Auctions with Resale Markets: An Application to U.S. Forest Service Timber Sales

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When bidders anticipate an opportunity for resale trade, the value of winning an auction is determined in part by the option values of buying and selling in the secondary market. One implication is that a bidder’s willingness to pay at an auction increases with the expected level of competition between resale buyers. Empirical evidence from auctions of timber contracts supports this prediction and rejects standard models that ignore resale. The estimated effect is smaller after policy changes expected to diminish the prevalence of resale. Additional evidence supports the predicted presence of a common value element introduced by the resale opportunity. (JEL D44, D82, C52, L73)  

A great deal of attention in the economics literature has focused on auctions, both as important economic institutions in their own right and as models of centralized competitive markets. Auctions have clear rules that often are easily adapted to game-theoretic models, yielding insights into strategic issues likely to arise in more complicated market structures. Consequently, auctions have provided some of the most promising empirical applications of strategic models with asymmetric information.  

Empirical studies of auctions have been used to evaluate competing models, test the predictive value of game theory, estimate distributions of private information, determine which selling mechanisms generate higher revenues, test for collusive behavior, and evaluate the effects of asymmetric information.  

A potentially important factor in bidding that has been ignored in nearly all of the auction literature is the existence of a secondary market in which an auction winner can sell the object he has won. While such resale opportunities frequently exist (more frequently than not), empirical studies have either ignored them or relied on conjectures regarding the effects of resale. Recent theoretical work has shown that resale can fundamentally change the interpretation of bidding data, a seller’s optimal choice of auction, the effects of a reserve price, and even existence of a separating (i.e., efficient) equilibrium.  

Hence, empirical evidence supporting the presence and significance of the effects of resale will have important implications for many applications. This paper shows that a resale opportunity can reverse a key testable implication of models ignoring resale. The prediction is then tested on data from sales of timber harvesting contracts held by the U.S. Forest Service between 1974 and 1989.  

While several previous studies have examined bidding at timber auctions, none has...
considered the effects of resale opportunities. However, resale is likely to have been an important factor determining bids in these auctions. Forest Service contracts in this period typically allowed several years’ delay before harvesting, and logs were usually harvested near the end of a contract term. The specialized lumber mills bidding for these contracts were likely to have had imperfect signals of their idiosyncratic demand (or other determinants of the values they place on a tract) several years into the future. Hence, gains to resale trade were likely to arise as uncertainty was resolved in the period between the auction and the harvest deadline. In fact, subcontracting of logging and sale of harvested logs prior to processing were common, and firms sometimes transferred entire contracts.

In the theoretical model, this resolution of uncertainty after the auction (as well as the potential entry of new firms in the interim period) accounts for the existence of an active resale market. The key effect of the resale opportunity is the endogenous determination of bidder valuations. A firm’s willingness to pay at an auction depends not only on the value it would obtain from harvesting and processing the timber itself, but also on the opportunities to buy and sell in secondary markets. A firm’s option to later sell a contract it wins raises its willingness to pay at the auction—the resale seller effect. However, the opportunity to buy in the resale market reduces the importance (value) of winning the auction—the resale buyer effect. This key feature of the model drives the empirical prediction taken to the data. In particular, when the number of bidders is a signal of the likely competition between buyers in the resale market, increasing the number of bidders raises the expected surplus extracted by the seller in the secondary market, magnifying the resale seller effect. Similarly, added competition diminishes the resale buyer effect. These effects imply that an increase in the number of bidders raises the value of winning the auction.

Many models predict that the winning bid will increase with the number of bidders or even that competition makes bidders more aggressive; however, the prediction here concerns not the outcome of the auction nor the strategies followed in equilibrium, but a bidder’s willingness to pay, i.e., his valuation. Indeed, this prediction distinguishes this model from standard models of auctions without resale, where a bidder’s willingness to pay is either unaffected by the number of bidders (in a private values model) or declining in the number of bidders (with common values). The empirical analysis uses a structural model that reveals the predicted relationship between the number of bidders and the distribution of valuations. This result is found in two different geographic regions, with stronger effects before policy changes that effectively banned contract transfers and substantially reduced the expected gains from subcontracting.

While this paper includes new theoretical results, it is most closely related to the empirical literature investigating the predictions of strategic bidding models. Like the seminal work of Hendricks and Robert H. Porter (1988), the analysis reveals evidence inconsistent with equilibrium predictions of the simplest models but supporting those of a more complex model capturing important features of the actual bidding environment. A resale opportunity is a particularly pervasive feature of auction markets, and these results show that its effects on bidding can be significant in practice. Hence, the evidence here suggests that the effects of resale should be taken seriously in many other applications as well. Finally, the paper makes a methodological contribution by developing a structural empirical model of English auctions that allows incorporation of instrumental variable techniques and accounts for information inferred by bidders from opponents’ behavior during the auction. The latter feature also makes possible a test for the common value element introduced to the auction by the resale opportunity.

Section I provides an overview of U.S. Forest Service timber sales, focusing on features that motivate the specification of the model. The theoretical model is presented in Section II, where equilibrium bidding is characterized and testable predictions are derived. Section III provides a brief discussion of the data. Section IV presents the estimation approach and empirical
results. In Section V, I examine the robustness of the results to variations in distributional assumptions. Section VI concludes. Proofs are given in the Appendix.

I. U.S. Forest Service Timber Sales

The U.S. Forest Service manages the majority of federal timber lands in the United States and frequently sells harvesting contracts by auction. A contract requires the purchaser to remove all included timber from the tract within a specified time period, usually between two and six years. Lumber mills are the primary bidders and are highly specialized by species, grades of timber, and the end products they produce. Hence, while there typically are many mills near a tract being offered, only those with an appropriate specialization will bid.

I focus on oral (English) auctions, the sale method used most frequently. Before a sale the Forest Service conducts a “cruise” of the tract to prepare an appraisal of the contract value and set a reserve price. The cruise report is published when the sale is advertised. Bidders must submit sealed bids of at least the advertised reserve price to qualify to participate in the actual auction. However, these reserve prices are widely viewed as nonbinding. Bids at the auction are made for each species on a per-unit (thousand board-feet) basis; i.e., bids are unit prices. The winner is the firm making the highest total bid, based on these prices and the Forest Service volume estimates. After the auction, all bids become public information.

In the sales considered here, known as “scaled sales,” payments (other than a deposit) are not made by the winner until the harvest and are then based on actual timber volumes rather than the original Forest Service estimates. The Forest Service also insures bidders against price fluctuations by indexing payments to timber prices at the time of harvest. These practices leave little common uncertainty about the value of a contract and, more important, little room for private information regarding any common elements in the values firms place on a given contract. However, because bidders are specialized, they have private information about their own sales and inventories of end products, contracts for future sales, and inventories of uncut timber from private timber sales.

This information structure suggests that a private values model is most appropriate. As we will see below, with a resale market it is important to make a distinction between bidders’ valuations (the values they place on winning the auction) and their use values (the values they place on the contract, ignoring resale opportunities). With a resale market, valuations are endogenous and may contain common components even when use values are purely private. I model bidders’ use values as independent and private and focus the empirical analysis on a subset of data for which this assumption is most compelling a priori.

Because contract terms are generally much longer than the time needed to harvest a tract (Randal R. Rucker and Leffler, 1988; Cummins, 1994) and most harvesting occurs at the end of the contract term (U.S. Forest Service, 1995), bidders are likely to have only noisy estimates of their use values at the time of the auction. Sawmills are likely to be uncertain of the timing and volume of future demand for end products, their inventories from other sales, and, in some cases, even the production technology they will possess over a contract term spanning several years. Such uncertainty would have no effect in standard auction models. However, this uncertainty and its subsequent resolution provide a motivation for resale trade.

Resale of Forest Service contracts may take at least two different forms. First, in some cases, contracts could be transferred between firms. These “third-party transfers” appear to have

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4 Bidders sometimes conduct their own cruises of a tract before the auction, although usually not for the type of sale studied here, where (as discussed below) bids are effectively prices per unit (National Resource Management Corporation, 1997). Private cruises are much more common for “tree measurement” sales, where bidders make lump-sum payments at the auction, bearing considerable risk due to uncertainty over the actual volume of timber on the tract.

5 Haile (1996) includes a discussion of reasons for this and provides supporting evidence.

occurred for only a small share of the contracts sold.\footnote{See Haile (1996). Third-party transfers have always been prohibited except in approved circumstances. However, before 1981, Forest Service policy identifying these circumstances included vague guidelines allowing transfers which “protect the interest of the United States” (U.S. Forest Service, 1976, 1981). This left room for liberal interpretation and it was widely believed that firms were buying contracts on a “speculative” basis.} Second, and more important, is subcontracting. Purchasers have always been permitted to subcontract any amount (including all) of the required harvesting and/or production of the final timber products. Logging itself was frequently subcontracted, and purchasers often sold a large share of the harvested logs to other mills for processing (U.S. Forest Service, 1990; Baldwin et al., 1997).

A recession in 1980–1981 and the ensuing defaults on many Forest Service contracts resulted in several important policy changes. Since May 4, 1981, third-party transfers have been permitted only in extreme circumstances—a purchaser going out of business, leaving an area permanently, or being taken over by another firm (U.S. Office of the Federal Register, 1980, 1981). Other policy changes implemented at approximately the same time are likely to have affected subcontracting as well. Contract lengths were shortened considerably, reducing the informational motivation for resale trade. Among the types of sales considered here, the average contract length declined from 55 months before 1981 to 33 months after. The Forest Service also began requiring larger deposits, raised the penalties for default, and made it more difficult to obtain contract extensions (Mead et al., 1983). If these changes achieved their intended effect of reducing the attractiveness of speculative bidding on Forest Service contracts, they should weaken the observed impacts of resale opportunities on bidding. Hence, this change in policy regime provides an opportunity to sharpen the empirical tests of the model below.

**II. The Model**

Consider a two-stage game in which an auction is followed by an opportunity for resale trade. In the first stage, \( n \) risk-neutral firms compete for a contract in an English auction with a reserve price of zero. Each bidder \( i \) has a noisy signal \( X_i \in [0, 1] \) of his use value \( U_i \in [0, 1] \).\footnote{For clarity I refer to each player as “he” despite the fact that bidders are actually firms.} After the auction, each bidder learns his own use value. In the second stage, the winning bidder holds another English auction without an announced reserve price, but is free to reject the high bid and keep the contract himself. There are \( m \) bidders at this resale auction, including the \( n - 1 \) losing bidders from the original sale and any firms that have entered in the interim.\footnote{Strictly speaking, the model does not allow bidders to exit, although any probability less than one can be placed on a bidder’s having a use value arbitrarily close to zero in the second stage.} Each of the \( m + 1 \) firms knows its use value at the resale auction. I assume \( n \) and \( m \) are strictly affiliated, implying that \( n \) is an informative signal of \( m \).\footnote{One plausible possibility is that only losing bidders will be interested in buying in the resale market, implying \( m = n - 1 \). See Paul R. Milgrom and Robert J. Weber (1982) for a discussion of affiliation. The key implication of strict affiliation here is that given any (strictly) increasing function \( \kappa(m) \), \( E[\kappa(m) \ln n] \) is (strictly) increasing in \( n \).} After the resale auction, the holder of the contract, firm \( j \) say, carries out the contractual requirements and obtains payoff \( u_j - p \), where \( p \) is the price he paid to obtain the contract.

