Money Illusion and Housing Frenzies[*]

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Abstract

A reduction in inflation can fuel run-ups in housing prices if people suffer from money illusion. For example, basing the decision on whether to rent or buy a house simply on monthly rent relative to current monthly mortgage payments, agents do not properly take into account that inflation lowers future real mortgage payments, therefore systematically misevaluating real estate. After empirically decomposing the price-rent ratio in a rational component and an implied mispricing, we find that (i) inflation and the nominal interest rate explain a large share of the time-series variation of the mispricing, (ii) the run-ups in housing prices starting in the late 1990s is reconcilable with the contemporaneous reduction in inflation and nominal interest rates, and (iii) the tilt effect cannot rationalize these findings.

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JEL classification: G12, R2.

1 Introduction

Housing prices have reached unprecedented heights in recent years. The sharp run-up in housing prices has been so striking that academics and non-academics alike have begun referring to it as housing bubble. Figure 1 illustrates different real house price indexes and shows that this phenomenon has been observed in several OECD countries.
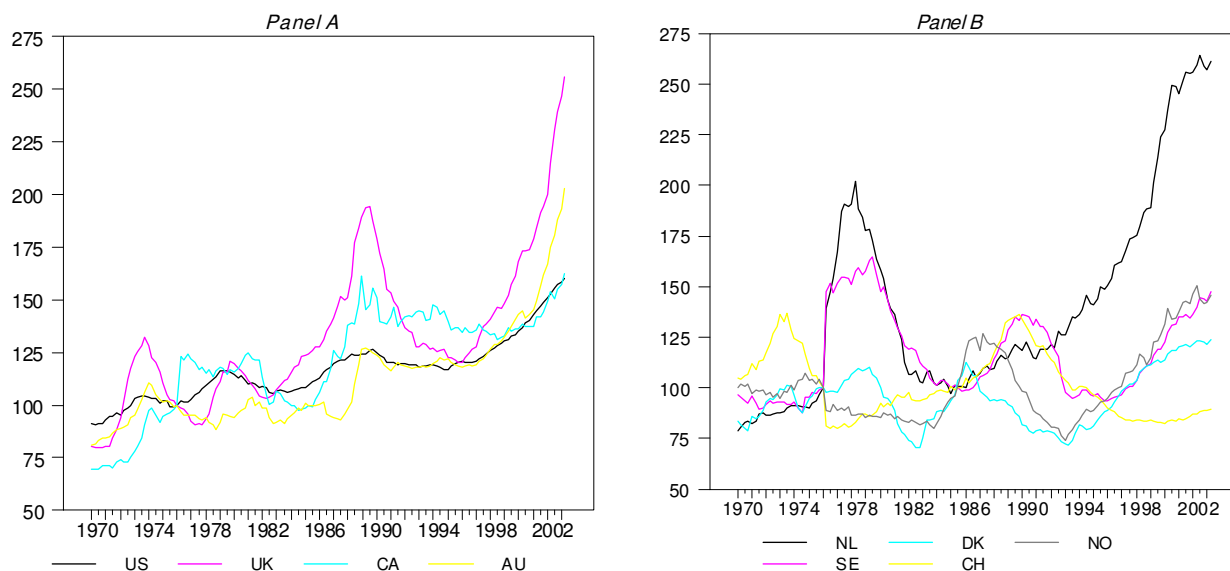

Figure 1: Residential property (real) price indices for a group of Anglo-Saxon countries (left panel) and for Scandinavian countries and other European countries (right panel). Base period is 1976, first quarter.

All the countries for which we have data show a sharp increase in housing prices with the exception of Switzerland (CH). The figure also shows that in the past sharp increases were typically followed by sharp downturns. Shiller (2005) documents similar patterns for other countries and cities over shorter samples. This suggests the presence of an underlying common factor that causes these large swings in housing prices. Indeed, since these swings lead to large wealth effects, a thorough understanding of the underlying mechanism leading to these run-ups is needed.

Most of these countries have also experienced a decline in the nominal interest rate over the last decade. Since the real interest rate has not registered such a sharp reduction, inflation seems to play an important role that cannot be fully explained by rational reasons.

In this paper we identify an empirical proxy for the mispricing in the housing market and show that it is largely explained by movements in inflation. Inflation matters and

it matters in a particular way. Our analysis shows that a reduction in inflation can generate substantial increases in housing prices in a setting in which agents are prone to money illusion. For example, people who base their decisions whether to rent or buy a house simply on monthly rent relative to the current monthly payment of a fixed nominal interest rate mortgage suffer from money illusion. They mistakenly assume that real and nominal interest rates move in lockstep. Hence, they wrongly attribute a decrease in inflation to a decline in the real interest rate and consequently underestimate the real cost of future mortgage payments. Therefore, they cause an upward pressure on housing prices when inflation declines.

To identify whether the link between house price movements and inflation is due to money illusion, we first have to isolate the rational components of price changes that are due to movements in fundamentals like land and construction costs, housing quality, property taxes, demographics (Mankiw and Weil (1989)) and time-varying risk premia.[1] We do so in two stages. First, by focussing on the price-rent ratio we insulate our analysis from fundamental movements that affect housing prices and rents symmetrically. Even though renting and buying a house are not perfect substitutes, the price-rent ratio implicitly controls for movements in the underlying service flow. Second, we try to identify rational channels through which inflation could influence the price-rent ratio. For this purpose, using a modified Campbell and Shiller's (1988) method, we decompose the price-rent ratio into two rational components (expected future returns on housing investment and rent growth rates)[2] and a mispricing component. After controlling for these rational channels, we find that inflation has a substantial explanatory power for the sharp run-ups and downturns of the housing market.

Figure 2 depicts the standardized time series of the (estimated) mispricing of the price-rent ratio in the U.K. housing market and the log of the reciprocal of the nominal interest rate, $\log\left(1/i_t\right)$. The first thing to notice is that the mispricing shows sharp and persistent run-ups during the sample period. Moreover, the movements in the *nominal* interest rate reciprocal closely match the momentum of the mispricing, even though, as shown in Section 3.1 below, the *real* interest rate does not seem to have any explanatory power for housing prices. The ability of the raw time series of the nominal interest rate reciprocal to track the trends in the mispricing is remarkable.

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[1] These variables alone are generally not able to capture the sharp run-ups in housing prices. It has become common in the empirical literature to add cubic 'frenzy' terms in the housing price regressions (see Hendry (1984) and Muellbauer and Murphy (1997)) and the rational expectations hypothesis has been rejected by the data (Clayton (1996)).

[2] First, inflation could be disruptive for the economy as a whole. This would lower agents' expectations of future real rent growth rates, thus reducing today's price-rent ratio. Second, an increase in inflation could make the economy riskier (or the agents more risk averse), thereby increasing the equilibrium risk-premium, which in turn reduces the price-rent ratio. Third, increase in inflation reduces the after-tax user cost of housing, potentially driving up housing demand (Poterba (1984)).
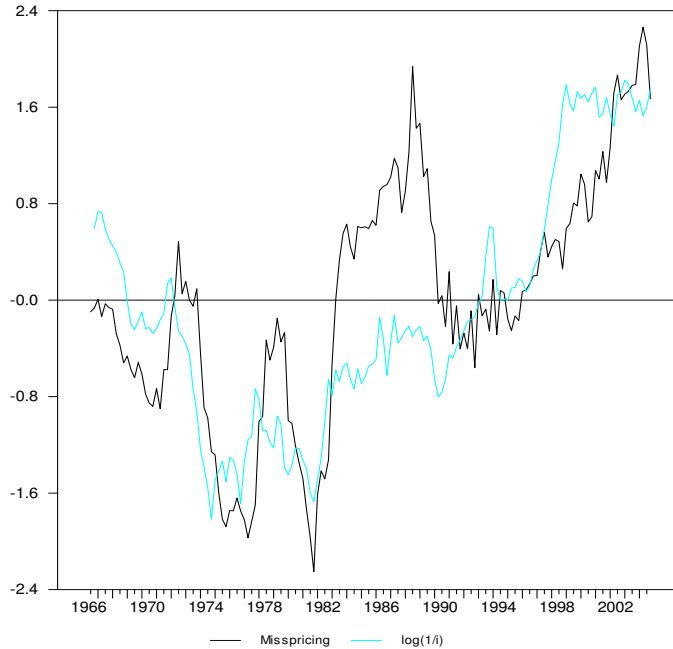
Figure 2: Mispricing and the log of the nominal interest rate reciprocal in the U.K.

The close link between inflation and housing prices could be due to the following departure from rationality and/or financing frictions. First, as argued by Modigliani and Cohn (1979), if agents suffer from money illusion their evaluation of an asset will be inversely related to the overall level of inflation in the economy. Moreover, as shown in Section 3.1 below, the nominal interest rate reciprocal that tracks trends in the mispricing of the housing market surprisingly well in Figure 2, corresponds to the valuation of the price-rent ratio of an agent that suffers from money illusion. This explanation of house price run-ups would also be in line with the finding of McCarthy and Peach (2004) that the sharp run-up in the U.S. housing market since the late 1990s can be largely explained by taking into account the contemporaneous reduction of nominal mortgage costs. Second, in an inflationary environment, the nominal payments on a fixed-payment mortgage are higher by a factor that is roughly proportional to the nominal interest rate reciprocal. This causes the real financing cost to shift towards the early periods of the mortgage, therefore causing a potential reduction in housing demand and prices. This is the so called *tilt effect* of inflation (see Lessard and Modigliani (1975) and Tucker (1975)). Nevertheless, why the tilt effect should matter cannot be fully explained in a rational setting since financial instruments that are not affected by this shift in the real cost of financing, like the price level

adjusted mortgage (PLAM) or the graduate payment mortgage (GPM), have been available to house buyers since at least the 1970s. Moreover, in Section 3.3 we show that the tilt effect is unlikely to be the driving force of the sharp run-ups in the housing market. Third, if fixed interest rate mortgages are not portable, individuals that have bought a house and locked in a low nominal interest rate might be less willing to sell their current house to buy a better one when nominal interest rates are higher. Hence, an increase in inflation that raises the nominal interest rate might depress the price of better quality residential properties. On the other hand, a reduction in inflation and nominal interest rates would free current home owners from this "lock in" effect.

