{(1)}, \dots, Z_{(m)}$ denote the ordered (highest to lowest) values of Z_{n+1}, \dots, Z_{n+m} . Then

$$\begin{aligned} w(x, y, \tilde{x}) &= w^e(x, \Omega(y, D_k)) \\ &= E [\max \{X_i, \text{2nd highest of } \{Z_{(1)}, Z_{(2)}, Y_1, r(X_i)\}\} | X_i = x, Y_1 = y]. \end{aligned}$$

Hence $w(x, y, \tilde{x})$ is nondecreasing in x and y for all x, y, \tilde{x} . If $y < r(x)$,

$$\begin{aligned} w(x, y, \tilde{x}) &= xG(r(x))^m + r(x)m[1 - G(r(x))]G(r(x))^{m-1} \\ &\quad + \int_{r(x)}^1 z m(m-1)g(z)(1 - G(z))G(z)^{m-2} dz. \end{aligned}$$

(14) then implies that for $y < r(x)$

$$\frac{\partial w(x, y, \tilde{x})}{\partial x} = G(r(x))^m > 0$$

implying part (ii) of MON. When $y \geq r(x)$,

$$\begin{aligned} w(x, y, \tilde{x}) &= r(x)G(r(x))^m + \int_{r(x)}^y z m g(z)G(z)^{m-1} dz + y m [1 - G(y)] G(y)^{m-1} \\ &\quad + \int_y^1 z m(m-1)g(z)(1 - G(z))G(z)^{m-2} dz \end{aligned}$$

implying that for $y \geq r(x)$

$$\frac{\partial w(x, y, \tilde{x})}{\partial x} = r'(x)G(r(x))^m > 0. \quad (15)$$

This implies part (i) of DIF. If $x < r(y)$,

$$\ell(x, y, \tilde{x}) = \ell_1(x, y, \tilde{x}) = 0. \quad (16)$$

Since $r(x) \leq y$ implies $x < r(y)$, (15) and (16) imply that $v_1(x, y, \tilde{x}) > 0$ when $y \geq r(x)$.

When $x \geq r(y)$

$$\ell(x, y, \tilde{x}) = xG(x)^m - r(y)G(r(y))^m - \int_{r(y)}^x m z g(z)G(z)^{m-1} dz$$

which strictly decreases in y . This ensures that $v(x, y, \tilde{x})$ is nondecreasing in y for all x, y, \tilde{x} . Since for $y < r(x)$

$$\begin{aligned} \frac{\partial w(x, y, \tilde{x})}{\partial x} &= G(r(x))^m \\ &> G(x)^m \geq \frac{\partial \ell(x, y, \tilde{x})}{\partial x} \end{aligned}$$

$v_1(x, y, \tilde{x}) > 0$ for $y < r(x)$ giving part (i) of MON. Finally, because no beliefs regarding bidders' types affect outcomes in the resale market, SM, BIL, and part (iii) of DIF and MON are all trivially satisfied.

Example 3. A Competitive Resale Market with Pure Common Values

Consider a single-object version of Bikhchandani and Huang's (1989) model of the resale market for Treasury bills. Specifically, suppose $m \geq 2$ and that the value of the object to any of the entrant buyers is equal to S_0 , which is unknown. Entrant buyers have no private information. First-stage bidders have use values of zero but informative signals, X_i , of S_0 . The price in the resale market is determined by an auction or competitive market and is, therefore, equal to the expected value of S_0 conditional on all public information. This public information includes all bids as well as S_1 , a signal of S_0 announced by the seller or otherwise revealed exogenously after the auction. Thus, if player i bids b_i and $\{b_1, \dots, b_{n-1}\}$ is the ordered set of his opponents' first-stage bids, the resale price when $b(\cdot)$ is the equilibrium bid function is

$$E[S_0 | X_i = b^{-1}(b_i), Y_1 = b^{-1}(b_1), \dots, Y_{n-1} = b^{-1}(b_{n-1}), S_1]. \quad (17)$$

Since this price will always be higher than the winning bidder's use value, the object will always be sold. All signals are strictly affiliated and are *information complements* with respect to S_0 ; i.e., defining

$$H(X_i, Y_1, \dots, Y_{n-1}, S_1) = E[S_0 | X_i, Y_1, \dots, Y_{n-1}, S_1],$$

the cross-partial H_{kj} is nonnegative for all k and $j \in \{1, \dots, n+1\}$, $j \neq k$.²⁹

Here,

$$w(x, y, \tilde{x}) = E[E[S_0 | X_i = \tilde{x}, Y_1, \dots, Y_{n-1}, S_1] | X_i = x, Y_1 = y] \quad (18)$$

while

$$\ell(x, y, \tilde{x}) = 0 \quad \forall x, y, \tilde{x}. \quad (19)$$

(19) implies that SYM and BIL are trivially satisfied. Strict affiliation of $X_i, Y_1, \mathbf{Y}_{-1}, S_1$, and S_0 ensures that MON is satisfied. DIF follows from the assumed existence of continuous marginal densities, since this implies that the conditional density underlying (18) is continuous in the conditioning variables. Finally, SM is implied by the information complementarity assumption.

²⁹ See Bikhchandani and Huang (1989) for examples.

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Figure 1