All use values for a given contract are drawn independently from a distribution \( G(\cdot) \) while each bidder’s signal has unconditional distribution \( F(\cdot) \). Conditional on his private signal \( X_i \), the distribution of bidder \( i \)’s use value is given by \( G(\cdot | X_i) \), with support \([0, 1]\) and associated density \( g(\cdot | X_i) \). For simplicity I assume that 

\[
\int_0^1 G(u|t) f(t) \, dt = \int_0^1 g(u|t) f(t) \, dt 
\]

is nondecreasing in \( u \in (0, 1) \) for all \( x \in [0, 1] \). Each \( X_i \) is strictly affiliated with \( U_i \) (i.e., \( g(\cdot | \cdot) \) has the strict monotone likelihood ratio property) but independent of \( U_j \) and \( X_j \) for all \( j \neq i \). Hence, without the resale market, bidders in the first-stage auction would have independent private values \( V_i = E[U_i | X_i] \).

Let \( Y_j \) denote the \( j \)th highest signal among a representative bidder’s first-stage opponents and define \( Y = \{Y_1, \ldots, Y_{n-1}\} \) and \( Y_{-1} = Y \backslash Y_1 \). Let \( \{U_1, \ldots, U_m\} \) denote the set (in descending order) of use values of a given play-
er’s second-stage opponents and let $G_{j,m}(\cdot | Y)$ be the distribution of $U_j$ conditional on information $Y$. Finally, let $H_{1,m}(\cdot | Y)$ represent the conditional (on information $Y$) distribution of the highest use value among a given bidder’s second-stage opponents excluding the opponent who had the highest first-stage signal.

While the first-stage game is obviously motivated by the application, there are many ways one might model the resale market. The second-stage game considered here may be a reasonable model of multilateral bargaining in some contexts; however, it is not intended to be descriptive. This specification is meant to provide a relatively simple strategic model of the secondary market that captures important features common to many market structures. Indeed, the analysis below relies on only a few natural properties of the resale game.

A. The Resale Auction

I restrict attention to perfect Bayesian equilibria of the two-stage game in symmetric strictly increasing bidding strategies. Hence, I first address equilibrium outcomes in the second-stage continuation game. Following Milgrom and Weber (1982), I model each English auction as a “button auction” taking place in a sequence of phases. An auction begins in phase 0 and enters a new phase each time a bidder exits. For simplicity I retain Milgrom and Weber’s assumption that exits are observable and irreversible. However, I do not assume the auction ends immediately when the next-to-last bidder exits. Usually such an assumption is without loss of generality. Here the assumption would bite in the second-stage auction, where the first-stage winner will reject the high bid if it is less than his own use value. Realizing this, the last remaining bidder in the resale auction effectively must choose an ultimatum offer with a constraint that this be no lower than the price $b_0$ at which his last opponent dropped out.

His optimal offer depends on his beliefs regarding the first-stage winner’s type. We will see below that the first-stage winner’s signal $x_w$ is never revealed by his behavior in the first stage. However, a lower bound $x_w$ on this signal will be inferred from the price the winner pays, which reveals the second highest signal in equilibrium. Therefore, all second-stage bidders have beliefs on the resale seller’s use value given by the distribution $G(\cdot | x_w) = \int_{x_w}^{1} G(\cdot | x_w) \frac{f(x_w)}{1 - F(x_w)} dx_w$. The final bidder in the second stage chooses an ultimatum offer $b$ to solve

$$\max_{b \geq b_0} (u - b) \tilde{G}(b|x_w).$$

The solution is $b^* = u - \frac{\tilde{G}(b^*|x_w)}{\bar{g}(b^*|x_w)}$.

We will see below that this solution is unique. Let $u^*(u; t) = \sup\{s \in [0, 1]: b^*(s; t) \leq u\}$ give the lowest type whose optimal offer is no larger than $u$ when the bound $x_w = t$ is observed.

Analysis of the earlier phases of the second-stage auction is straightforward. In each phase a bidder’s dominant strategy is to remain in the auction as long as his continuation value is positive, i.e., as long as the price remains below his use value. Let $\Sigma$ represent the equilibrium strategies for the entire second-stage continuation game. These strategies define bids $b_i^k$ for each player $i = 1 \ldots m + 1$ for each phase $k$ of the second-stage auction, as well as a cutoff value $a_i$ defining the offers $i$ will accept when he is the seller in the second stage (he accepts any offer above his use value). Hence

$$b_i^k = u_i, \quad \forall \ k < m - 1$$

$$b_i^k = \max\{b_0, b^*(u_i|x_w)\}, \quad k = m - 1$$

$$a_i = u_i.$$

I assume the price paid in the initial auction is known to all resale buyers. This is not essential, but is consistent with the fact that all bids at Forest Service auctions are public information. In fact, there are businesses that collect auction data to sell to firms in the industry.
Based on these strategies, we can specify bidders’ expectations of their second-stage profits conditional on the information available to them in the first stage and the outcome of the first-stage auction. These are the expectations needed to determine bidding strategies for the first stage. Consider a bidder \( i \) with signal \( X_i = x \) who wins the first-stage auction. Since \( x_w = y \), in this case, the high bid in the second stage will be \( \max \{ \tilde{u}_2, b^*(\tilde{u}_1; y_1) \} \). Bidder \( i \)'s expected payoff (gross of the price paid in the first stage) is the expected maximum of this second-stage bid and his own use value. Letting \( \rho(m|n) \) give the conditional probability of there being \( m \) buyers in the second stage, this expectation can be represented by \( W(X_i, Y_1, \ldots, Y_{n-1}) \), where

\[
W(x, y_1, \ldots, y_{n-1}) = \int \sum_{m=n-1}^{\infty} \rho(m|n) \times \left\{ u_i + \int_{u_i}^{u^*(u_i; y_1)} \int_{u_i}^{\tilde{u}_1} (\tilde{u}_2 - u_i) \times dG_{2:m}(\tilde{u}_2|y, \tilde{u}_1) \times dG_{1:m}(\tilde{u}_1|y) \right. \\
+ \int_{u^*(u_i; y_1)}^{u^*(u_i; y_1) + b^*(\tilde{u}_1; y_1)} (\tilde{u}_2 - u_i) \times dG_{2:m}(\tilde{u}_2|y, \tilde{u}_1) \times dG_{1:m}(\tilde{u}_1|y) \\
\left. + \int_{u^*(u_i; y_1)}^{\bar{u}_i} \left[ b^*(\tilde{u}_1; y_1) - u_i \right] \times G_{2:m}(b^*(\tilde{u}_1; y_1)|y, \tilde{u}_1) \times dG_{1:m}(\tilde{u}_1|y) \right\} dG(u_i|x). \]

The expected payoff to \( i \) conditional on losing can be derived similarly. The only complication is that the value of \( x_w \), the lower bound on the winner’s first-stage signal, is not necessarily equal to \( y_2 \). This is because \( i \)'s own bid may be second highest in the first-stage auction. Hence 1 condition directly on the value of \( x_w \) in the expectation \( L(X_i, Y_1, \ldots, Y_{n-1}; x_w) \), where

\[
L(x, y_1, \ldots, y_{n-1}; x_w) = \int \sum_{m=n-1}^{\infty} \rho(m|n) \times \left\{ G_{1:m}(b^*(u_i|x_w)|y)[u_i - b^*(u_i|x_w)] \\
+ \int_{u^*(u_i|x_w)}^{\bar{u}_i} (u_i - s)G(s|y_1) \, dH_{1:m}(s|y_1) \right\} \times dG(u_i|x). \]

Here the first term in braces gives the expected profit to \( i \) when both the first-stage winner and all the other second-stage buyers have use values less than \( i \)'s optimal take-it-or-leave-it offer; the second term gives his expected profit when he buys at a price equal to another buyer’s use value.

**B. First-Stage Bidding**

A player’s bid in phase \( k \) of the first-stage auction is a maximum price at which to remain in the auction (assuming no other bidder drops out, initiating another phase), conditional on the information revealed by previous exits. With symmetric strictly increasing bidding strategies, these exits reveal the corresponding bidders’ types in equilibrium. Thus, conditioning on the \( k \) exits observed prior to phase \( k \) amounts to conditioning on

\[
Y_{k-1} = \{ Y_{n-k}, \ldots, Y_{n-1} \}. \]

This conditioning is important because \( Y_{k-1} \) provides information about the opponents a player would face in the resale market. Hence the realization of \( Y_{k-1} \) affects the value a bidder places on winning the auction in equilibrium. In this sense, the resale opportunity adds
a common value element to the first-stage auction.\textsuperscript{13} Let

\begin{equation}
W_n^k(x, y, y_{-1}^k) = E[W(X_i, Y_1, ..., Y_{n-1})]
\end{equation}

\begin{equation}
X_i = x, Y_{n-1-k} = y, Y_{-1}^k = y_{-1}^k.
\end{equation}

For \( k < n - 2 \) let

\begin{equation}
l_n^k(x, y, y_{-1}^k; \bar{x}) = E[L(X_i, Y_1, ..., Y_{n-1}; x_w)]
\end{equation}

\begin{equation}
X_i = x, Y_{n-1-k} = y, Y_{-1}^k = y_{-1}^k; x_w = Y_2.
\end{equation}

while

\begin{equation}
l_n^{n-2}(x, y, y_{-1}^k; \bar{x})
= E[L(X_i, Y_1, ..., Y_{n-1}; x_w)]
\end{equation}

\begin{equation}
|X_i = x, Y_{n-1-k} = y, Y_{-1}^k = y_{-1}^k; x_w = \bar{x}].
\end{equation}

These equations give expectations of the gross payoffs from winning and losing the auction, conditional on the information bidder \( i \) has in phase \( k \) and an assumption that the lowest signal among \( i \)'s remaining opponents is the same as his own signal. These expectations are taken conditional on the assumption that all opponents follow equilibrium strategies and that all players (including \( i \) himself) follow equilibrium strategies in future phases—hence the conditioning on \( x_w = Y_2 \) when \( i \) loses in phase \( k < n - 2 \). In phase \( n - 2 \), losing implies that \( i \)'s own bid, parameterized by \( \bar{x} \), will reveal the lower bound \( \bar{x}_w \).