The balance of the paper is organized as follows. The next section reviews the related literature on money illusion, borrowing constraint and speculative trading. Section 3 formally analyzes the link between inflation and housing prices using the U.K. housing market as a case study.[3] In particular: Section 3.1 derives a proxy for the valuation of the price-rent ratio of an agent that is affected by money illusion and provides a first assessment of the empirical link between housing prices and inflation; Section 3.2 provides a method, based on the difference between the objective and subjective measure of the market, to identify the mispricing in the price-rent ratio, and shows that the estimated mispricing is largely explained by changes in the rate of inflation; Section 3.3 shows that it is unlikely that the tilt effect is the driving force of the link between inflation and mispricing on the housing market. In Section 4 we extend our empirical analysis to a cross-country setting and show that the strong link between housing price mispricing and inflation holds across countries. A final section concludes and a full description of the data sources is provided in the appendix.

2 Related Literature

2.1 Money Illusion

"An economic theorist can, of course, commit no greater crime than to assume money illusion." Tobin (1972)

"In fact, I am persuadable – indeed, pretty much persuaded – that money illusion is a fact of life." Blinder (2000)

In this section we sketch the links to the existing literature. In particular, we review previous definitions of money illusion, relate it to the psychology literature and summarize the empirical evidence on the effect of money illusion on the stock market.

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[3]We first focus on the U.K. market since the longer sample period (1966:Q2–2004:Q4), the better quality of the data, the availability of PLAM mortgage schemes, and the fact that most U.K. mortgages are portable, allow for sharper and more robust inference.

Definition of Money Illusion. Fisher (1928, p. 4) defines money illusion as "the failure to perceive that the dollar, or any other unit of money, expands or shrinks in value."[4] Patinkin (1965, p. 22) refers to money illusion as any deviation from decision making in purely real terms: "An individual will be said to be suffering from such an illusion if his excess-demand functions for commodities do not depend [...] solely on relative prices and real wealth..." Leontief (1936) is more formal in his definition by arguing that there is no money illusion if demand and supply functions are homogenous of degree zero in all nominal prices.

Related Psychological Biases. Money illusion is also very closely related to other psychological judgement and decision biases. In a perfect world money is a veil and only real prices matter. Individuals face the same situation after doubling all nominal prices and wages. The *framing effect* states that alternative representations (framing) of the same decision problem can lead to substantially different behavior (Tversky and Kahneman (1981)). Shafir, Diamond, and Tversky (1997) document that agents' preferences depend to a large degree on whether the problem is phrased in real terms or nominal terms. This framing effect has implications on (i) time preferences as well as on (ii) risk attitudes. For example, if the problem is phrased in nominal terms, agents prefer the nominally less risky option to the alternative which is less risky in real terms. That is, they avoid nominal risk rather than real risk. If on the other hand the problem is stated in real terms, their preference ranking reverses. The degree to which individuals ignore real terms depends on the relative *saliency* of the nominal versus real frame.

Anchoring is a special form of framing effect. It refers to the phenomenon that people tend to be unduly influenced by some arbitrary quantities when presented with a decision problem. This is the case even when the quantity is clearly uninformative. For example, the nominal purchasing price of a house can serve as an anchor for a reference price even when the real price can be easily derived.[5]

While individuals understand well that inflation increases the prices of goods they buy, they often overlook inflation effects which work through indirect channels, e.g. general equilibrium effects. For example, Shiller (1997a) documents survey evidence that the public does not think that nominal wages and inflation comove over the long-run. Shiller (1997b) provides evidence that less than a third of the respondents in his survey study would have expected their nominal income to be higher if the U.S. had experienced higher inflation over the last five years. The impact of inflation on wages

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[4]Most authors use the terms "money illusion" and "inflation illusion" interchangeably. Sometimes the latter is also used to refer to a situation where households ignore changes in inflation.

[5]Fisher (1928) provides several interesting examples of inflation illusion due to anchoring. For example on pages 6-7 he writes about a conversation he had with a German shop woman during the German hyperinflation period in the 1920s: "That shirt I sold you will cost me just as much to replace as I am charging you [...] But I have made a profit on that shirt because I bought it for less."

is more indirect. Inflation increases the nominal profits of the firm, therefore it will increase nominal wages. Similarly, the reduction in mortgage rates due to a decline in expected future inflation expectations is direct, while the fact that it will also lower future nominal income is indirect. This inattention to indirect effects can be related to two well known psychological judgement biases: mental accounting and cognitive dissonance. *Mental accounting* (Thaler (1980)) is a close cousin of narrow framing and refers to the phenomenon that people keep track of gains and losses in different mental accounts. By doing so, they overlook the links between them. In our case, they ignore the fact that higher inflation affects the interest rate of the mortgage and the labor income growth rate in a symmetric way. *Cognitive dissonance* might be another reason why individuals do not see that inflation increases future nominal income. They have a tendency to attribute increases in nominal income to their own achievements than simply to higher inflation.[6]

Inflation Illusion and the Stock Market. To the best of our knowledge, we are the first who empirically assess the link between money illusion and house prices. However, there are a list of papers that empirically document the impact of money illusion on stock market prices, often referred to as the "Modigliani-Cohn" hypothesis. Modigliani and Cohn (1979) argue convincingly that prices significantly depart from fundamentals since investors make two inflation-induced judgement errors: (i) they tend to capitalize equity earnings at the nominal rate rather than the real rate and (ii) they fail to realize that firms' corporate liabilities depreciate in real terms. Hence, stock prices are too low during high inflation periods. Ritter and Warr (2002) document that the value-price ratio is positively correlated with inflation and that this effect is more pronounced for leveraged firms. Moreover, they show that the inflation and the value-price ratios are negatively correlated with future market returns. Using the Campbell and Shiller's (1988) dynamic log-linear evaluation method and a subjective proxy for the equity risk premium, Campbell and Vuolteenaho (2004) show in the time-series that a large part of the mispricing in the dividend-price ratio can be explained by inflation illusion.[7] Our methodology builds on their approach with the advantage that we do not have to arbitrarily specify a proxy for the risk premium on the housing investment. In contrast, Cohen, Polk, and Vuolteenaho (forthcoming) focus on the cross-sectional implications of money illusion on asset returns and find supportive evidence for the "Modigliani-Cohn" hypothesis.

Basak and Yan (2005) show, within a dynamic asset pricing model, that even though the utility cost of money illusion (and hence the incentive to monitor real values) is

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[6]Shiller (1997a) also noted that "Not a single respondent volunteered anywhere on the questionnaire that he or she benefited from inflation. [...] There was little mention of the fact that inflation redistributes income from creditors to debtors."

[7]Additional evidence on the time-series link between market returns and inflation can be found in Asness (2000, 2003) and Sharpe (2002).

small, its effect on equilibrium asset prices can be substantial. In the same spirit, Fehr and Tyran (2001) show that (under strategic complementarity) even if only a small fraction of individuals suffer from money illusion, the aggregate effect can be large.

2.2 Borrowing Constraint and Speculation

Tilt effect. Lessard and Modigliani (1975) and Tucker (1975) show that under nominal fixed payment and fixed interest rate mortgages, inflation shifts the real burden of mortgage payments towards the earlier years of the financing contract. This limits the size of the mortgages agents can obtain. This tilt effect could lead to a reduction in housing demand. Kearl (1979) and Follain (1982) find an empirical link between inflation and housing prices and argue that liquidity constraints could rationalize their finding. Wheaton (1985) questions this simple argument in a life-cycle model and shows that several restrictive assumptions are needed for this to be the case.

Speculative Trading and Short-Sale Constraints. Borrowing constraints might also limit the amount of speculation. Harrison and Kreps (1978) show that speculative behavior can arise if agents have different opinions, i.e. non-common priors. Said differently, even if they could share all the available information, they would still disagree about the likelihood of outcomes. Scheinkman and Xiong (2003) put this model in a continuous-time setting and show that transaction costs dampen the speculative component of trading, but only have limited impact on the size of the bubble. Models of this type rely on the presence of short-sale constraints – which is a natural constraint in the housing market – to preempt the ability of rational agents to correct the mispricing. Other factors that limit arbitrage include noise-trader risk (DeLong, Shleifer, Summers, and Waldmann (1990)) and synchronization risk (Abreu and Brunnermeier (2003)).

3 Housing Prices and Inflation

We focus on the link between inflation and the price-rent ratio. In principle, an agent could either buy or rent a house to receive the same service flow. However, renting and buying a house are not perfect substitutes since households might derive extra utility from owning a house (e.g. ability to customize the interior, pride of ownership). Moreover, properties for rent might on average be different from properties for sale.[8]

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[8]The house price index reflects all types of dwellings while rents tend to overweight smaller and lower quality dwellings. Given that high quality houses fluctuate more over the business cycle, the data might show a spurious link between inflation, nominal interests rate and the price-rent ratio if inflation and/or nominal interest rates had a clear business cycle pattern. We address this concern formally in Section 3.2.2 and show that this does not affect our main findings.

Nevertheless, long-run movement in the rent level should capture long-run movements in the service flow. Furthermore, changes in construction cost, demographic changes, and changes in housing quality should at least in the long-run affect house prices and rent symmetrically. As a consequence, in studying mispricing on the housing market, we focus on the price-rent ratio. Gallin (2004) finds that house prices and rents are cointegrated and that the price-rent ratio is a good predictor of future price and rent changes. Compared to the price-income ratio, the price-rent ratio has the advantage of being less likely to increase dramatically due to changes in fundamentals (e.g. in demography or property taxes). Moreover, Gallin (2003) empirically rejects the hypothesis of cointegration between prices and income using panel-data tests for cointegration, that have been shown to be more powerful than the time-series analog. This implies that the commonly used error correction representation of prices and income would lead to erroneous frequentist inference. Finally, studying the price-rent ratio is analogous to the commonly used price-dividend ratio to analyze the mispricing in the stock market.