To characterize the equilibrium price, we may assume that the last remaining bidder in the first-stage auction does not raise his bid above the exit price of his last opponent.\textsuperscript{14} Therefore, the auction ends in phase \( n - 2 \). Suppose \( b(\cdot) \) is the equilibrium bid function for this phase. Exits in prior phases have revealed the realization of \( Y_{n-1}^{x_{-1}} = Y_{-1} \). Hence a bidder with signal \( X_i = x \) chooses \( \bar{x} \approx y_2 \) to maximize

\begin{equation}
\pi_n^{n-2}(x, \bar{x}; y_{-1})
= \int_{y_2}^{\bar{x}} [w_n^{n-2}(x, y, y_{-1}) - b(y)] \frac{f(y)}{1 - F(y_2)} dy
\end{equation}

\begin{equation}
+ \int_{\bar{x}}^{1} l_n^{n-2}(x, y, y_{-1}; \bar{x}) \frac{f(y)}{1 - F(y_2)} dy.
\end{equation}

As in other models of English auctions, each bidder optimally bids his expected valuation, conditional on an assumption that his remaining opponents have the same type he does. Here a bidder's valuation

\begin{equation}
u_n^{n-2}(x, y, y_{-1}; \bar{x})
= w_n^{n-2}(x, y, y_{-1}) - l_n^{n-2}(x, y, y_{-1}; \bar{x})
\end{equation}

is endogenously determined by the difference between the gross payoffs from winning and losing.

**LEMMA 1:** \( \nu_n^{n-2}(x, y, y_{-1}; y) \) strictly increases in \( x \).

**PROOF:**

See Appendix.

**THEOREM 1:** In any perfect Bayesian equilibrium in symmetric strictly increasing bidding strategies, the seller's revenue is \( \nu_n^{n-2}(x, x, y_{-1}, x) \).

**PROOF:**

See Appendix.

In phases \( k = 0, 1, ..., n - 3 \), a bidder wins the auction only if all \( n - k - 1 \) of his remaining opponents exit simultaneously (since otherwise a new phase would begin). Since this occurs with probability zero, bidders consider only the trade-off between the expected payoff from losing the auction and the option value of continuing to the next phase.

\textsuperscript{13} This issue is discussed further in Haile (2000b).

\textsuperscript{14} See the detailed discussion in Haile (2000b).

\textsuperscript{15} Any bid above \( b(1) \) [below \( b(0) \)] is equivalent to a bid of \( b(1) \) [of \( b(0) \)]. Hence we can restrict attention to bids in the range of the equilibrium bid function and ignore the specification of beliefs off the equilibrium path.
phase. Let \( \{b^k(\cdot; y^k_{k-1})\}_{k=0}^{n-2} \) represent a sequence of symmetric strictly increasing equilibrium bid functions, one for each phase \( k \) of the first-stage auction. Define \( y_n \equiv 0 \) and let

\[
F_k(s|y^k_{k-1}) = 1 - \left( \frac{1 - F(s)}{1 - F(y^k_{n-k})} \right)^{n-k-1}.
\]

This gives the distribution of the lowest signal among a given bidder’s remaining opponents in phase \( k \). The expected payoff to a bidder with signal \( x \) who follows a strategy of remaining in the auction up to a price \( b^k(\bar{x}; y^k_{k-1}) \) (with \( \bar{x} \geq y^k_{n-k} \)) is

\[
\pi_n^k(x, \bar{x}; y^k_{n-k}) = \int_{\bar{x}}^{1} l_n^k(x, y, y^k_{k-1}; \bar{x}) \, dF_k(y|y^k_{k-1}) + \int_{y^k_{n-k}}^{\bar{x}} \pi_{n+1}^k(x, x; y) \, dF_k(y|y^k_{k-1}).
\]

Define

\[
V(X_i, Y_{i-1} \ldots Y_{n-1}) = W(X_i, Y_{i-1} \ldots Y_{n-1}) - L(X_i, Y_{i-1} \ldots Y_{n-1}; X_i)
\]

and let

\[
\tilde{v}_n^k(x, y^k_{k-1}) = V(X_i, Y_{i-1} \ldots Y_{n-1})|_{x = y_1 = \ldots = y_{n-1-k} = x, y^k_{k-1} = y^k_{k}}.
\]

This expectation is the analog of that used by players to make their bids in Milgrom and Weber’s (1982) equilibrium of an English auction with affiliated values. With the adaptation to the endogenous valuations environment here, the result is the same. Let \( b \) represent the sequence of bid functions \( \{b^k(\cdot; y^k_{k-1})\}_{k=0}^{n-2} \), where

\[
b^k(x; y^k_{k-1}) = \tilde{v}_n^k(x, y^k_{k-1}) \quad \forall \; k.
\]

THEOREM 2: The strategies \( \{b, \Sigma\} \) form a perfect Bayesian equilibrium of the two-stage game.

PROOF: See Appendix.

C. Empirical Implications

The bidding strategies above appear quite similar to those for an English auction without resale. An important distinction, however, is the endogeneity of valuations. This feature provides the basis for an empirical test. The key empirical prediction comes from two simple observations. First, adding bidders magnifies the resale seller effect: when there are more bidders there will also be more potential buyers in the resale market (at least in expectation), making gains to resale trade more likely to exist and increasing competition between bidders in the second stage. This increases the expected surplus extraction of the resale seller, raising the value of winning the auction. Second, raising the number of bidders diminishes the resale buyer effect: the expected value of attempting to buy in the secondary market shrinks when the number of competitors increases, since added competition makes it less likely that a given loser will buy in the secondary market and raises the price he pays when he does buy. Theorem 3 gives a formal statement of this result.

THEOREM 3: For any \( n, k \leq n - 2, \hat{k} \in \{k, k+1\}, \hat{y} \in \{0, 1\}, \) let \( y^\hat{k}_{\hat{y}} \) denote the ordered set of types \( \{y^k_{k-1}, \hat{y}\} \) when \( \hat{k} \neq k \). Then \( \tilde{v}_{n+1}^\hat{k}(x, y^\hat{k}_{\hat{y}}) > \tilde{v}_n^k(x, y^k_{k-1}) \).

PROOF: See Appendix.

The interpretation of Theorem 3 is simple: adding a bidder to the first-stage auction will cause some of the original bidders to drop out one phase later than they would have without the added competition; however, regardless of whether a given bidder does this, his exit price is strictly higher, due to his anticipation of the added competition in the second stage. Indeed, every bidder in an \( n \)-bidder auction would have a higher willingness to pay in every phase of an \((n + 1)\)-bidder auction and would therefore exit at a higher price. Note that although Theorem 3 is proven only for the resale game modeled here, the key effects of \( n \) on the magnitudes of the resale seller effect and resale buyer effect would hold in almost any natural specification.
of the second-stage game. The following corollary provides the main empirical implication taken to the data.

**COROLLARY 1:** \( \hat{d}_n^{x,y,-1} \) strictly increases in \( n \) for all \( x \).

This result states that the willingness to pay (bid) of a given bidder in the final phase of the first-stage auction is strictly increasing in the number of bidders at the auction. Using the notation of Theorem 3, the corollary focuses on the case in which \( k = n - 2 \) and \( \tilde{k} = k + 1 \); i.e., the case in which the added bidder exits before the final phase (phase \( n - 2 \)) of the auction.

It should be emphasized that this result refers to the willingness to pay of a given bidder. Adding a bidder to an auction will raise the expected winning bid in many models. However, in standard models without resale, the willingness to pay (exit price) of a given bidder does not increase with \( n \). In private values auctions without resale, a bidder’s willingness to pay is unaffected by the number of bidders. In any other affiliated values auction without resale, a player’s willingness to pay declines in the number of bidders because adding competitors intensifies the winner’s curse. Here, although the resale opportunity adds a common component to bidders’ valuations, the comparative statistics prediction of standard common values models does not hold—indeed it is reversed. Therefore, the prediction of Corollary 1 distinguishes bidding with a resale opportunity from bidding in both private and common value models of auctions without resale.

It is interesting to note that there is a winner’s curse effect in this model. However, in contrast to standard common value models (e.g., Milgrom, 1981), there is no sense in which winning is especially bad news when the number of competitors is large. As always, each bidder must account for the information that his winning the auction will reveal (that his signal was highest) in determining his willingness to pay. For the comparative statics, however, this means only that adding a bidder raises the value of winning by less than a bidder would think if he ignored the fact that he wins only when his own signal is higher than that of the added bidder. This winner’s curse limits the positive effect of adding a bidder on the gross payoff to the auction winner, but this payoff is unambiguously higher when there are more bidders. 

### III. Data

Data are taken from U.S. Forest Service records for timber sales held between 1974 and 1989 in regions 1 (Montana, northern Idaho, North Dakota, and northwestern South Dakota) and 5 (California). For each auction, the data include the date and location of the sale, the length of the contract (months between the sale and the harvest deadline), as well as the cruise estimates of volume, density (volume per acre), selling value of end products, harvesting costs, manufacturing costs, and road construction costs. In addition, the highest total bid offered by each firm is recorded. All dollar-denominated variables are converted to constant 1983 dollars per thousand board-feet of timber.

I consider only scaled sales of live sawtimber with contracts incorporating price indexing. Salvage sales, sales set aside for small businesses, and sales with only one bidder were excluded. Sales with contract lengths under 12 months were excluded in order to focus on sales where the model of information revelation between the auction and harvest is most sensible. For consistency, sales were included only when appraisals were conducted using the dominant appraisal method in this time period, known as the “residual value method.”

As a measure of supply conditions I calculated the total volume of timber sold by the Forest Service in each region in the six-month period prior to each sale. I also construct a Herfindahl index of the concentration of total volume across the species on each tract, since specialized mills may value tracts more highly when the timber volume is concentrated in one

---

16 These effects would also be present in sealed-bid auctions, although an additional signaling motive complicates bidding strategies (Haile, 2000b). As a result, econometric identification of the effect examined here could be obtained only through an assumption on the precise form of the game played in the resale market and the distributions \( F(\cdot) \) and \( G(\cdot | \cdot) \).

17 See the related discussion in Haile (2000b).
TABLE 1—SUMMARY STATISTICS

<table>
<thead>
<tr>
<th>Region 1</th>
<th>Region 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of auctions</td>
<td>262</td>
</tr>
<tr>
<td>Number of bidders</td>
<td>3.27</td>
</tr>
<tr>
<td>Winning bid</td>
<td>138.27</td>
</tr>
<tr>
<td>Contract length</td>
<td>52.87</td>
</tr>
<tr>
<td>Volume (1,000 MBF)</td>
<td>8.14</td>
</tr>
<tr>
<td>Acres (1,000s)</td>
<td>1.70</td>
</tr>
<tr>
<td>Density (volume/acre)</td>
<td>7.72</td>
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<tr>
<td>Selling value</td>
<td>456.59</td>
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<tr>
<td>Harvesting cost</td>
<td>161.72</td>
</tr>
<tr>
<td>Manufacturing cost</td>
<td>184.99</td>
</tr>
<tr>
<td>Road construction value</td>
<td>40.98</td>
</tr>
<tr>
<td>Species concentration</td>
<td>0.367</td>
</tr>
<tr>
<td>Six-month inventory</td>
<td>0.500</td>
</tr>
<tr>
<td>Housing starts</td>
<td>1,661</td>
</tr>
</tbody>
</table>

Note: All dollar figures are in 1983 dollars per thousand board-feet (MBF).

or only a few species. Added to the Forest Service data set are monthly U.S. housing starts (seasonally adjusted, lagged one month). Variations in housing starts tend to lead variations in lumber prices, so this variable may provide a measure of bidders' expectations about future demand not captured in current market prices. Together these variables constitute the set of observable characteristics of each sale to be conditioned on in the empirical analysis. Table 1 provides summary statistics for these variables in the sample of auctions studied.