In this section we show first that a simple non-linear function of the nominal interest rate is a proxy for the valuation of the price-rent ratio by an agent prone to money illusion. Empirically, we first document the correlation between nominal values and future price-rent ratios. To gain further understanding of this empirical link, we then decompose the price-rent ratio into a rational component and an implied mispricing and study its comovements with inflation. In this section we conduct our empirical analysis focusing on U.K. data because the longer sample period (1966:Q2–2004:Q4) and the better quality of the data allow us to obtain a sharper and more robust inference. The subsequent Section 4 expands the analysis to a cross-country setting, confirming the results of the U.K. data.

3.1 Housing Prices and Money Illusion - A First-Cut [9]

In a dynamic optimization setting the equilibrium real price an agent is willing to pay for the house, P_t, should be equal to the present discounted value of future real rents, $\{L_t\}$, and the discounted resale value of the house.

$$P_t = \tilde{E}_t \left[\sum_{\tau=t+1}^{T-1} m_{t,\tau} L_\tau + m_{t,T} P_T \right]$$

where $m_{t,\tau}$ is the stochastic discount factor between t and $\tau > t$, T is the time of resale and $\tilde{E}_t$ is the expectations operator given agents' subjective beliefs at time t.

In order to present a first insight into the role of inflation bias, we start by considering a simple setting without uncertainty and with constant real rent as in Modigliani

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[9]Readers who are familiar with the empirical link between inflation and housing prices can skip this section without loss of continuity.

and Cohn (1979). In this case the equilibrium price-rent ratio for an economy with rational agents is

$$\frac{P_t}{L_t} = E_t \left[\sum_{\tau=t+1}^{T-1} \frac{1}{(1+r_\tau)^{\tau-t-1}} \right] \simeq \frac{1}{r}, \tag{1}$$

where we let $T \to \infty$, r is the *real* risk-free rate and we assume that $\lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T P_T = 0$. The last equation holds exactly if the real risk-free rate, r_t, is constant.[10]

Instead, if the agent suffers from money illusion, she treats the (constant) nominal risk-free rate as real, $\tilde{E}_t[r_\tau] = E_t[i_\tau]$. This implies the inflation biased evaluation

$$\frac{P_t}{L_t} = \tilde{E}_t \left[\sum_{\tau=t+1}^{T-1} \frac{1}{(1+r_\tau)^{\tau-t-1}} \right] \simeq E_t \left[\sum_{\tau=t+1}^{T-1} \frac{1}{(1+i_\tau)^{\tau-t-1}} \right] \simeq \frac{1}{i}, \tag{2}$$

where the first approximation ignores the Jensen's inequality term and the second approximation is exact if the nominal interest rate, i_t, is constant.[11] This derivation parallels the one in Modigliani and Cohn (1979) for the stock market. Equations (1) and (2) suggest to that $1/i_t$, $1/r_t$ and inflation π_t should be used as alternative regressors to test for money illusion. It is also worth emphasizing that $1/i_t$ is highly non-linear in i_t for low i_t – a fact independently emphasized for the real interest rate by Himmelberg, Mayer, and Sinai (2005).

Note that the tilt effect leads to the same regressor, since a mortgage with fixed nominal annual payment of 1 dollar forever is currently valued at $1/i_t$. Hence, the maximum size of mortgage a household can afford is determined by $1/i_t$. We devote Section 3.3 to discriminate between money illusion and the tilt effect.

To take a first look at the empirical link between inflation, nominal interest rates and the price-rent ratio, we explore wether i_t, r_t, π_t, $1/i_t$ and $1/r_t$ have forecasting power

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[10]Note that strictly speaking L_t reflects all payoffs from owning a house. This includes not only the service flow from living in the house but also tax benefits, property tax etc. For our empirical analysis we focus only on the main component: the market price of the service from living in the house. The standard user cost approach in real estate economics takes the other components into account as well. The user cost is stated in terms of per dollar of house value. More specifically, $u_t = r_t^f + \omega_t - \tau(r_t^m + \pi_t + \omega_t) + \delta_t - g_{t+1} + \gamma_t$, where r_t^f is the risk-free real interest rate, ω_t the property tax per dollar house value, the third term captures the fact that nominal interest payments and property tax are deductible form the income tax with marginal tax rate τ, δ_t reflects maintenance costs and g_{t+1} is the capital gain (loss) per dollar of house value, γ_t is the risk premium. Note that since nominal mortgage interest payments are income tax deductible, inflation lowers user cost and, since the price-rent ratio should be equal to the reciprocal of the user cost, this suggests higher house prices (see Poterba (1984, 1991)). This is exactly the opposite inflation effect of the one caused by money illusion. A major drawback of the user cost approach is that the house price appreciation is assumed to be exogenous and is not derived from a consistent dynamic equilibrium. In particular, by assuming that the price appreciation follows historical patterns, one implicitly assume "irrational" positive feedback trading phenomena.

[11]Equation (2) makes clear that money illusion matters independently of whether the mortgage contract is a flexible rate or a fixed rate one.

for the price rent ratio. In assessing the forecasting performance of these variables, one faces several econometric issues. First, Ferson, Sarkissian, and Simin (2002) use a simulation exercise to argue that the in-sample regression results may be spurious, and both R^2 and statistical significance of the regressor are biased upward if both the *expected* part of the regressand and the predictive variable are highly persistent (see also Torous, Valkanov, and Yan (2005)). Therefore, since P_t/L_t is highly persistent, this could lead to spurious results. Second, in exploring the forecastability of the price-rent ratio, the choice of the control variables is problematic and to some extent arbitrary since the literature on housing prices has suggested numerous predictors. Moreover, Poterba (1991) outlines that the relation between house prices and forecasting variables often used in the literature has not been stable across sub-samples.

We address both issues jointly. For the first problem, we remove the persistent component of the price-rent ratio by constructing the forecasting errors

$$\hat{\delta}_{t+1,t+1-\tau} = \begin{cases} P_{t+1}/L_{t+1} - \hat{E}_{t-\tau}\left[P_{t+1}/L_{t+1}\right] & \text{for } \tau > 0 \\ P_{t+1}/L_{t+1} & \text{for } \tau = 0 \end{cases} \tag{3}$$

where τ is the forecasting horizon and $\hat{E}_{t-\tau}\left[P_t/L_t\right]$ is the (estimated) persistent component of the price-rent ratio and we introduce the convention that for $\tau = 0$, $\hat{\delta}_{t+1,t+1} = P_{t+1}/L_{t+1}$. Second, we estimate $\hat{E}_{t-\tau}\left[P_t/L_t\right]$ by fitting a reduced form vector auto regressive model (VAR) for P_t/L_t, the log gross return on housing, $r_{h,t}$, the rent growth rate Δl_t and the log real return on the twenty-year Government Bonds, r_t (constructed as the nominal rate, i_t, minus quarterly inflation).

Following Campbell and Shiller (1988), for small perturbations around the steady state, the variables included in the VAR should capture most of the relevant information for the price-rent ratio. Indeed, the R^2 of the VAR equation for P_t/L_t is about 99 percent, which is consistent with previous studies that have outlined the high degree of predictability of housing prices (see, among others, Kearl (1979), Follain (1982) and Muellbauer and Murphy (1997)). This approach for constructing forecast errors, $\hat{\delta}_{t+1,t+1-\tau}$, is parsimonious since it allows us to remove persistency from the dependent variable without assuming a structural model. It is also conservative since the reduced form VAR is likely to over-fit the price-rent ratio. We use quarterly data over the sample period 1966 third quarter to 2004 fourth quarter.[12]

Figure 3 summarizes the results about the predictability of the price-rent ratio. The figure plots the t-statistics[13] (Panel A) and measures of fit (Panel B) of five univariate regressions of $\hat{\delta}_{t+1,t+1-\tau}$ on r_t, i_t, $1/r_t$, $1/i_t$ and a smoothed moving average of inflation,

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[12] The VAR is estimated with one lag since this is the optimal lag length suggested by both the Bayesian and Akaike information criteria

[13] The t-statistics are constructed using Newey and West (1987) corrected standard errors.
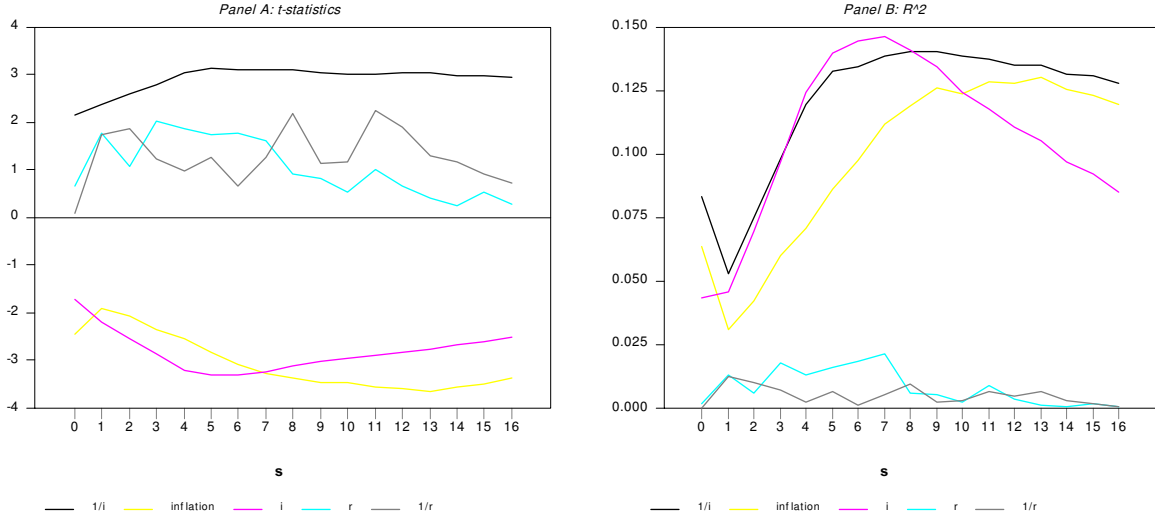
Figure 3: t-statistics and R^2 of univariate regressions of the forecast error $\hat{\delta}_{t+1,t+1-\tau}$ on interest rates and interest rate reciprocals (both nominal and real) as well as inflation.