IV. Estimation and Results

A. Estimation Issues

1. Interpretation of the Bids.—Theorem 3 implies that the bid of any given firm will be higher when it is competing against more opponents. If the prices at which each bidder exits each auction were recorded, this prediction could be tested with a regression specifying bids as the dependent variable. Unfortunately, these exits cannot be precisely inferred from the available data. At the Forest Service auctions, bidders call out prices on their own and need not regularly indicate whether they are "in" or "out." This is common in the practice of English auctions but contrasts with usual models (including that above) in which the price is viewed as rising exogenously while bidders continuously indicate their participation until observably dropping out. Consequently, interpreting the recorded bids as the intended exit prices in an idealized button auction model is likely to be misleading.

I take a standard approach (e.g., Paarsch, 1992b; Hendricks and Paarsch, 1995; Baldwin et al., 1997) to dealing with this problem by using only the highest recorded bid at each auction. This winning bid is interpreted as the intended exit price of the bidder with the second-highest signal, i.e., the exit price for phase \( n - 2 \). This interpretation would be exactly correct if the auction were conducted in the idealized button auction form envisioned in the theory. It should be a close approximation for an oral auction as well, since when only two bidders remain (even if they are not aware this is the case) each must respond to the other's bid to avoid losing at a price he is willing to beat. An additional virtue of this approach is the fact that Theorem 1 provides a unique characterization of this bid.

2. Unobserved Heterogeneity.—The number of bidders at an auction may be correlated with tract characteristics that are observed by bidders
but not by the researcher. Instrumental variables are used here to avoid the omitted variable bias that could result. As instruments I use the number of sawmills in the county of each sale and its contiguous counties. These sawmill counts are clearly determined prior to the announcement of a sale. Furthermore, the counts are relatively stable across time, whereas the cruisers responsible for the appraisal data used as covariates move every few years as a matter of Forest Service policy (Baldwin et al., 1997). Finally, the same local sawmill count applies to a wide range of tracts, which are typically quite heterogeneous even within small geographic areas. Hence, while there may be considerable unobserved heterogeneity across tracts, it is unlikely that this would be correlated with the sawmill counts.

3. Functional Form.—The empirical model must allow the distribution of bidder valuations to vary with the number of bidders as well as a fairly large set of observable sale characteristics. This makes a fully nonparametric analysis infeasible. I adopt a simple parameterization in which the location of the distribution of valuations is a linear function of sale characteristics. In addition, while the monotonicity predicted by Theorem 3 could be derived under many specifications of the resale market, different resale games will imply different functional forms for this relationship. Since little is known about the true structure of the resale market, I specify a simple reduced form for this relationship which nests both the null and alternative hypotheses. Adding a subscript $t$ to the variables $n$, $x$, and $y$,

$$
\log \tilde{v}^{n-2}_{ni}(x_{it}, y_{-1t}) = \alpha + \beta \log n_i + c_i \gamma + h_i + \zeta(x_{it}, y_{-1t}, c_i, n_i, h_i).
$$

The left side of (5) is the (natural) logarithm of the phase-$(n_i - 2)$ valuation referred to in Corollary 1. On the right side, $c_i$ is a vector of covariates reflecting observed market conditions and tract characteristics, while $h_i$ reflects sale characteristics observed by bidders but not by the researcher. Consistent with the discussion above, I assume $h_i$ is independent of $c_i$ but allow correlation of $n_i$ and $h_i$. The stochastic term $\zeta(x_{it}, y_{-1t}, c_i, n_i, h_i)$ accounts for (i) bidder $i$’s private information $x_{it}$, which is assumed independent of $n_i$, $h_i$, and $c_i$; and (ii) the information $y_{-1t}$ revealed prior to phase $n_i - 2$ of auction $t$. Below I consider two specifications of $\zeta(x_{it}, y_{-1t}, c_i, n_i, h_i)$: one that accounts for the information revealed by previous exits (i.e., the realization of $y_{-1t}$) and one that ignores this information, effectively assuming that exits during the auction are unobserved by bidders.

B. Observed Exits

Assuming exits during the auction are observed by bidders, let

$$
\zeta(x_{it}, y_{-1t}, c_i, n_i, h_i) = \Omega(x_{it}, y_{-1t}, c_i, n_i, h_i) + \varepsilon(x_{it})
$$

where $\varepsilon(\cdot)$ is strictly increasing and

$$
\Omega(x_{it}, y_{-1t}, c_i, n_i, h_i) \equiv \log \tilde{v}^{n-2}_{ni}(x_{it}, y_{-1t}) - E[\log \tilde{v}^{n-2}_{ni}(x_{it}, y_{-1t})|x_{it}, c_i, n_i, h_i].
$$

$\Omega(x_{it}, y_{-1t}, c_i, n_i, h_i)$ reflects the adjustment to bidder $i$’s willingness to pay in phase $n_i - 2$ resulting from the revelation of the actual realization of $y_{-1t}$. By construction $\Omega(x_{it}, y_{-1t}, c_i, n_i, h_i)$ has expectation zero, both unconditionally and conditional on $\{x_{it}, n_i, c_i, h_i\}$. Note that when $n_i = 2$, $y_{-1t} = \emptyset$, implying that $\Omega(x_{it}, y_{-1t}, c_i, n_i, h_i) = 0$.

---

18 Sawmill counts are from the U.S. Census Bureau’s County Business Patterns series. Lists of contiguous counties were prepared by the Bureau of Economic Analysis. Other candidate instruments are dummy variables indicating the species of trees present on each tract. Similar results are obtained when these instruments are used.
Henceforth I will suppress the arguments of \( \Omega(x_{it}, y_{-1t}, c_t, n_t, h_t) \) and write \( \Omega_{it} \). To further simplify notation, let \( w_i = [1, \log n_t, c_t, \theta] \), \( \theta = [\alpha, \beta, \gamma]' \), and \( \varepsilon_{it} = \varepsilon(x_{it}) \). Equation (5) can then be rewritten

\[
\log \tilde{\Omega}_{n_{it}}^{-2}(x_{it}, y_{-1t}) = w_i \theta + h_t + \Omega_{it} + \varepsilon_{it}.
\]

Assume without loss of generality that \( b_{1t}, \ldots, b_{n_{it}} \) and \( x_{1t}, \ldots, x_{n_{it}} \) are listed in descending order. Interpreting the winning bid \( b_{1t} \) as suggested above then gives

\[
(6) \quad \log b_{1t} = \log \tilde{\Omega}_{n_{it}}^{-2}(x_{2t}, y_{-1t}) = w_i \theta + u_t
\]

where

\[
(7) \quad u_t = h_t + \Omega_{2t} + \varepsilon_{2t}.
\]

Because \( \varepsilon_{2t} \) is an order statistic, in general

\[
E[\log \tilde{\Omega}_{n_{it}}^{-2}(x_{2t}, y_{-1t})|w_i] \neq w_i \theta
\]

even if \( E[h_t|w_i] = 0 \). Imposing a distributional assumption on \( \varepsilon_{it} \) nails down this expectation, making it possible to identify \( \theta \).

I assume each set \( \{\varepsilon_{1t}, \ldots, \varepsilon_{nt}\} \) is an (ordered) random sample of \( n_t \) draws from a normal distribution with mean zero and variance \( \sigma^2 \). The lognormal distribution has frequently been used in the analysis of bidding data (e.g., Jean-Jacques Laffont et al., 1995). Indeed, up to the choice of covariates and the account made here for unobserved heterogeneity and the information learned from opponents’ exits, the specification here is identical to that in Baldwin et al. (1997). They provide evidence supporting the suitability of assuming lognormally distributed values at other Forest Service auctions. Some additional evidence (at least regarding the moment assumptions made here) will be provided below.

Letting \( \Phi(\cdot) \) and \( \phi(\cdot) \) denote the standard normal distribution and density functions, the probability density function for \( \varepsilon_{2t} \) is

\[
\phi_{2}(\varepsilon_{2t}|n_t, \sigma_t) = n_t(n_t - 1) \frac{1}{\sigma_t} \phi\left(\frac{\varepsilon_{2t}}{\sigma_t}\right)
\]

\[
\times \left[ 1 - \Phi\left(\frac{\varepsilon_{2t}}{\sigma_t}\right)\right] \Phi\left(\frac{\varepsilon_{2t}}{\sigma_t}\right)^{n_t-2}.
\]

Define

\[
(6) \quad \xi_t = E[\varepsilon_{2t}|n_t, \sigma_t] = \int_{-\infty}^{\infty} \varepsilon \phi_{2}(\varepsilon|n_t, \sigma_t) \, d\varepsilon
\]

and let \( z_t = \{c_t, u_t\} \), where \( u_t \) are the instruments discussed above, which are assumed independent of \( h_t \) and \( \{x_{it}\} \). The assumption that \( h_t \) is independent of \( c_t \) then implies that \( h_t \) has expectation zero conditional on \( z_t \). By construction \( \Omega_{2t} \) has expectation zero conditional on \( c_t, x_{2t}, n_t, \) and \( h_t \). Therefore,

\[
E[u_t|z_t] = \xi_t.
\]

This gives a set of moment conditions identifying \( \theta \) and \( \sigma_t \).

Let \( \mu^t(n_t) \) denote the \( r \)th moment of a random variable that is the second highest of \( n_t \) draws from a standard normal distribution. Then \( \xi_t = \sigma_t \mu^t(n_t) \). In principle, one could calculate \( \mu^t(n_t) \) for each auction \( t \) and apply standard linear instrumental variable techniques to the equation

\[
\log b_{1t} = w_i \theta + \sigma_t \mu^t(n_t) + \eta_{2t}
\]

where the mean zero “error” term is

\[
\eta_{2t} = \varepsilon_{2t} - \xi_t + h_t + \Omega_{2t}.
\]

However, \( \mu^t(n_t) \) and \( n_t \) are both monotonic functions of \( n_t \) and are highly collinear.\(^{21}\) This

\(^{20}\) More precisely, I assume only that the first and second moments of \( \varepsilon_{2t} \) are given by the first and second moments of a random variable which is the second highest of \( n_t \) draws from a normal distribution. Two specifications of \( \sigma_t \) are considered: (i) \( \sigma_t = \sigma_t v_{it} \) and \( \sigma_t = \sigma_t^* + \sigma_t v_{it} \), where \( \sigma_t^* \) is a 2 \times 1 parameter vector and \( v_{it} \) consists of two observable tract characteristics likely to affect the dispersion of bidder valuations per unit of timber; the share of the estimated contract value coming from road construction and the total volume of the tract. Other specifications of \( \sigma_t \) yield similar results.