π_t.[14] (Recall that we introduced the convention that for $\tau = 0$, $\hat{\delta}_{t+1,t+1} = P_{t+1}/L_{t+1}$). That is, the first point in each of the plotted series corresponds to the regression output of a standard forecasting regression for the price-rent ratio.

Focusing first on $\tau = 0$ – the standard forecasting regression – it is apparent that the real interest rate, r, has no forecasting power for the price-rent ratio with a t-statistic (Panel A) of 0.741 and a R^2 (Panel B) of about 0 percent. This is consistent with the finding of Muellbauer and Murphy (1997) that the real interest rate has no explanatory power for movements in the real price of residential housing. The sign of the slope coefficient of the nominal interest rate, i, is negative suggesting that an increase in the nominal interest rate reduces the price-rent ratio. The regressor is statistically significant only at the 10 percent level and explains about 5 percent of the variation in the price-rent ratio. The figure also shows that lagged inflation is a significant predictor of the price-rent ratio and that the estimated slope coefficient has a negative sign, which is consistent with the Modigliani and Cohn (1979) argument that inflation causes a negative mispricing in assets. This is also consistent with the findings of Kearl (1979) and Follain (1982) that housing demand is reduced by greater inflation. The regressor explains about 7 percent of the time variation in P_t/L_t. From the predictive regression of the price-rent ratio on $1/r_t$ – as suggested by equation (1) – we learn that this variable is not significant nor has any forecasting power for the future price-rent ratio, reinforcing the conjecture that house prices do not tend to respond

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[14]Note that the measure of inflation we use is the CPI index *without* housing. The smoothing window is of sixteen quarters and we take .9 as smoothing parameter.

to changes in the real interest rate. However, the reciprocal of the nominal interest rate, $1/i_t$, is highly statistically significant and has a positive sign implying that the price-rent ratio tends to comove with the valuation of agents prone to money illusion. Moreover, this regressor is able to explain about 9 percent of the time variation in the price-rent ratio. Consistently with money illusion, inflation π_t shows a significant negative correlation with housing prices.

Focusing on $\tau > 0$, we can assess whether the regressors considered have forecasting power for the unexpected component of price-rent changes. It is clear from Figure 3 that the real interest rate (both in terms of r and $1/r$) generally has no explanatory power for the unexpected movements in the price-rent ratio. To the contrary, the nominal interest rate, inflation and the reciprocal of the nominal interest rate are statistically significant forecasting variables of unexpected movements in the price-rent ratio, and explain a substantial share of the time series variation of this variable.

For robustness we check our results using the real interest rate implied by the inflation protected 10 years government securities, instead of using nominal interest rate minus inflation, and using the implied inflation instead of our smoothed inflation. Unfortunately, this data is available only since 1982:Q1. Consistently with the previous results, we find that this measure of the real interest rate also has no explanatory power for the price rent ratio: the regressor is not statistically significant for any horizon τ and its point estimates changes sign at some horizons. Moreover, using implied inflation instead of smoothed inflation we obtain similar patterns as in Figure 3. The only difference is that implied inflation is not statistically significant at two horizon levels, $\tau = 1$ and 2; this is likely to be due to the fact that we lose 16 years of quarterly data using implied inflation.

These results suggest the presence of a strong empirical link between nominal values and the price-rent ratio but do not clarify whether this link is the consequence of rational behavior or money illusion. We disentangle the role of money illusion in the next subsection.

3.2 Decomposing the Inflation Effect

Inflation can affect the price-rent ratio for rational reasons. In this subsection we differentiate the rational effects of inflation on the price-rent ratio – through expected future rent growth rates and expected future returns on housing – from the effect of inflation on the mispricing. Note also that inflation can influence expected future returns directly or through the taxation effect mentioned earlier.

3.2.1 Methodology

We follow the Campbell and Shiller (1988) methodology, but also allow agents to have subjective beliefs. Letting P be the price of housing and L be the rental payment, the

gross return on housing, R_h, is given by the following accounting identity:

$$R_{h,t+1} = \frac{P_{t+1} + L_{t+1}}{P_t}.$$

Following Campbell and Shiller (1988), we log-linearize this relation around the steady state but, given our focus on mispricing, we allow traders to have a probability measure for the underlying stochastic process that is different from the objective one. As a consequence, the steady state depends on the underlying measure of the traders. Under the assumption that the price-rent ratio is stationary, we can log-linearize the last equation as

$$r_{h,t+1} = (1 - \rho) k + \rho (p_{t+1} - l_{t+1}) - (p_t - l_t) + \Delta l_{t+1},$$

where $\rho := 1/(1 + \exp(l - p))$, $r_h := \log R_h$, $p := \log P$, $l := \log L$, $\Delta l_{t+\tau} := l_{t+\tau} - l_{t+\tau-1}$, k is a constant, and variables without time subscript are evaluated at their time series average. The log price-rent ratio can be therefore rewritten (disregarding a constant term) as a linear combination of future rent growth, future returns on housing and a terminal value

$$p_t - l_t = \lim_{T \to \infty} \left[\sum_{\tau=1}^{T-1} \rho^{\tau-1} (\Delta l_{t+\tau} - r_{h,t+\tau}) + \rho^T (p_{t+T} - l_{t+T}) \right]. \tag{4}$$

Moving to excess rent growth rates, $\Delta l_{t+\tau}^e = \Delta l_t - r_t$, and excess returns (risk premia) on housing, $r_{h,t}^e = r_{h,t} - r_t$, where r_t is the real return on the long-term government bond (with maturity of 10 or 20 years), the price-rent ratio can be expressed as

$$p_t - l_t = \sum_{\tau=1}^{\infty} \rho^{\tau-1} \left[\Delta l_{t+\tau}^e - r_{h,t+\tau}^e \right] + \lim_{T \to \infty} \rho^T (p_{t+T} - l_{t+T}). \tag{5}$$

This equality also has to hold for any realization and hence, holds in expectation for any measure.

Conservative Mispricing Measure. Our conservative mispricing measure, ε_t, is designed to reflect deviations between objective and subjective expectation about excess rent growth rates and future risk premia. More formally, we define the mispricing, ε_t, as the difference between observed log price-rent ratio and the log price-rent ratio that would prevail if (i) all agents were computing expectations under the objective measure and (ii) the transversality condition under the objective measure holds, i.e. $E_t \left[\lim_{T \to \infty} \rho^T (p_{t+T} - l_{t+T}) \right] = 0$. That is, the mispricing captures the difference in expectations about future excess rent growth rates and housing investment risk premia plus $\tilde{E}_t \left[\lim_{T \to \infty} \rho^T (p_{t+T} - l_{t+T}) \right]$,

$$\varepsilon_t := \sum_{\tau=1}^{\infty} \rho^{\tau-1} \left(\tilde{E}_t - E_t \right) \left[\Delta l_{t+\tau}^e - r_{h,t+\tau}^e \right] + \tilde{E}_t \left[\lim_{T \to \infty} \rho^T (p_{t+T} - l_{t+T}) \right],$$

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where we use the convention $\left(\tilde{E}_t - E_t\right)[x] := \tilde{E}_t[x] - E_t[x]$. The second term in the last equation is zero if the transversality condition holds under the subjective measure. If traders have the objective measure and they do not believe in explosive paths, then ε_t is always zero. Otherwise, this need not be the case. Note that we assume that all traders have the same subjective measure. If traders have heterogeneous measures and face short-sale constraints (as for example in Harrison and Kreps (1978)), ε_t would also be affected by a speculative component.

Taking objective expectations of Equation (5) and assuming that we can interchange the order of limit, summation and expectation operator, the price-rent ratio can be restated as

$$p_t - l_t = \sum_{\tau=1}^{\infty} \rho^{\tau-1} E_t\left[\Delta l_{t+\tau}^e - r_{h,t+\tau}^e\right] + \underbrace{E_t\left[\lim_{T\to\infty} \rho^T\left(p_{t+T} - l_{t+T}\right)\right]}_{=\varepsilon_t}.$$

The ε-mispricing can then be expressed as a violation of the transversality condition under the objective measure. To see this, take subjective expectation of equation (5) and subtract the above equation from it. Therefore, the ε-mispricing captures bubbles which are due to potentially exploding paths, including as a particular case the intrinsic bubbles analyzed in Froot and Obstfeld (1991). Our assumption that the price-rent ratio is stationary also guarantees that ε_t is finite (provided that rent growth rates and housing returns are stationary).

So far our analysis applies to any form of belief distortion and is not specific to money illusion. In order to see how our definition of mispricing can capture money illusion, let's consider the following example: as in Modigliani and Cohn (1979) individuals fail to distinguish between nominal and real rates of returns. They mistakenly attribute a decrease (increase) in inflation π_t to a decline (increase) in real returns, $r_{h,t}$ – or equivalently ignore that a decrease in inflation also lowers nominal rent growth rate $(\Delta l_t + \pi_t)$, i.e. $\tilde{E}_t\left[\Delta l_{t+\tau} - r_{h,t+\tau}\right] = E_t\left[\Delta l_{t+\tau} - r_{h,t+\tau} - \pi_{t+\tau}\right]$. For simplicity, let us also assume that the transversality condition holds under the subjective measure, i.e. $\tilde{E}_t\left[\lim_{T\to\infty} \rho^T\left(p_{t+T} - l_{t+T}\right)\right] = 0$. In this case, our mispricing measure reduces to

$$\varepsilon_t = -\sum_{\tau=1}^{\infty} \rho^{\tau-1} E_t\left[\pi_{t+\tau}\right].$$

That is, the mispricing and hence the price-rent ratio are increasing as expected inflation declines. Note that in this particular case money illusion always causes a negative mispricing error. However, if individuals have a reference level of inflation, say $\bar{\pi}$, this is not necessarily true. In this case the last equation becomes

$$\varepsilon_t = -\sum_{\tau=1}^{\infty} \rho^{\tau-1} E_t\left[\pi_{t+\tau} - \bar{\pi}\right].$$

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In this setting, even though the level of mispricing is different, its correlation with inflation is unchanged.

To construct the empirical counterpart of ε_t we follow Campbell (1991) and compute the objective expectations of future returns and rent growth rates using a reduced form VAR. The variables included in the VAR are the log excess return on housing, $r_{h,t}^e$, the log price-rent ratio, $p_t - l_t$, the excess rent growth rate, Δl_t^e, and the exponentially smoothed moving average of inflation, π_t. The VAR is estimated using quarterly data and the chosen lag length is one (both the Bayesian and the Akaike information criteria prefer this lag length for the estimated model).

With the estimated VAR at hand we then decompose the observed log price-rent ratio into three components: the implied pricing error, $\hat{\varepsilon}$, the discounted expected future rent growth, and the discounted expected future returns

$$p_t - l_t = \sum_{\tau=1}^{\infty} \rho^{\tau-1} \hat{E}_t \Delta l_{t+\tau}^e - \sum_{\tau=1}^{\infty} \rho^{\tau-1} \hat{E}_t r_{h,t+\tau}^e + \hat{\varepsilon}_t, \tag{6}$$

where $\hat{E}_t$ denotes conditional expectations computed using the estimated VAR. Equation (6) allows us to break the economic link between inflation and the rent-price ratio into its rational part, given by the first two terms on the right hand side, and the mispricing, $\hat{\varepsilon}$.

Our specification can be viewed as modification of the Campbell and Vuolteenaho (2004) approach for the U.S. stock market. We do not rule out explosive paths. Moreover, our mispricing $\hat{\varepsilon}_t$ has the advantage that we do not need to rely on an exogenously constructed risk factor to construct subjective expectations. In the next section we construct a less stringent mispricing measure which relies on an subjective risk factor like in Campbell and Vuolteenaho (2004), but we do not impose transversality condition to be equal to zero under both measures.

Less Conservative Mispricing Measure. Our measure of mispricing ε_t might be too conservative, since it only captures belief distortions that cause mispricing that leads to a violation of the transversality condition under the objective measure.

If we model the subjective risk-premium, $\tilde{E}_t \left[r_{h,t+\tau}^e \right]$, directly, we obtain a less conservative measure of mispricing, ψ_t.

$$p_t - l_t = \sum_{\tau=1}^{\infty} \rho^{\tau-1} E_t \left[\Delta l_{t+\tau}^e \right] - \sum_{\tau=1}^{\infty} \rho^{\tau-1} \tilde{E}_t \left[r_{h,t+\tau}^e \right] \tag{7}$$

$$+ \underbrace{\tilde{E}_t \left[\lim_{T \to \infty} \rho^T \left(p_{t+T} - l_{t+T} \right) \right] + \sum_{\tau=1}^{\infty} \rho^{\tau-1} \left(\tilde{E}_t - E_t \right) \left[\Delta l_{t+\tau}^e \right]}_{=:\psi_t}.$$

Equation (7) can easily be derived by taking subjective expectations of Equation (4) and by adding and subtracting $\sum_{\tau=1}^{\infty} \rho^{\tau-1} E_t \left[\Delta l_{t+\tau}^e \right]$. Note that ψ_t coincides with the belief distortion about futures rent growth rates if the transversality condition under the subjective measures is zero.

One way to model $\tilde{E}_t \left[r_{h,t+\tau}^e \right]$ is to assume that it can be represented as a linear function of a subjective risk factor λ_t, that is

$$\sum_{\tau=1}^{\infty} \rho^{\tau-1} E_t \left[r_{h,t+\tau}^e \right] = \underbrace{\alpha + \beta \lambda_t}_{=: \sum_{\tau=1}^{\infty} \rho^{\tau-1} \tilde{E}_t \left[r_{h,t+\tau}^e \right]} - \eta_t$$

where η_t is the difference between objective and subjective expected future risk premia.

Note that equating objective and subjective expectation of Equation (4) we have, after rearranging, that

$$\sum_{\tau=1}^{\infty} \rho^{\tau-1} E_t \left[r_{h,t+\tau}^e \right] = \sum_{\tau=1}^{\infty} \rho^{\tau-1} \tilde{E}_t \left[r_{h,t+\tau}^e \right]$$

$$\underbrace{- \sum_{\tau=1}^{\infty} \rho^{\tau-1} \left(\tilde{E}_t - E_t \right) \left[\Delta l_{t+\tau}^e \right] - \tilde{E}_t \left[\lim_{T \to \infty} \rho^T \left(p_{t+T} - l_{t+T} \right) \right] + E_t \left[\lim_{T \to \infty} \rho^T \left(p_{t+T} - l_{t+T} \right) \right]}_{= -\psi_t + \varepsilon_t}.$$

This implies that we can recover ψ_t as a linear combination of our baseline mispricing measure ε_t and the residual η_t

$$\psi_t = \varepsilon_t + \eta_t.$$

In order to construct the empirical counterpart of ψ_t we have to chose a subjective risk factor, λ_t, for the risk premia on housing investment. As suggested in Campbell and Vuolteenaho (2004) we use as risk proxy the conditional volatility of an investment that is long on housing market and short on the 10 years government bonds. That is, we construct $\hat{\eta}_t$ as the OLS residual of the following linear regression

$$\sum_{\tau=1}^{\infty} \rho^{\tau-1} \hat{E}_t \left[r_{t+\tau}^e \right] = \hat{\alpha} + \sum_{\tau=0}^{7} \hat{b}_\tau \hat{h}_{t-\tau} - \hat{\eta}_t. \tag{8}$$

where the regressors $\hat{h}_{t-\tau}$ are GARCH-estimates of the conditional volatility[15] and $\sum_{\tau=0}^{7} \hat{b}_\tau \hat{h}_{t-\tau} = \widehat{\beta \lambda_t}$. We therefore implement the following decomposition of the price rent ratio

$$p_t - l_t = \sum_{\tau=1}^{\infty} \rho^{\tau-1} \hat{E}_t \left[\Delta l_{t+\tau}^e \right] - \hat{\alpha} - \widehat{\beta \lambda_t} + \hat{\psi}_t,$$

where $\hat{\psi}_t = \hat{\varepsilon}_t + \hat{\eta}_t$.

---

[15] The fitted model is a GARCH(2,2) with an AR(1) component for the mean to take into account the persistence in housing returns.

Some note of caution is appropriate about this particular decomposition. First, the measure of mispricing ψ_t depends crucially on the chosen subjective risk factor λ_t – which is arbitrary. Second, $\hat{\psi}_t$ is likely to overestimate the actual mispricing, since $\widehat{\beta\lambda_t}$ might not fully capture the subjective expected risk premia. This in turn would deliver a measure of mispricing with a lower signal to noise ratio. Third, for the OLS construction in Equation (8) to be correct, λ_t should be orthogonal to η_t. Fourth, since $\hat{\psi}_t$ will inherit the time series properties of $\widehat{\beta\lambda_t}$ through $\hat{\eta}_t$, this could deliver a spurious link with inflation.

3.2.2 Empirical Evidence

In this subsection we focus on the empirical links between mispricing measures and inflation. As in the previous methodological subsection, we first focus on the ε-mispricing and then proceed with the ψ-mispricing.

ε-mispricing. Analyzing the first empirical decomposition

$$ p_t - l_t = \sum_{\tau=1}^{\infty} \rho^{\tau-1} \hat{E}_t \left[\Delta l_{t+\tau}^e \right] - \sum_{\tau=1}^{\infty} \rho^{\tau-1} \hat{E}_t \left[r_{h,t+\tau}^e \right] + \hat{\varepsilon}_t, $$

is fundamental in assessing the role of inflation on housing prices, since the negative correlation between the price-rent ratio and inflation observed in the data could be solely the outcome of fully rational behavior. There are several rational channels through which inflation could affect housing prices. First, if inflation damages the real economy, $\sum_{\tau=1}^{\infty} \rho^{\tau-1} \hat{E}_t \left[\Delta l_{t+\tau}^e \right]$ should be negatively related with inflation. For example, this could be the case of stagflation caused by a cost-push shock. Second, $\sum_{\tau=1}^{\infty} \rho^{\tau-1} \hat{E}_t \left[r_{h,t+\tau} \right]$ could tend to rise if inflation makes the economy riskier (or investors more risk averse), therefore driving up the required excess return on housing investment. If any of these were the case, the negative correlation between price-rent ratio and inflation could simply be the outcome of negative real effects of inflation or of time varying risk premia on the housing investment. Most importantly, if there were no inflation illusion, we would expect $\hat{\varepsilon}$ to be uncorrelated with π_t, $\log(1/i_t)$, and i_t (unless reduction in inflation fuels speculative frenzies). Instead, the Modigliani and Cohn (1979) hypothesis of money illusion would predict a negative correlation between the mispricing, $\hat{\varepsilon}$, and inflation (and the nominal interest rate), and a positive correlation between the mispricing and $\log(1/i_t)$.

Table 1 Panel A reports the regression output of the three components of the log price-rent ratio in equation (6), on the exponentially smoothed moving average of inflation, π_t, the nominal interest rate, i_t, and the log of its reciprocal, $\log(1/i_t)$.

Dependent Variables:			Regressors:			
	π_t		i_t		$\log\left(1/i_t\right)$	
	Slope coeff.	R^2	Slope coeff.	R^2	Slope coeff.	R^2
Panel A:						
$\hat{\varepsilon}_t$	-3.904 (7.946)	.65	-6.295 (6.927)	.55	.129 (5.991)	.52
$\sum_{\tau=1}^{\infty}\rho^{\tau-1}\hat{E}_t\Delta l_{t+\tau}^e$	-2.577 (2.390)	.12	-3.962 (1.938)	.09	.093 (2.083)	.12
$-\sum_{\tau=1}^{\infty}\rho^{\tau-1}\hat{E}_t r_{h,t+\tau}^e$	1.949 (1.242)	.03	3.581 (1.050)	.04	$-.052$ (0.687)	.02
Panel B:						
$\hat{\psi}_t$	-6.15 (2.48)	.17	-10.848 (2.66)	.17	.241 (2.82)	.19

Table 1: Univariate Regressions on inflation, nominal interest rate and illusion proxy. Newey and West (1987) corrected t-statistics in brackets.