\(^{21}\) In unreported linear IV regressions, the estimated values of \( \beta \) and \( \sigma_t = \sigma \) are extremely large in absolute value.
problem can be overcome by using the second moment of $\varepsilon_{1t}$, which places a restriction on the variance of $\eta_{1t}$ for any given value of $\sigma_t$. Define

$$u_t(\theta) = \log b_t - w_t \theta.$$  

From (6) and (7), we know that the second moment of $u_t(\theta)$ depends on the variances of $h_t$ and $\Omega_{2t}$, as well as the second moment of $\varepsilon_{2t}$. In auctions with only two bidders, however, no exits are observed before the auction ends, implying $\Omega_{2t} = 0$. Letting $\tau^2$ denote the variance of $h_t$, standard Generalized Method of Moments (GMM) techniques can then be applied using the moment conditions

$$(8) \, \, E[z_t(u_t(\theta) - \sigma_t \mu^1(n_t))] = 0 \quad \forall \, t$$

$$(9) \, \, E[z_t(u_t(\theta)^2 - \tau^2 - \sigma_t^2 \mu^2(n_t))] = 0 \quad \forall \, t \text{ such that } n_t = 2.$$  

Since $n_t$ is fixed in equation (9), it is clear that $\tau$ and $\sigma_t$ are not separately identified from (9) alone. The fact that $\tau$ does not appear in (8) ensures identification; however, given that $\sigma_t$ was poorly identified from the first equation alone, it should not be surprising that in practice the separate identification of $\sigma_t$ and $\tau$ is weak. While (9) aids considerably in the estimation of $\theta$ (by doubling the number of moment conditions) and also enables precise estimation of $[\tau^2 + \sigma_t^2 \mu^2(n_t)]$, (8) and (9) fail to give precise and stable estimates of $\sigma_t$ and $\tau$ separately. However, because $\tau$ is a nuisance parameter, a useful approach yielding precise estimates of $\theta$ and $\sigma_t$ is to fix the value of $\tau$. Of course, fixing $\tau$ at the wrong value could give misleading results. I therefore use a sequence of trial values for $\tau$. Zero is a lower bound on $\tau$, while an upper bound on $\tau^2$ is the total variance of $u_t$. This suggests two approaches for estimating an upper bound. First, I calculate the residual variance from an OLS regression of $\log b_t$ on $c_t$. If $n_t$ and $u_t$ are nonnegatively correlated, this provides an upper bound on the total variance of $u_t$. A second estimate is obtained from the residual variance from the OLS regression of $\log b_t$ on $c_t$ and $\log n_t$. The maximum of these two bounds is approximately 0.48 for region 1 and 0.57 for region 5. Below I report estimates obtained with values of $\tau$ between 0.05 and 0.65.

Tables 2 and 3 summarize results for regions 1 and 5 respectively. For each trial value of $\tau$, three specifications were estimated. In the first, $\sigma_t$ is assumed constant across auctions, and the number of bidders at each auction is assumed exogenous ($\log n_t$ is included in $z_t$ instead of the instruments). The other two specifications employ instrumental variables, with the last specification also allowing $\sigma_t$ to vary across auctions. For each specification, the table reports the estimates of $\beta$ and $\sigma_t$, the corresponding estimate of $\sigma_t$, and the results of key hypothesis tests. As we would expect, as the value of $\tau$ increases, the estimate of $\sigma_t$ shrinks, usually hitting zero when $\tau$ is near the estimated upper bounds noted above.

The parameter of primary interest, $\beta$, is estimated to be positive and significantly different from zero in every case, as predicted by Corollary 1. This estimate varies substantially between the specifications with and without instrumental variables, with Lars Peter Hansen’s (1982) $J$-test of overidentifying restrictions uniformly rejecting the specifications without instruments. The estimated derivative $d \, \hat{\varepsilon}_a^{-2}(x, y_{-1})$ for the full sample period ranges from $13$ to $64$, although the estimates vary much less if one ignores the specifications without instruments and those in which a large value of $\tau$ forces $\hat{\sigma}_t$ to zero. Ignoring these specifications, the estimates of $\beta$ (and, in fact, of all parameters except $\sigma_t$) vary little as $\tau$
changes. In the IV specifications, the estimates of \( \frac{d}{dn} \hat{v}^2(u, y_{-1}) \) are significantly larger before the policy changes, as predicted. A Wald test of equality of these derivatives before and after 1981 (labeled “\( \frac{d}{dn} \) constant”) rejects at a 10-percent level or better in every case.25

25 In region 5 this test is based on estimates of all parameters obtained separately in the pre-1981 and post-1981 subsamples. In region 1 there are too few observations after 1981 for this “separate regressions” approach, so the test is based on a specification allowing only \( \beta \) to differ across the two time periods. For the same reason, none of the negative point estimates of \( \beta \) for the post-1981 period in region 1 is statistically significant.
of overidentifying restrictions fails to reject any of the IV specification except when $\tau$ is so large that $\delta_T = 0$.

C. Unobserved Exits

The assumption that exits are perfectly observed by bidders during the auction may seem at odds with the concern that the bid data do not correctly reveal this information. Of course, exits might be observed during the auction in other ways—a bidder might pack his briefcase or even walk out of the room. More subtle behavior may also convey information, although imperfectly. Neither the theory nor the empirical specification above relies on exits being perfectly revealed. However, it is interesting to ask whether the data reveal evidence that any significant information is observed before the auction ends. When exits are unobserved the empirical model changes only in that $a_i$ is identically zero for all auctions. This gives

$$\xi(x_{it}, y_{-it}, e_i, n_i, h_i) = e_{it}$$

26 With an analogous simplification of the theoretical model, all of the results above carry through.
and implies that the moment condition (9) can be applied to all auctions.27

27 With \( n_t \) varying in this case, \( \tau \) and \( \sigma \) are separately identified from (9). Because the mean of the order statistic \( \varepsilon_{2t} \) increases in \( n_t \) while its variance decreases in \( n_t \), the second moment is a nonmonotonic function of \( n_t \). In region 1, however, there is still too little variation in this second moment to obtain precise estimates. In region 5, where the sample is much larger, this problem disappears and precise robust results are obtained: \( \hat{\beta} = 0.49 \), which is significantly greater than zero (p-value < 0.0001), and the J-test statistic has a p-value of 0.9981.

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>0.05</th>
<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
<th>0.25</th>
<th>0.30</th>
<th>0.35</th>
<th>0.40</th>
<th>0.45</th>
<th>0.50</th>
<th>0.55</th>
<th>0.60</th>
<th>0.65</th>
</tr>
</thead>
<tbody>
<tr>
<td>No instruments, ( \sigma = \sigma )</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>( \hat{\beta} ) &amp; (0.07) &amp; (0.07) &amp; (0.07) &amp; (0.07) &amp; (0.07) &amp; (0.07) &amp; (0.07) &amp; (0.07) &amp; (0.07) &amp; (0.07) &amp; (0.07) &amp; (0.07) &amp; (0.07)</td>
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<td>Test p-values:</td>
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<tr>
<td>( \beta = 0 ) &amp; 0.300</td>
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</tr>
</tbody>
</table>

| Instrumental variables, \( \sigma_t = \sigma \) |      |      |      |      |      |      |      |      |      |      |      |      |      |
| \( \hat{\beta} \) & 1.53 | 1.53 | 1.54 | 1.55 | 1.56 | 1.57 | 1.59 | 1.60 | 1.52 | 1.71 | 1.90 | 2.10 | 2.30 |
| \( d\hat{\beta} \) & 72.82 | 72.61 | 72.27 | 71.79 | 71.15 | 70.35 | 69.31 | 67.85 | 60.47 | 67.71 | 76.05 | 85.33 | 95.64 |
| \( \sigma_t \) & 0.59 | 0.58 | 0.56 | 0.53 | 0.50 | 0.46 | 0.40 | 0.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Test p-values: |      |      |      |      |      |      |      |      |      |      |      |      |      |
| \( \beta = 0 \) & 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| J-test & 0.212 | 0.207 | 0.200 | 0.191 | 0.180 | 0.168 | 0.160 | 0.124 | 0.000 | 0.005 | 0.030 | 0.001 | 0.282 |

| Instrumental variables, \( \sigma_t = \sigma \) |      |      |      |      |      |      |      |      |      |      |      |      |      |
| \( \hat{\beta} \) & 1.40 | 1.41 | 1.41 | 1.42 | 1.42 | 1.43 | 1.44 | 1.26 | 1.45 | 1.65 | 2.05 | * |      |
| \( d\hat{\beta} \) & 66.29 | 66.03 | 65.59 | 64.95 | 64.10 | 63.00 | 61.48 | 51.20 | 58.90 | 67.20 | * | 84.85 | * |
| \( \sigma_t \) & 0.57 | 0.56 | 0.54 | 0.51 | 0.47 | 0.43 | 0.36 | 0.01 | 0.01 | 0.01 | * | 0.01 | * |
| Test p-values: |      |      |      |      |      |      |      |      |      |      |      |      |      |
| \( \beta = 0 \) & 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| J-test & 0.311 | 0.313 | 0.316 | 0.320 | 0.325 | 0.332 | 0.341 | 0.413 | 0.457 | 0.411 | * | 0.151 | * |

Notes: Standard errors are in parentheses. An asterisk (*) indicates results omitted due to numerical problems arising when \( \sigma_t \) is driven to zero.
Table 5—Unobserved Exits, Region 5

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>0.05</th>
<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
<th>0.25</th>
<th>0.30</th>
<th>0.35</th>
<th>0.40</th>
<th>0.45</th>
<th>0.50</th>
<th>0.55</th>
<th>0.60</th>
<th>0.65</th>
</tr>
</thead>
<tbody>
<tr>
<td>No instruments, ( \sigma_r = \sigma )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\beta} )</td>
<td>0.15</td>
<td>0.17</td>
<td>0.20</td>
<td>0.25</td>
<td>0.33</td>
<td>0.46</td>
<td>0.72</td>
<td>0.70</td>
<td>0.72</td>
<td>0.75</td>
<td>0.81</td>
<td>0.85</td>
<td>0.87</td>
</tr>
<tr>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.04)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>( d\hat{\sigma}<em>n^2(x, y</em>{-1})/dn )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\sigma}_r )</td>
<td>0.56</td>
<td>0.54</td>
<td>0.51</td>
<td>0.46</td>
<td>0.38</td>
<td>0.25</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.04)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.02)</td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>Test p-values:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\beta} = 0 )</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>J-test</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

| \( \sigma_r = \sigma \) |
| \( \hat{\beta} \) | 1.33 | 1.34 | 1.34 | 1.35 | 1.36 | 1.41 | 1.42 | 1.43 | 1.45 | 1.48 | 1.55 | 1.70 | 1.86 |
| (0.21) | (0.21) | (0.21) | (0.21) | (0.21) | (0.25) | (0.25) | (0.25) | (0.25) | (0.25) | (0.25) | (0.25) | (0.09) |
| \( d\hat{\sigma}_n^2(x, y_{-1})/dn \) |
| \( \hat{\sigma}_r \) | 0.68 | 0.67 | 0.67 | 0.66 | 0.66 | 0.60 | 0.57 | 0.51 | 0.45 | 0.37 | 0.25 | 0.10 | 0.00 |
| (0.07) | (0.07) | (0.07) | (0.08) | (0.08) | (0.10) | (0.11) | (0.13) | (0.16) | (0.21) | (0.32) | (0.29) | (0.15) |
| Test p-values: |
| \( \hat{\beta} = 0 \) | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| d\hat{\sigma}/dn constant | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Exits unobserved | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| J-test | 0.890 | 0.896 | 0.906 | 0.918 | 0.932 | 0.992 | 0.994 | 0.996 | 0.997 | 0.997 | 0.997 | 0.996 | 0.988 |