The first row of Table 1 Panel A reports the univariate regression output of regressing the pricing errors on the proxies that are meant to capture inflation illusion. All the regressors are highly statistically significant and the estimated signs are the one we would expect under money illusion: the mispricing of the price-rent ratio tends to rise as inflation and nominal interest rates decrease and $\log\left(1/i_t\right)$ rises. Moreover, our proxies for inflation bias are able to explain between one half and two thirds of the time series variation of the mispricing of the price-rent ratio. Ideally, we would like to regress $\hat{\varepsilon}_t$ on the objective expectation of future inflation. One way to capture variations in expected inflation is to use the series of implied inflation from the inflation protected 10 years government securities. Using this measure as explanator of $\hat{\varepsilon}_t$ we obtain an R^2 of 37 percent and a $t-$statistics of 3.017.

The second row shows that expected future real rent growth rates seem to be negatively correlated with inflation and nominal interest rate (this last variable is significant only at the 10 percent level), and positively correlated with $\log\left(1/i_t\right)$. Nevertheless, only a small share (between 9 percent and 12 percent) of the time variation in expected rent growth are explained by the regressors considered. These results are consistent with a view in which inflation influences the rent to price ratio partially due to the fact that an increase in inflation damages the real economy. On the other hand, this could simply be the outcome of housing rents being more sticky than the general price level.

The third row outlines that there is no significant link between inflation and risk premia on the housing investment. The regressors considered are not statistically significant and explain only between 2 percent and 4 percent of the time series variation in expected future returns on housing. Moreover, the estimated signs of the regressors imply that inflation is associated with a lower risk premium on housing investment, i.e. in times of high inflation the housing investment is considered to be less risky

than investing in long-horizon government bonds. Since we use a before-tax measure of returns on housing, this result could also be due to the fact that an increase in inflation increases the after-tax return on housing (see Poterba (1984)), therefore requiring a lower before-tax risk premium.

The sum of the slope coefficients associated with each of the regressors in Table 1 Panel A is an estimate of the elasticity of the price-rent ratio with respect to that regressor. Our results therefore imply that, on average, a one percent increase in inflation (nominal interest rate) maps into a 4.5 (6.7) percent decrease in the price of housing relative to rent, and that the largest contribution to this negative elasticity is given by the effect of inflation (nominal interest rate) on the mispricing

The results in Table 1 suggest that inflation illusion can explain a large share of the mispricing in the housing market and that the negative correlation between inflation and the rent-price ratio is mainly due to the effect of inflation illusion on the mispricing.

Figure 2 in the introduction plots the standardized series of estimated pricing errors $\hat{\varepsilon}$ and $\log(1/i_t)$. It is apparent that movements in the log of the nominal interest rate reciprocal closely tracks the changes in the pricing errors $\hat{\varepsilon}$. Even though in the mid–eighties we do a poor job in capturing the level of mispricing, the overall apparent link between both series is remarkable.

ψ-**mispricing.** The first thing of interest is to compare the sizes of the mispricing. Figure 4 plots the price-rent ratio, the ε-mispricing measure and the ψ-mispricing measure over our sample period.

First, notice that both measures of mispricing have generally the right pattern of correlation with the price-rent ratio. Second, the ε-mispricing captures a non-negligible fraction of the variation in the price-rent ratio. Third, as argued in the methodological section, the ψ-mispricing measure seems to attribute too large a fraction of the movements in the price-rent ratio to the mispricing.

Next, we analyze the explanatory power of the inflation illusion proxies for the ψ-mispricing. Panel B of Table 1 shows that $\hat{\psi}_t$ – as inflation illusion would imply – covaries negatively (and significantly) with inflation π_t. Similarly, the univariate regressions with nominal interest rate i_t and $\log(1/i_t)$ also deliver significant results consistent with money illusion. Overall, the explanatory power of the inflation illusion proxies is reduced for the ψ-mispricing. This is not surprising, since $\hat{\psi}_t$ seems to overstate the time-variation of the mispricing.

3.2.3 Robustness Analysis

Assessing Uncertainty. To assess the robustness of these results, we next consider the uncertainty due to the fact that we do not directly observe expected future returns on housing and rent growth rates, but instead we use the estimated VAR to construct their proxies.
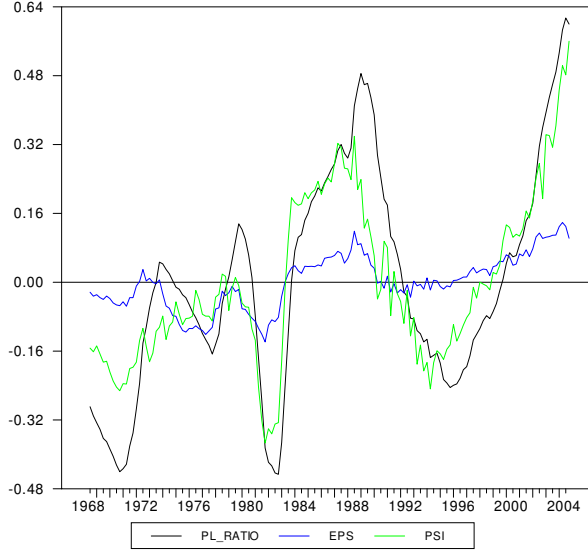
20

Figure 4: Price-rent ratio and mispricing measures

Under a diffuse prior, the posterior distribution of the estimated VAR can be factorized as the product of an inverse Wishart and, conditional on the covariance matrix, a multivariate normal distribution

$$
\begin{aligned}
\beta|_{\Sigma} &\sim N\left(\hat{\beta}, \Sigma \otimes (X'X)^{-1}\right) \\
\Sigma^{-1} &\sim \text{Wishart}\left(\left(n\hat{\Sigma}\right)^{-1}, n-m\right)
\end{aligned}
$$

where β is the vector of slope coefficients in the VAR system, Σ is the covariance matrix of the residuals, the variables with a hat denote the corresponding estimates, X is the matrix of regressors, n is the sample size and m is the number of estimated parameters (see Zellner (1971), Schervish (1995) and Bauwens, Lubrano, and Richard (1999)).[16] To assess the robustness of the results in Section 3.2.2 we compute 10,000 draws from the posterior distribution of the VAR coefficients and, for each draw, we construct expected excess returns, expected rent growth rates and implied mispricing, and use this variables to repeat the regressions reported in the previous section (the procedure is described in detail in Appendix A.2). Table 2 reports the results of this Monte Carlo exercise.

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[16]This result is exact under normality and the Jeffrey's prior $f(\beta, \Sigma) \propto |\Sigma|^{-(p+1)/2}$ (where p is the number of left hand side variables), but can also be obtained, under mild regularity conditions, as an asymptotic approximation around the posterior MLE.

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Dependent Variables:	Regressors:					
	π_t		i_t		$\log\left(1/i_t\right)$	
	Slope coeff.	R^2	Slope coeff.	R^2	Slope coeff.	R^2
$\hat{\varepsilon}_t$	-3.9 $[-11.1, -.185]$	.64 $[.05, .94]$	-6.28 $[-17.4, -.68]$	.54 $[.05, .75]$	.129 $[.01, .372]$	.52 $[.05, .67]$
$\sum\limits_{\tau=1}^{\infty} \rho^{\tau-1}\hat{E}_t\Delta l_{t+\tau}^e$	-2.6 $[-11.8, 9.08]$	.27 $[0, .85]$	-4.01 $[-18.1, 13.9]$	.2 $[0, .64]$	.095 $[-.303, .392]$	.21 $[0, .58]$
$-\sum\limits_{\tau=1}^{\infty} \rho^{\tau-1}\hat{E}_t r_{h,t+\tau}^e$	1.94 $[-7.71, 12.9]$	.11 $[0, .74]$	3.57 $[-11.2, 20.4]$	.09 $[0, .59]$	$-.052$ $[-.413, .281]$	.08 $[0, .52]$

Table 2: Median and 95 percent confidence intervals for slope coefficients and R^2.

Each row of the table reports the median slope coefficient associated with the regressor, the median R^2 and (in squared brackets) their 95 percent confidence intervals. The first row of Table 2 shows that the relation between inflation illusion and the mispricing of the rent-price ratio is a robust one: inflation and nominal interest rate show a significantly negative correlation with the mispricing while the inflation-biased valuation shows a significantly positive correlation. Moreover, even though the distribution of the estimated R^2 has a heavy left tail, there seems to be a very high posterior probability that these variables explain a large share of the time series variation in the mispricing. The second and third row of Table 2 show instead that there is substantial uncertainty about the correlation between inflation, nominal interest rate and expected future returns on housing and expected future rent growth rates. Overall, these results confirm an empirically strong link between nominal values and the mispricing of the housing market, and suggest that this mechanism is the main source of the negative correlation between the price-rent ratio and inflation and the nominal interest rate.

Assessing the Role of the Business Cycle. Unlike the price-dividend ratio in the stock market, the observed price-rent ratio is a less precise measure since the house price index reflects all types of dwellings while the rent index tends to overweight smaller and lower quality dwellings.

The prices of high quality houses appreciate at a higher rate during booms, and depreciate more during recessions, than cheaper houses do (see, among others, Poterba (1991) and Earley (1996)). This might cause the measured price-rent ratio to comove with the business cycle. Hence, if inflation and the nominal interest rate had a clear business cycle pattern, our estimated mispricing measures could show a spurious correlation with these variables.