| \( \sigma_r = \sigma \) |
| \( \hat{\beta} \) | 1.26 | 1.27 | 1.28 | 1.29 | 1.30 | 1.32 | 1.34 | 1.37 | 1.42 | 1.46 | 1.56 | 1.73 | 1.89 |
| (0.20) | (0.20) | (0.19) | (0.19) | (0.19) | (0.19) | (0.18) | (0.18) | (0.18) | (0.17) | (0.17) | (0.12) | (0.11) | (0.10) |
| d\hat{\sigma}_n^2(x, y_{-1})/dn |
| \( \hat{\sigma}_r \) | 0.66 | 0.65 | 0.64 | 0.61 | 0.58 | 0.54 | 0.49 | 0.43 | 0.34 | 0.23 | 0.10 | 0.04 | 0.05 |
| (0.06) | (0.06) | (0.07) | (0.07) | (0.07) | (0.08) | (0.09) | (0.10) | (0.12) | (0.16) | (0.22) | (0.06) | (0.06) |
| Test p-values: |
| \( \hat{\beta} = 0 \) | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| d\hat{\sigma}/dn constant | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| \( \sigma_r = \sigma \) | 0.375 | 0.392 | 0.422 | 0.465 | 0.523 | 0.595 | 0.681 | 0.776 | 0.982 | 0.996 | 0.982 | 0.979 | 0.917 |
| Exits unobserved | 0.028 | 0.028 | 0.028 | 0.027 | 0.027 | 0.026 | 0.026 | 0.026 | 0.018 | 0.021 | 0.004 | 0.003 | 0.001 |
| J-test | 0.960 | 0.962 | 0.966 | 0.970 | 0.974 | 0.978 | 0.982 | 0.986 | 0.997 | 0.997 | 0.997 | 0.993 | 0.973 |

Note: Standard errors are in parentheses.

Identifying restrictions again reject the specifications without instruments and fail to reject any of the IV specifications. In both regions, the hypothesis that \( d\hat{\sigma}_n^2(x, y_{-1})/dn \) is the same before and after the 1981 policy changes is rejected in every case, with the pre-1981 derivatives always larger. Here, none of the tests of the hypothesis \( \sigma_r = \sigma \) reject.

While the standard J-tests of overidentifying restrictions fail to reject the unobserved exits specification, casual comparisons of results for the observed and unobserved exits models reveal large differences in the parameter estimates, suggesting that the unobserved exits assumption may be inappropriate. Under the null hypothesis of unobserved exits, using both sets of moments (8) and (9) on all observations with an optimal weighting matrix yields consistent and asymptotically efficient estimates. Under the alternative of observed exits, however, this approach yields inconsistent results.
estimated. The observed exits specification gives consistent estimates under the null and alternative, although under the null this approach is inefficient. Hence, a Hausman test of the unobserved exits hypothesis could be constructed. Unfortunately, the asymptotic ordering (under the null) of the relevant covariance matrices fails to hold for the estimated covariance matrices in almost every specification, resulting in negative numbers for the (asymptotically chi-square) test statistics. While this is consistent with an incorrect null hypothesis, a different approach is needed for formal testing.

One possible approach exploits the fact that a test of overidentifying restrictions in the observed exits model (which hold under both the null and alternative) can be constructed using \( \sqrt{T} \)-consistent estimates of \( \theta \) and \( \sigma \) [see Whitney K. Newey and Daniel McFadden (1994) for details]. Under the null hypothesis that exits are unobserved, the unobserved exits model provides such consistent estimates. Tables 4 and 5 include \( p \)-values for the resulting tests. The tests reject (with \( p \)-values no larger than 0.03) in every case except for the region 1 specifications with \( \sigma_t = \sigma \forall t \).

Note that rejecting this null also means rejecting the hypothesis that bidders’ valuations are purely private—i.e., that even if bidders observe opponents’ exits, this does not affect their willingness to pay. Hence these results support the predicted presence of a common value element introduced by the resale opportunity.

Tables 6 and 7 present the complete set of parameter estimates for \( \tau \) fixed at 0.25. Results are very similar for other values of \( \tau \). Parameter estimates are generally precisely estimated with the anticipated signs and plausible magnitudes. Assuming the model is correctly specified, the marginal effects of the estimated tract selling value, manufacturing costs, and harvesting costs, for example, should have an absolute value of 1.0 if the Forest Service estimates were perfect. This is unlikely but provides a baseline for comparison. In region 1 the estimated marginal effects are indeed fairly close to 1 in absolute value, except in one case. In region 5, where there are considerably more data, the absolute values of these estimates are all quite close to 1.

### V. Specification Testing and Alternative Distributions

The moment restrictions on \( \varepsilon_{2t} \) play a key role in identifying the effect of the number of bidders on valuations. Changing the number of bidders at an auction changes the expected

---

### Table 6—Full Results with \( \tau = 0.25 \)

<table>
<thead>
<tr>
<th>Region 1</th>
<th>Observed exits</th>
<th>Unobserved exits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Parameter estimate</td>
<td>Standard error</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log of number of bidders</td>
<td>0.9697</td>
<td>0.2283</td>
</tr>
<tr>
<td>Selling value</td>
<td>0.0042</td>
<td>0.0010</td>
</tr>
<tr>
<td>Manufacturing cost</td>
<td>-0.0051</td>
<td>0.0022</td>
</tr>
<tr>
<td>Harvesting cost</td>
<td>-0.0086</td>
<td>0.0017</td>
</tr>
<tr>
<td>Road construction value</td>
<td>0.0143</td>
<td>0.0092</td>
</tr>
<tr>
<td>Road/selling value</td>
<td>-6.65</td>
<td>4.28</td>
</tr>
<tr>
<td>Density</td>
<td>-0.0018</td>
<td>0.0059</td>
</tr>
<tr>
<td>Housing starts</td>
<td>0.0008</td>
<td>0.0004</td>
</tr>
<tr>
<td>Species concentration</td>
<td>-0.4712</td>
<td>0.2313</td>
</tr>
<tr>
<td>Six-month inventory</td>
<td>0.4947</td>
<td>0.2130</td>
</tr>
<tr>
<td>Log of volume</td>
<td>0.0119</td>
<td>0.0827</td>
</tr>
<tr>
<td>Contract length</td>
<td>0.0035</td>
<td>0.0046</td>
</tr>
<tr>
<td>Standard deviation (( \sigma ))</td>
<td>0.3367</td>
<td>0.0783</td>
</tr>
<tr>
<td>J-test ( p )-value</td>
<td>0.6061</td>
<td>0.2830</td>
</tr>
<tr>
<td>Observations</td>
<td>296</td>
<td>296</td>
</tr>
</tbody>
</table>
A. Specification Tests

The identifying assumptions above are that the first and second moments of \( \sigma_2 \) are given by the first and second moments of a random variable which is the second highest of \( n_t \) draws from a normal distribution. The tests of overidentifying restrictions above fail to reject (in the IV specifications) the moment conditions derived from these assumptions. Unfortunately, additional testing is difficult. Seemingly natural approaches might be to test restrictions on higher moments implied by normality or to test the distribution of the estimated residuals for deviations from the distribution predicted by a normality assumption. These approaches suffer from two problems. One is the fact that consistency of GMM estimates does not rely on a correct distributional assumption, only on correct moment conditions. Hence, tests of normality other than tests of the first and second moments used in estimation are not tests of overidentifying restrictions but tests of related (but irrelevant) hypotheses. A second problem with these approaches is that even when \( \sigma_2 \) is a normal order statistic, the difference

\[
\hat{\alpha}_t = \log b_{2t} - w_t \hat{d}
\]

does not have a known distribution. From (6) and (7), only if \( h_t = \Omega_{2t} = 0 \) for all \( t \) does normality of \{\( e_{2t} \)\} imply that the residual \( \hat{\alpha}_t \) is (asymptotically) the second largest of \( n_t \) normal deviates; otherwise, it has a distribution which is the convolution of those of \( e_{2t}, \Omega_{2t}, h_t \). This precludes a residuals-based test even for two-bidder auctions.

Nonetheless, casual inspection may suggest whether the distributions of residuals accord

### Table 7—Full Results with \( \tau = 0.25 \)

<table>
<thead>
<tr>
<th>Region 5</th>
<th>Observed exits</th>
<th>Unobserved exits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Parameter estimate</td>
<td>Standard error</td>
</tr>
<tr>
<td>Constant</td>
<td>1.97</td>
<td>0.3045</td>
</tr>
<tr>
<td>Log of number of bidders</td>
<td>0.8814</td>
<td>0.0651</td>
</tr>
<tr>
<td>Selling value</td>
<td>0.0061</td>
<td>0.0005</td>
</tr>
<tr>
<td>Manufacturing cost</td>
<td>-0.0052</td>
<td>0.0013</td>
</tr>
<tr>
<td>Harvesting cost</td>
<td>-0.0063</td>
<td>0.0005</td>
</tr>
<tr>
<td>Road construction value</td>
<td>-0.0001</td>
<td>0.0051</td>
</tr>
<tr>
<td>Road/selling value</td>
<td>0.5077</td>
<td>1.76</td>
</tr>
<tr>
<td>Density</td>
<td>0.0004</td>
<td>0.0001</td>
</tr>
<tr>
<td>Housing starts</td>
<td>-0.1548</td>
<td>0.0848</td>
</tr>
<tr>
<td>Species concentration</td>
<td>-0.0024</td>
<td>0.0019</td>
</tr>
<tr>
<td>Six-month inventory</td>
<td>-0.0517</td>
<td>0.0372</td>
</tr>
<tr>
<td>Log of volume</td>
<td>-0.0045</td>
<td>0.0879</td>
</tr>
<tr>
<td>Contract length</td>
<td>0.0040</td>
<td>0.0021</td>
</tr>
<tr>
<td>Standard deviation (( \sigma_t ))</td>
<td>0.3958</td>
<td>0.0400</td>
</tr>
<tr>
<td>J-test p-value</td>
<td>0.1369</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>1,559</td>
<td></td>
</tr>
</tbody>
</table>

29 Other studies have relied on similar assumptions for identification in an MLE context, for example in estimating optimal reserve prices, which depend on the shape of the underlying distribution of valuations (Paarsch, 1997), or in distinguishing competitive from collusive behavior based on distributions of order statistics (Baldwin et al., 1997).
with our expectations given the assumed stochastic structure. If \( h_t = \Omega_{2t} = 0 \) for all \( t \), \( \frac{\varepsilon_{2t}}{\sigma_t} \) would have density \( \phi_2(\cdot | n_t, 1) \). The distribution of the full sample of standardized residuals would be a mixture (over \( n_t \)) of these densities. Figures 1 and 2 show, for each region and several trial values of \( \tau \), a kernel density (Epanechnikov) for the standardized residuals (based on the IV specifications in Tables 2 and 3 with \( \sigma_t = \sigma \)) as well as the predicted density

\[
\phi_2^*(\varepsilon) = \sum_{n=2}^{12} q(n) n(n-1) \phi(\varepsilon) \times (1 - \Phi(\varepsilon)) \Phi(\varepsilon)^{n-2}.
\]

The mixing weight \( q(n) \) is the share of auctions in the sample for which \( n_t = n \). Given the results of the Hansen chi-square tests, we know that the means of these distributions match well and that the differences in variances are largely accounted for by \( \tau^2 \). As one would expect if the model is correctly specified, the residual distri-

---

Notes: Solid lines—kernel density of residuals \( \log b_{1t} - w_t \hat{\theta} \). Dotted lines—predicted density under normality and \( \Omega_{2t} = h_t = 0 \).