Figure 5 plots the time series of the U.K. exponentially smoothed quarterly inflation, the return on the twenty-year Government Bonds, and the Hodrick and Prescott (1997) filtered estimate of the GDP business cycle. Clearly, there does not seem to be a strong contemporaneous correlation of inflation and nominal interest rates with the business
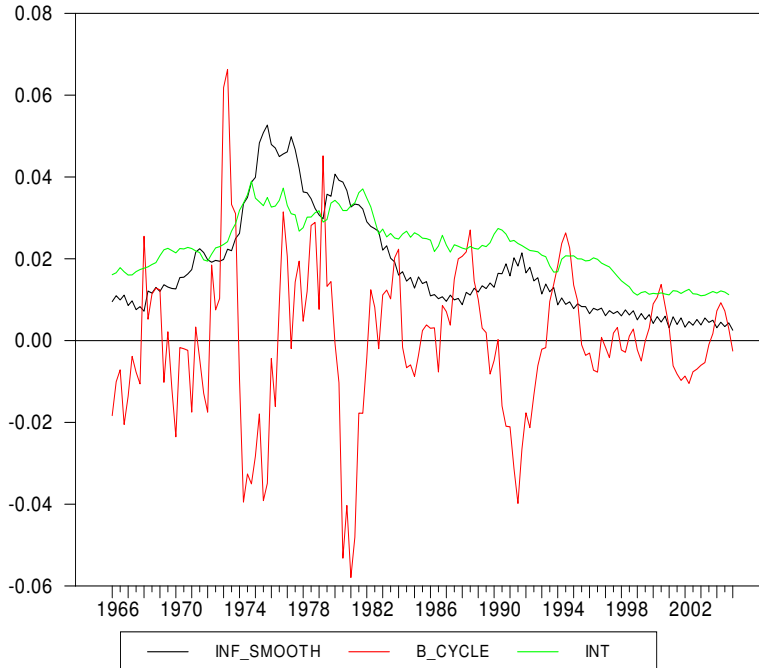
Figure 5: U.K. business cycle and inflation

cycle (the correlation coefficients are $-.16$ and $-.15$ respectively). This suggests that the high degree of explanatory power that inflation and the nominal interest rate have for the housing market mispricing is unlikely to be due to the comovement of these variables with the business cycle. In Appendix A.3 we address this issue formally, and we find that the inclusion of the business cycle in the OLS regressions for the mispricing measures (i) does not drive out the statistical significance of π_t, i_t and $\log(1/i_t)$, (ii) does not significantly change the point estimates of the elasticities of the mispricing reported in Table 1, (iii) does not increase significantly our ability to explain the time variation in the mispricing, (iv) and that the business cycle alone has very little (in the case of $\hat{\varepsilon}_t$) or no (in the case of $\hat{\psi}_t$) explanatory power for the mispricing measures.

3.3 Tilt Effect

Our empirical results are consistent with money illusion. Nevertheless, we could also be capturing the tilt effect of inflation. Recall from Section 3.1 that the reciprocal of the nominal interest rate, $1/i$, is proportional to the amount agents can borrow under a fixed nominal payment mortgage. Such a contract generates a financing constraint that varies with the nominal interest rate and hence with inflation. However, agents

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could use multiple alternative financing schemes available on the market, that are not affected by the tilt effect. This is for example the case for flexible interest rate mortgages, price level adjusted mortgages (PLAM) or the graduate payment mortgages (GPM).[17] This is especially true in the United Kingdom, where PLAM and GPM were available at least since the early 1970's. Furthermore, over the years, new more flexible mortgage products were introduced in all major countries. In the US for example, interest only mortgages, which substantially lower the initial payments, have become very popular recently.[18] Hence, we would expect that the importance of the tilt effect – if it ever was there – declines over time. That is, the negative elasticity of the mispricing to inflation should become less negative over the sample period. We empirically assess this hypothesis. Figure 6 depicts point estimates and Newey and West (1987) 95 percent confidence intervals of the univariate regressions of the estimated mispricing on π_t, i_t, and $1/i_t$ over a time-varying sample. We use the first ten years of data to obtain an initial estimate of the slope coefficient associated with each regressor, and we then add one data point at a time and update our estimates. For example, the point corresponding to 1992 first quarter is the estimated slope coefficient over the sample 1966 second quarter to 1992 first quarter.
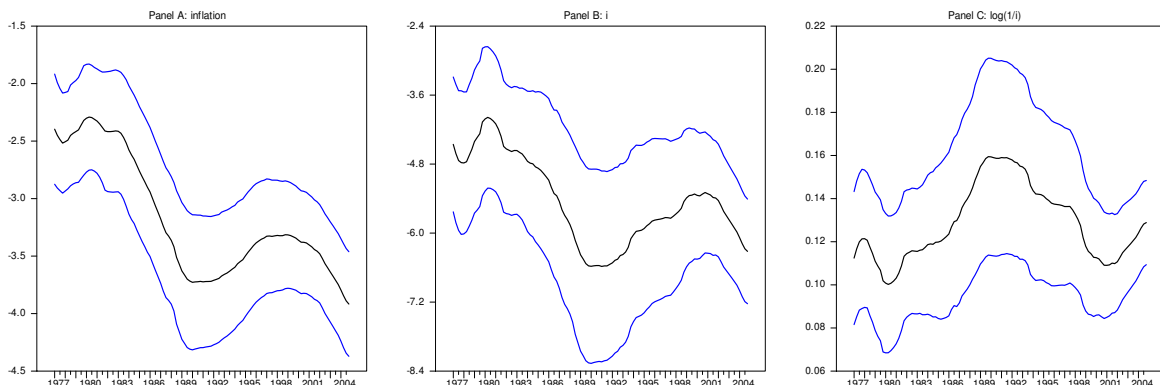
Figure 6: Point estimates and 95 percent Newey and West (1987) corrected confidence bounds of slope coefficients as sample size increases.

Figure 6 clearly reveals that the trend goes in the opposite direction of what we would

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[17]On the other hand, Spiegel (2001) provides a rationale for endogenous credit rationing in the housing market due to moral hazard.

[18]See e.g. Lowenstein's article in the New York Times on June 5, 2005 which cites the Lehman Brother report "The Changing Landscape of the Mortgage Market" for describing the recent increase in interest-only mortgages.

expect if the tilt effect were the driving mechanism behind the empirical link between housing prices and inflation. Over time, the negative relation between mispricing and inflation (nominal interest rate) becomes even more negative. In addition, the relationship between mispricing and the log of the nominal interest rate reciprocal is clearly not decreasing over time. All three findings show that it is unlikely that the tilt effect is the driving force of the empirical link between housing mispricing and inflation.

How does this finding square with money illusion? Money illusion does not have a clear implication whether the elasticity of mispricing to inflation should vary over time. Nevertheless, the estimated increase (decrease in the slope coefficient) is consistent with a setting in which households attention to inflation depends on the recent history of inflation: after and during a period of high inflation money illusion is very costly, hence households are more attentive to inflation and less prone to money illusion; after and during a period of low inflation – as in the last part of our sample – the cost of money illusion is perceived to be low and hence money illusion is more wide-spread increasing the elasticity of the mispricing to inflation.

4   US-Evidence[19]

In this section we examine the link between housing market mispricing and nominal values in the United States. Following the same procedure as in Section 3.2 we decompose the movements in the price-rent ratio into changes in expected future rent growth rates, $\sum_{\tau=1}^{\infty} \rho^{\tau-1} \hat{E}_t \Delta l_{t+\tau}^e$, expected future excess returns, $-\sum_{\tau=1}^{\infty} \rho^{\tau-1} \hat{E}_t r_{h,t+\tau}^e$, and implied mispricing, $\hat{\varepsilon}_t$. We then regress these three components on the three proxies meant to capture the effect of inflation on the price-rent ratio. The sample period available runs form 1970 first quarter to 2004 third quarter. Univariate regression results are reported in Table 3. The first row shows that the proxies considered are all significant explanators of the mispricing. Moreover, the sign of the estimated elasticity is the one we would expect under inflation illusion: the mispricing of the price-rent ratio tends to rise as inflation and nominal interest rates decrease. The coefficient estimates for the U.S. data are slightly higher than for the U.K. which might be due to the fact that mortgages are not portable in the U.S. The measures of fit are somehow smaller compared to the U.K. case, but this is likely to be due to the shorter sample period and poorer quality of U.S. data. For a review of the measurement problems in U.S. data on housing see McCarthy and Peach (2004). Nevertheless, the R^2 ranges form 15 percent when the explanatory variable is the nominal interest rate to 48 percent when we use inflation as the explanatory variable of the mispricing.

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[19]We are currently in the process of extending the analysis to more OECD countries. We have already obtained housing price time series for the countries mentioned in Figure 1, and we are currently in the process of constructing time series of housing investment returns.

Dependent Variables:	Regressors:					
	π_t		i_t		$\log\left(1/i_t\right)$	
	Slope coeff.	R^2	Slope coeff.	R^2	Slope coeff.	R^2
$\hat{\varepsilon}_t$	-10.2 (5.148)	.48	-6.86 (2.648)	.15	.159 (3.238)	.21
$\sum_{\tau=1}^{\infty} \rho^{\tau-1} \hat{E}_t \Delta l_{t+\tau}^e$	-2.65 (7.336)	.62	-3.18 (6.493)	.62	.060 (4.989)	.57
$-\sum_{\tau=1}^{\infty} \rho^{\tau-1} \hat{E}_t r_{h,t+\tau}^e$	4.21 (1.984)	.11	6.11 (2.201)	.12	$-.099$ (1.583)	.12

Table 3: Univariate Regressions on nominal interest rate, inflation and illusion proxy. U.S. data. Newey and West (1987) corrected t-statistics in brackets.

The second row shows that there is a significantly negative (positive) correlation between inflation and nominal interest rate (log of the nominal interest rate reciprocal) and expected future rent excess growth rates. This could either be a consequence of a negative effect of inflation on the real economy or due to a higher degree of stickiness in housing rents than in the general price level. The regressors considered are able to explain between 57 percent and 62 percent of the time series variation in expected future growth rates. The last row shows that there is a statistically significant link between inflation and nominal interest rate and future risk premia on housing investment (but not between the log of the nominal interest rate reciprocal and risk premia), which is consistent with the fact that, under distortionary taxes, an increase in inflation reduces the user cost of housing. These results imply a negative elasticity of the price-rent ratio to inflation (nominal interest rates) of about 8.6 (3.9) and that the largest contribution to this comes from the effect of inflation (nominal interest rate) on the mispricing.

Table 4 reports the results of a Monte Carlo exercise (described in Section A.2 of the Appendix) analogous to the one presented in Section 3.2 and which, as in the case of U.K. data, confirms the soundness of the empirical link between mispricing in the housing market and inflation, nominal interest rate and the log of the nominal interest rate reciprocal. On the other hand, it shows that there is substantial uncertainty about the rational links between inflation (nominal interest rate) and the price-rent ratio, even though both variables show a significantly negative correlation with the risk premium on the housing investment.

Dependent Variables:			Regressors:			
	π_t		i_t		$\log(1/i_t)$	
	Slope coeff.	R^2	Slope coeff.	R^2	Slope coeff.	R^2
$\hat{\varepsilon}_t$	-10.2 $[-16.2,\,-7.25]$	$.48$ $[.36,\,.62]$	-6.83 $[-10,\,-4.79]$	$.15$ $[0.11,\,0.21]$	$.159$ $[.115,\,.25]$	$.21$ $[.16,\,.26]$
$\displaystyle\sum_{\tau=1}^{\infty}\rho^{\tau-1}\hat{E}_t\Delta l^e_{t+\tau}$	-2.69 $[-10.4,\,3.15]$	$.13$ $[.0,\,.91]$	-3.18 $[-0.67,\,8.74]$	$.44$ $[.0,\,.69]$	$.06$ $[-.025,\,.164]$	$.44$ $[.0,\,.68]$
$\displaystyle-\sum_{\tau=1}^{\infty}\rho^{\tau-1}\hat{E}_t r^e_{h,t+\tau}$	4.15 $[-3.63,\,16.6]$	$.55$ $[.0,\,.97]$	6.07 $[0.92,\,14.5]$	$.17$ $[.0,\,.53]$	$-.1$ $[-.272,\,.011]$	$.11$ $[.0,\,.50]$

Table 4: Median and 95 percent confidence intervals for slope coefficients and R^2. U.S. data.

5   Conclusion

This paper studies the close link between inflation and housing prices. It provides supportive evidence that agents are prone to money illusion since movements in the mispricing in the housing market are largely explained by changes in inflation, the nominal interest rate and a variable meant to capture money illusion. We also show that the tilt effect cannot explain our findings. These results hold for both the U.K. and the U.S. housing markets and help us understand the run-ups starting in the 1990's.

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A Appendix

A.1 Data Description

A.1.1 U.K. Data

The house price series is from the Nationwide Building Society, and covers the sample period 1966:Q2–2005:Q1. Over the period 1966:Q2–2005:Q5 the index is constructed as a weighted average using floor area, therefore allowing to control for the influence of house size. Over the periods 1974:Q1–1982:Q4, and 1983:Q1–1992:Q1 additional controls (for region, property type, etc.) have been added in the construction of the index. Since 1993 the index also takes into account changes in the neighborhood classification. The rent series is constructed combining several sources available through the Office of National Statistics. Over the period 1966:01–1987:01 we use the CTMK LA:HRA series of rents on dwellings paid by tenants in the UK and we combine it with the data on the stock of housing available trough the Office of the Deputy Prime Minister. Over the period 1987:02–1987:12 we use the RPI-SGPE rent series of monthly percent changes over one month. Over the period 1988:01–2005:02 we use the CZCQ - RPI series of percent changes in rent over one year. The rent-free tenancies are excluded from the calculation of average rents. To obtain a series in levels for the price-rent ratio we scale the index series to match the level of the average price-rent ratio in 1990. As interest rate we use the 20-year par yield on British Government Securities available over the sample 1963:Q4–2004:Q4. All the results presented in the paper are based on the longest possible sample given the data at hand (1966:Q2–2004:Q4).

The implied inflation series, available over the period 1982:Q1-2005:Q1, is from the Bank of England and is constructed using the inflation protected 10 years government securities.

The real GDP measure is the seasonally adjusted, chained volume measures with constant 2002 prices and is available over the period 1955:Q1-2005:Q1 from the Office of National Statistics

A.1.2 U.S. Data

To construct the house price index series we use (i) the weighted repeat-sale housing price index form the Office of Federal Housing Enterprise Oversight over the sub-sample 1976:01–2004:03 and the (ii) Census Bureau housing price index (obtained through the Bank of International Settlements) over the period 1970:01–1975:04. To construct the rent index we use the CPI-Rent from the Bureau of Labor Statistics. We re-scale the indexes to levels to match the historical average of the U.S. price-rent ratio over the same sample (as reported in Ayuso and Restoy (2003)). As long-run interest rate we use the return on the 10-year Treasury bill. As measure of inflation we use the CPI index without housing.

A.1.3 Australian Data

The residential property price index is obtained from the Bank of International Settlements and available over the sample 1970:01–2005:01. The rental price index is obtained through Datastream for the period 1972:03–1980:01 and from the Bank of International Settlements for the period 1980:02–2005:01. The CPI series is obtained from Datastream. To construct the price-rent ratio and housing return series we rescale the rental and property indexes so that their ratio matches the ratio of median price to median rent over the period 2000-2001available in the "2005 year book of Australia" from the Australian bureau of statistics (Tables 8.7 and 8.17).The estimation reported are performed using the longest available sample given the data at hand: 1972:03-2005:01.

A.2 Assessing Uncertainty

To assess uncertainty in the regression results in Table 2, we report 95 percent confidence intervals for the estimated slope coefficients and R^2 constructed via Monte Carlo integration by drawing form the posterior distribution of the estimated VAR coefficients. We proceed as follows:

1. We draw covariance matrices $\grave{\Sigma}$ from the inverse Wishart with parameters $\left(n\hat{\Sigma}\right)^{-1}$ and $n - m$.

2. Conditional on $\grave{\Sigma}$ we draw a vector of coefficients for the VAR, $\grave{\beta}$, from

$$\grave{\beta} \sim N\left(\hat{\beta}, \grave{\Sigma} \otimes (X'X)^{-1}\right).$$

3. Using the draws of the VAR slope coefficients, $\grave{\beta}$, we construct expected discounted sums of rent excess growth rates ($\sum_{\tau=1}^{\infty} \rho^{\tau-1} \grave{E}_t \Delta l_{t+\tau}^e$) and excess housing

returns $(\sum_{\tau=1}^{\infty} \rho^{\tau-1} \grave{E}_t r^e_{h,t+i})$, and we compute pricing errors $(\grave{\varepsilon})$ as

$$\grave{\varepsilon}_t = p_t - l_t - \sum_{\tau=1}^{\infty} \rho^{\tau-1} \grave{E}_t \Delta l^e_{t+\tau} + \sum_{\tau=1}^{\infty} \rho_t^{\tau-1} \grave{E}_t r^e_{h,t+\tau}$$

We then regress $\grave{\varepsilon}_t$, $\sum_{\tau=1}^{\infty} \rho^{\tau-1} \grave{E}_t \Delta l^e_{t+\tau}$ and $\sum_{\tau=1}^{\infty} \rho_t^{\tau-1} \grave{E}_t r^e_{h,t+\tau}$ on π_t, i_t and the log of the inflation-biased evaluation $1/i_t$, and we store the estimated slope coefficients and measures of fit.

4. We repeat this procedure 10,000 times and compute confidence intervals for the OLS slope coefficients associated with π_t, i_t and the log of $1/i_t$, and for the corresponding measures of fit, from the corresponding percentiles of the Monte Carlo iterations.

A.3   Assessing the Role of the Business Cycle

To construct a business cycle proxy for the U.K. we follow Hodrick and Prescott (1997), that is we estimated the following state-space model

$$
\begin{aligned}
\Delta y_t &= g_t + c_t \\
g_t &= 2g_{t-1} - g_{t-2} + v_t
\end{aligned}
\tag{9}
$$

where Δy_t is GDP growth from quarter $t-5$ to quarter t, g_t is the unobserved state variable meant to capture the smooth time varying trend, and c_t is the cyclical component. The variance of v_t is normalized to be $1/1600$ times the variance of the cyclical component, c_t, as it is customary with quarterly data. This state-space representation is estimated via Kalman filter and Kalman smoother.

Table A1 reports OLS regressions of our mispricing measures ($\hat{\varepsilon}_t$ and $\hat{\psi}_t$) on the variables meant to capture money illusion (π_t, i_t and $\log(1/i)$) and the business cycle component of GDP identified by the H-P filter ($\hat{c}_t$).

Row:	Dep. variable:	$\hat{c}_t$	π_t	i_t	$\log\left(1/i\right)$	R^2
(1)	$\hat{\varepsilon}_t$	0.85 (2.201)				.07
(2)		0.41 (2.281)	−3.80 (7.801)			.67
(3)		0.49 (2.158)		−6.10 (6.399)		.57
(4)		0.60 (2.462)			0.12 (5.769)	.56
(5)	$\hat{\psi}_t$	1.11 (0.963)				.01
(6)		0.36 (0.349)	−5.98 (2.279)			.17
(7)		0.41 (0.369)		−10.5 (2.436)		.17
(8)		0.55 (0.503)			0.24 (2.605)	.19

Table A1. Regressions on business cycle fluctuations, inflation, nominal interest rate, and illusion proxy. Newey and West (1987) corrected t-statistics in brackets.

It is clear from the first and fifth rows of Table A1 that the business cycle as little (in the case of $\hat{\varepsilon}_t$) or no (in the case of $\hat{\psi}_t$) explanatory power for the mispricing. The remaining rows clearly show that the inclusion of the business cycle in the OLS regressions for the mispricing a) does not drive out the statistical significance of π_t, i_t and $\log\left(1/i_t\right)$, b) does not significantly change the point estimates of the elasticities of the mispricing reported in Table 1, c) does not increase significantly our ability to explain the time variation in the mispricing (comparing Table A1 to Table 1, we have that the increase in R^2 is ranges from 0 to 4 percent, and there is virtually no increases in the – non reported – $\bar{R}^2$).