Likewise, one may gain some confidence from the fact that similar results are obtained using higher moments of the distribution of normal order statistics.

Residuals are not shown for specifications yielding \( \sigma_t = 0 \). Values of \( \tau \) between those included here give very similar results.
butions look like those for convolutions of the normal order statistics with other mean zero random variables.

B. Alternative Distributions

While the tests of overidentifying restrictions above fail to reject the model, we have no evidence that any distributional specification would be rejected by these tests, nor that the results are robust to variations in the distribution used to derive the identifying moment restrictions. To evaluate these concerns, I examine estimates obtained with different distributions. Two alternatives were considered: the log-logistic and Weibull. In the log-logistic specification, the logistic distribution is substituted for the normal distribution above. The normal and logistic densities have similar shapes, although the fatter tails of the logistic can give quite different distributions of the extreme order statistics. In the Weibull specification, the winning bid is specified in levels rather than logs, with each \( e_{it} \) drawn from a Weibull distribution with scale parameter \( \sigma \) and shape parameter \( \delta \), both to be estimated. The Weibull distribution is quite flexible and has been used in a number of other empirical studies of auctions (e.g., Paarsch, 1992a, b, 1997; Stephen G. Donald et al., 1997).

Table 8 presents the results from the logistic specification for the same range of \( \tau \) considered above. For simplicity, results are reported only for the observed exits model with \( \sigma = \sigma \). The results are very similar to those using the normal distribution. In every case (1) the estimate of \( \beta \) is positive and significant; (2) the estimate of \( \sigma \) is significantly larger prior to the 1981 policy changes; and (3) the test of overidentifying restrictions fails to reject except when \( \tau \) is so large that \( \hat{\sigma} = 0 \).

Table 9 reports the Weibull results. The same range of values for \( \tau \) is considered, although the estimated upper bounds on \( \tau \) are lower in this specification (0.29 and 0.48 for regions 1 and 5, respectively). In region 1, the point estimates of \( \beta \) are positive for most (but not all) values of \( \tau \), although the test of \( \beta = 0 \) rejects only when \( \hat{\beta} < 0 \). In nearly every case, however, the test of overidentifying restrictions rejects the model. In region 5, point estimates of \( \beta \) are always positive but marginally insignificant, with p-values for the test of \( \beta = 0 \) ranging from 0.11 to 0.15.
in most specifications. However, the tests of overidentifying restrictions uniformly reject the model.

An exhaustive analysis of the results one might obtain with other distributional assumptions is obviously impossible. However, the results here provide some additional confidence in the conclusions drawn from the normal model by showing that (1) the results are not unique to the normal specification; and (2) the tests of overidentifying restrictions, which fail to reject the normal (or logistic) model, will reject moment conditions derived from other distributions.

**VI. Conclusion**

The model developed in this paper provides an empirical prediction that distinguishes bidding at auctions with resale opportunities from that in standard private or common value models that ignore resale. With a resale opportunity, bidder valuations are endogenously determined by the option value the resale market provides to winners and losers of an auction. This key feature of auctions with resale opportunities drives the empirical prediction taken to the data. Using a structural empirical model that explicitly accounts for unobserved heterogeneity and for information revealed by opponents’ bids during an English auction, I find evidence that bidders’ valuations are higher when the option value of selling in the resale market is high and the option value of buying in the resale market is low. This finding is predicted by the model here but inconsistent with standard auction models that ignore resale. This empirical result is found in both of the geographic regions studied, and a change in policy regime that was expected (and intended) to reduce the importance of resale leads to an attenuation of this effect in both regions. Additional evidence is found for the effects of information revealed during an English auction through opponents’ bids and, therefore, for the common value element the resale market introduces to bidders’ valuations in the model.
Any model must abstract from certain features of a market to focus on others. However, the empirical evidence here suggests that ignoring a resale opportunity can lead to misleading results for researchers and policy makers. The effects of resale on bidding in the model here are somewhat subtle, with equilibrium strategies that bear close resemblance to those for the same auction without resale. This might instill greater confidence in standard models than is actually warranted. This paper makes clear that resale has important implications for the interpretation of bidding data. As the work of Bikhchandani and Huang (1989) and Haile (1999, 2000a, b) has shown, a resale opportunity (1) implies that bidder valuations are endogenous and potentially dependent on the selling mechanism itself; (2) introduces new options for sellers who might encourage or discourage an active secondary market; (3) can introduce signaling as a component of bidding strategies at sealed-bid auctions; (4) can lead to a reversal of standard results ranking auctions by expected revenues; and (5) can preclude the existence of an efficient bidding equilibrium. The evidence here suggests that the effects of resale markets are not merely theoretical possibilities and that careful attention must be paid to the broad range of effects of resale markets, particularly when questions of market design or interpretation of bidding data are at issue.

Some caveats are of course in order. While the theoretical model here appears to capture an important feature of Forest Service timber sales missing from previous models, it abstracts from other potentially important features. Two aspects of the Forest Service auctions in particular suggest directions for future work. The first is the fact that bidders at a given Forest Service auction are likely to compete against each other in future auctions. While one message of this paper is the importance of accounting for dynamic strategic effects in the specification of empirical models, this type of repeated interaction is clearly not captured by the

### Table 9—Observed Exits

#### Weibull Specification

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>0.05</th>
<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
<th>0.25</th>
<th>0.30</th>
<th>0.35</th>
<th>0.40</th>
<th>0.45</th>
<th>0.50</th>
<th>0.55</th>
<th>0.60</th>
<th>0.65</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\beta} )</td>
<td>-20.2</td>
<td>-13.9</td>
<td>7.2</td>
<td>35.2</td>
<td>57.8</td>
<td>82.6</td>
<td>94.5</td>
<td>103.7</td>
<td>110.8</td>
<td>115.1</td>
<td>117.6</td>
<td>120.4</td>
<td>122.6</td>
</tr>
<tr>
<td>( d\hat{\sigma}^{-2}(x, y_{-1})/dn )</td>
<td>(9.88)</td>
<td>(8.83)</td>
<td>(162)</td>
<td>(168)</td>
<td>(167)</td>
<td>(169)</td>
<td>(168)</td>
<td>(179)</td>
<td>(182)</td>
<td>(186)</td>
<td>(192)</td>
<td>(195)</td>
<td>(196)</td>
</tr>
</tbody>
</table>

- **Region 1**
  
  - Full sample: -6.89 - 4.73 2.43 11.97 19.70 28.14 32.20 35.34 37.73 39.20 40.04 41.00 41.75
  
  - Pre-1981: -4.98 - 2.10 6.33 13.33 13.01 29.63 - 7.63 36.37 38.70 40.38 41.41 42.01 41.58
  
  - Post-1981: 8.33 - 6.60 1.00 12.63 33.90 12.46 41.35 25.85 26.64 27.99 28.50 27.05 25.30

- **Region 5**
  
  - Full sample: 26.47 26.59 26.76 27.11 26.30 28.06 29.39 32.22 37.48 41.01 43.84 46.71 49.61
  
  - Pre-1981: 17.19 17.18 18.02 20.47 26.25 27.82 28.82 33.72 33.81 35.53 35.64 39.46 25.24
  
  - Post-1981: 18.77 18.79 18.79 18.70 20.54 24.31 27.18 29.66 31.98 31.43 32.26 33.04 33.59

- **Test p-values:**
  
  - \( \beta = 0 \): 0.041 0.115 0.965 0.833 0.730 0.625 0.573 0.561 0.542 0.537 0.539 0.537 0.532
  
  - \( d\hat{\beta}/dn \): 0.879 0.791 0.002 0.997 0.031 0.134 0.000 0.005 0.007 0.012 0.023 0.149 0.237
  
  - J-test: 0.109 0.096 0.111 0.033 0.004 0.004 0.005 0.007 0.012 0.018 0.020 0.025 0.027

- **Note:** Standard errors are in parentheses.
model presented here. This is a feature of many auctions that has been ignored throughout the empirical auction literature, but which deserves careful attention. Related to this point is the maintained assumption that bidding is not collusive. Repeated interaction and resale opportunities may enhance the ability of bidders to collude, and there have been charges of collusion in the timber industry. This paper makes no attempt to empirically distinguish bidding consistent with a noncollusive model of auctions with resale from bidding consistent with a model of collusion, although this is an issue worth serious consideration in the future.

APPENDIX

PROOF OF LEMMA 1:

Rewrite (2) and (3) as

\[ W(x, y_1, \ldots, y_{n-1}) = \int_0^1 \varphi_w(u) \, dG(u|x) \]

and

\[ L(x, y_1, \ldots, y_{n-1}, x_w) = \int_0^1 \varphi_l(u) \, dG(u|x). \]

Then

\[ \psi_n^{-2}(x, y_1, y_{-1}, y_1) = E \left[ \int_0^1 [\varphi_w(u) - \varphi_l(u)] \, dG(u|x) \right] \]

\[ Y_1 = y_1, \quad Y_{-1} = y_{-1}, \quad x_w = y_1. \]

Given the strict affiliation of \( U_i \) and \( X_i \), showing that \( \varphi_w(u) > \varphi_l(u) \) \( \forall u \in (0, 1) \) will prove the result. From (2)

\[ \varphi_w(u) = \sum_{m=n-1}^{\infty} \rho(m|n) \left\{ G_{1;m}(u|y) + \int_{u}^{u*(u; y_1)} G_{2;m}(u|\tilde{u}_1, y) \, dG_{1;m}(\tilde{u}_1|y) \right\}. \]

Similarly, since \( G_{1;m}(\cdot|y) = G(\cdot|y_1) \times H_{1;m}(\cdot|y_{-1}) \), differentiation inside the integral in (3) gives

\[ \varphi_l(u) = \sum_{m=n-1}^{\infty} \rho(m|n) \left\{ G_{1;m}(u|y) - \int_{b*(u; y_1)}^{u} H_{1;m}(s|y_{-1}) \, dG(s|y_1) + \frac{\partial b*(u; y_1)}{\partial u} H_{1;m}(b*(u; y_1)|y_{-1}) \times \left[ g(b*(u; y_1)|y_1)(u - b*(u; y_1)) - G(b*(u; y_1)|y_1) \right] \right\}. \]

The term in square brackets would be set to zero if \( b*(u; y_1) \) were chosen knowing that \( Y_1 = y_1 \); however, because \( b*(u; y_1) \) is chosen under the assumption \( Y_1 \geq y_1 \), strict affiliation implies that this term is nonpositive. Hence (since \( u*(u; y_1) > u \forall u > 0 \))

\[ \varphi_l(u) \leq G_{1;m}(u|y) < \varphi_w(u) \quad \forall u \in (0, 1). \]

PROOF OF THEOREM 1:

From (4),

\[ \psi_n^{-2}(x_1, \ldots, x_{n-1}; y_1) = E \left[ \int_0^1 [\varphi_w(u) - \varphi_l(u)] \, dG(u|x) \right] \]

\[ Y_1 = y_1, \quad Y_{-1} = y_{-1}, \quad x_w = y_1. \]
Differentiating (A1) gives
\[
\frac{\partial^2}{\partial \tilde{x} \partial x} \pi_n^{n-2}(x, \tilde{x}; y_{-1}) = \frac{\partial}{\partial x} \left[ w_n^{n-2}(x, \tilde{x}, y_{-1}) - l_n^{n-2}(x, \tilde{x}, y_{-1}; \tilde{x}) \right]
\]
which is strictly positive by Lemma 1. Hence
\[
\frac{\partial}{\partial \tilde{x}} \pi_n^{n-2}(x, \tilde{x}; y_{2}) \geq 0 \iff \tilde{x} \leq x
\]
implies that the first-order condition defines an optimum.

PROOF OF THEOREM 2:

First note that \( \frac{\partial}{\partial \tilde{x}} \pi_n^{n-2}(x, y_{-1}) = u_n^{n-2}(x, y_{-1}; x) \). So the strategy for phase \( n - 2 \) is that prescribed by Theorem 1. Second, it is straightforward to confirm that \( V(x, y_1, \ldots, y_{n-1}) \) strictly increases in \( y_j \) \( \forall j = 1, 2, \ldots, n - 1 \), ensuring that \( b \) specifies strictly increasing bids. Now consider a bidder \( i \) with signal \( X_i = x \) in phase \( k \leq n - 3 \) of the auction. Suppose that in phase \( k + 1 \), for any realization of \( Y_{k+1} \),
\[
(\text{A2}) \quad \frac{\partial}{\partial \tilde{x}} \pi_n^{n-1}(x, \tilde{x}; y_{k+1}) > 0 \quad \tilde{x} < x \leq 0 \quad \tilde{x} > x.
\]
This implies that if \( i \) were to deviate in phase \( k \) by remaining in the auction past his equilibrium bid, it would be optimal for him to exit immediately in phase \( k + 1 \). Hence, when \( i \) bids as if his signal were \( \tilde{x} \geq y_{n-k} \) he obtains an expected payoff
\[
\pi_n(x, \tilde{x}, y_{k+1}) = \int_{y_{k+1}}^{x} \pi_n^{k+1}(x, x; \{y, y_{k+1}\}) \, dF_k(y|y_{k+1})
\]
where \( \{y, y_{k+1}\} \) denotes the realization of \( Y_{k+1} = \{Y_{n-k-1}, Y_{k+1}\} \). Differentiating
\[
\frac{\partial}{\partial \tilde{x}} \pi_n^{n-2}(x, \tilde{x}; y_{-1}) = \frac{\partial}{\partial x} \left[ w_n^{n-2}(x, \tilde{x}, y_{-1}) - l_n^{n-2}(x, \tilde{x}, y_{-1}; \tilde{x}) \right]
\]
with respect to $\bar{x}$ yields (recall that $\frac{\partial}{\partial \bar{x}} l^k_n(x, y, y_{k-1}; \bar{x}) = 0$ for $k < n - 2$)

\begin{equation}
(A3) \quad \frac{\partial}{\partial \bar{x}} \pi^k_n(x, \bar{x}; y_{k-1}) = f_k(\bar{x}|y_{k-1})
\end{equation}

\begin{equation}
\times \left[ \pi^{k-1}_n(x, x; \{\bar{x}, y_{k-1}\}) - l^k_n(x, \bar{x}, y_{k-1}; \bar{x}) \right]
\end{equation}

giving the first-order condition

$$\pi^{k+1}_n(x, x; \{x, y_{k-1}\}) = l^k_n(x, x, y_{k-1}; x).$$

This equality holds for any strictly increasing bid function, since if an opponent exits in phase $k$ at price $b^k(x, y^k_{k-1})$, a bidder with signal $x$ will exit (lose) in phase $k + 1$ before any other bidders exit and, therefore, before any additional information is revealed. The same argument implies that $\pi^{k+1}_n(x, x; \{x, y_{k-1}\}) = 0$ for all $\bar{x} > x$. For $\bar{x} < x$, the right-hand side of (A3) can be written

$$f_k(\bar{x}|y^k_{k-1}) \left[ \pi^{k+1}_n(x, x; \{\bar{x}, y^k_{k-1}\})
- \int_{\bar{x}}^1 l^{k+1}_n(x, y, \{\bar{x}, y^k_{k-1}\}; \bar{x})
\times dF_{k+1}(y|\{\bar{x}, y_{k-1}\}) \right].$$

This expression must be positive since otherwise a deviant bid of $b^{k+1}_n(\bar{x}; \{\bar{x}, y^k_{k-1}\})$ is no worse than the equilibrium bid for phase $k + 1$, which is false by (A2). Hence

$$\frac{\partial}{\partial \bar{x}} \pi^k_n(x, \bar{x}; y_{k-1}) > 0 \quad \bar{x} < x$$

implying that $\pi^k_n(x, \bar{x}; y_{k-1})$ is maximized at $\bar{x} = x$ and that this choice of $\bar{x}$ is strictly preferred to any $\bar{x} < x$. The optimality of the proposed sequence of bid functions then follows by induction.

**PROOF OF THEOREM 3:**

Let

$$\hat{\omega}^k_n(x, y^k_{k-1}) = W(X, Y_1, ..., Y_{n-1})$$

and

$$\hat{\rho}^k_n(x, y^k_{k-1}) = L(X, Y_1, ..., Y_{n-1}; X_i)$$

both with the right-hand sides evaluated at $X_i = Y_{i-1} = Y_{n-1-k} = x$ and $Y^k_{k-1} = y^k_{k-1}$. Then

$$\hat{\omega}^k_n(x, y^k_{k-1}) = \hat{\omega}^{k-1}_n(x, y^k_{k-1}) - \hat{\rho}^k_n(x, y^k_{k-1}),$$

so it is sufficient to show

\begin{equation}
(A4) \quad \hat{\omega}^k_n(x, y^k_{k-1}) > \hat{\omega}^{k+1}_n(x, y^k_{k-1})
\end{equation}

\begin{equation}
(A5) \quad \hat{\rho}^k_n(x, y^k_{k-1}) < \hat{\rho}^{k+1}_n(x, y^k_{k-1}).
\end{equation}

Let $y^k(x, y^k_{k-1}) = Y$ evaluated at $Y_i = Y_{i-1} = Y_{n-1-k} = x$ and $Y^k_{k-1} = y^k_{k-1}$. To show (A4) observe that

\begin{equation}
(A6) \quad \hat{\omega}^k_n(x, y^k_{k-1}) = \sum_{m=n-1}^{\infty} \rho(m/n)
\times \left\{ \int_0^1 \int_0^1 \int_0^1 \nu_m(\mu, \bar{u}_1, \bar{u}_2; x)
\times d\Gamma_{12,m}(\bar{u}_1, \bar{u}_2|y^k(x, y^k_{k-1})) dG(u|x) \right\}
\end{equation}

where $\nu_m(u, \bar{u}_1, \bar{u}_2; x) = max\{u, \bar{u}_2, b^*(\bar{u}_1, x)\}$ and $\Gamma_{12;m}(\cdot, \cdot|y^k_{k-1})$ is the joint distribution of $\bar{u}_1$ and $\bar{u}_2$ given $Y^k_{k-1}$. Note that $\nu_m$ is non-decreasing in its first three arguments; furthermore, it is strictly increasing in $\bar{u}_2$ when $\bar{u}_2 > max\{u, b^*(\bar{u}_1, x)\}$ [an event given positive probability in (A6)] and strictly increasing in $\bar{u}_1$ when $b^*(\bar{u}_1, x) > max\{u, \bar{u}_2\}$ [also given positive probability in (A6)]. Since the distribution $\Gamma_{12;m}(\cdot, \cdot|y^k(x, y^k_{k-1}))$ strictly dominates $\Gamma_{12;m}(\cdot, \cdot|y^k(x, y^k_{k-1}))$ regardless of whether $k = k$ or $k = k + 1$ (the only
possibilities when one bidder is added, holding
the types of all other bidders fixed), this implies

$$\int_0^1 \int_0^1 \int_0^1 \nu_w(u, \bar{u}_1, \bar{u}_2; x) \times d\Gamma_{12m+1}(\bar{u}_1, \bar{u}_2|x, y^k) \, dG(u|x)$$

$$> \int_0^1 \int_0^1 \int_0^1 \nu_w(u, \bar{u}_1, \bar{u}_2; x) \times d\Gamma_{12m}(\bar{u}_1, \bar{u}_2|x, y^k) \, dG(u|x).$$

Strict affiliation of $m$ and $n$ then gives (A4).

To show (A5), note that

$$(A7) \quad \hat{p}_m(x, y^k) = \sum_{m=1}^{\infty} \rho(m|n)$$

$$\times \left\{ \int_0^1 \nu(t, x) \, dH_{1m}(t|y^k(x, y^k)) \right\}$$

where

$$\nu(t, x) = \int_t^{u^*(t;x)} (u - t)G(t|x) \, dG(u|x)$$

$$+ \int_t^{u^*(t;x)} [u - b^*(u, x)]$$

$$\times G(b^*(u, x)|x) \, dG(u|x).$$

Differentiation gives

$$(A8) \quad \frac{\partial}{\partial t} \nu_t(t, x)$$

$$= \int_t^{u^*(t;x)} [(u - t)g(t|x) - G(t|x)] \, dG(u|x)$$

$$- G(t|x) \, dG(u|x)$$

$$= [(u^*(t; x) - t)g(t|x) - G(t|x)]$$

$$\times [G(u^*(t; x)|x) - G(t|x)].$$

Since (1) implies that $\forall \ t > 0$

$$\int_x^1 G(t|y) \, dF(y) > t$$

the second term of the product in (A8) is positive for all $t > 0$. So showing that

$$(A9) \quad u^*(t; x) = t + \frac{\int_x^1 G(t|y) \, dF(y)}{\int_x^1 g(t|y) \, dF(y)} > t$$

will ensure that (A8) is negative. Given (A9),

$$(A10) \quad u^*(t; x) - t - \frac{G(t|x)}{g(t|x)} < 0$$

which is implied by strict affiliation. This shows that

$$\frac{\partial}{\partial t} \nu_t(t, x) < 0 \ \forall \ t > 0.$$ Since $\forall \ t > 0$

$$H_{1m+1}(t|y^k(x, y^k)) < H_{1m}(t|y^k(x, y^k))$$

the definition of first-order stochastic dominance implies

$$\int_0^1 \nu_t(t, x) \, dH_{1m+1}(t|y^k(x, y^k))$$

$$< \int_0^1 \nu_t(t, x) \, dH_{1m}(t|y^k(x, y^k)).$$

Strict affiliation of $m$ and $n$ then gives (A5).
REFERENCES


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