

# Advisors and Asset Prices: A Model of the Origins of Bubbles

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## Abstract

Many asset price bubbles occur during periods of excitement about new technologies. We focus on the role of advisors and the communication process with investors in explaining this stylized fact. Advisors are good-intentioned and want to maximize the welfare of their advisees (like a parent for a child). But only some understand the new technology (the tech-savvys); others do not and can only make a downward-biased recommendation (the old-fogies). While smart investors recognize the heterogeneity in advisors, naive ones mistakenly take whatever is said at face value. Tech-savvys inflate their forecasts to signal that they are not old-fogies since more accurate information about their type improves the welfare of investors in the future. A bubble arises for a wide range of parameters and its size is maximized when there is a mix of smart and naive investors in the economy. Our model yields a number of additional testable implications.

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# 1 Introduction

A striking stylized fact regarding asset price bubbles is that they tend to occur during periods of excitement about new technologies (see, e.g., Malkiel (2003), Nairn (2002), Shiller (2000)). Such speculative episodes in the U.S. include: (1) railroads, (2) electricity, (3) automobiles, (4) radio, (5) micro-electronics, (6) personal computers, (7) bio-technology, and most recently (8) the internet. While there are a number of theories of asset price bubbles, there are surprisingly few explanations as to why they tend to occur around technological innovations. In this study, we focus on the role of advisors and the communication process with investors in explaining this stylized fact.

In the aftermath of the internet bubble, many place the blame on biased advisors for manipulating the expectations of naive investors. This is certainly the preferred explanation of the media and regulators. They point out that sell-side analysts working for investment banks duped individual investors by hyping initial public offerings. There are several pieces of evidence in support of this view. First, numerous papers have documented the incentives for analysts to generate biased, optimistic forecasts (see, e.g., Lin and McNichols (1998), Hong and Kubik (2003)). Second, there is mounting evidence that individual investors cannot see that these biased recommendations are due to incentives to sell stocks (see, e.g., Malmendier and Shantikumar (2004)). And third, analysts' optimistic forecasts have an impact on prices (see, e.g., Michaely and Womack (1999))

In this study, we too focus on the role of advisors but point out that there is something deeper in the communication process between them and investors that leads to an upward bias in prices. We assume that all advisors are good-intentioned in that they want to honestly disclose their signals to investors and care about the welfare of their advisees. This is not to say, however, that we do not believe that biased advisors had an important role in the dot-com bubble. We only want to point out that such bias is not needed. Indeed, it is not clear that such bias can explain bubbles during earlier periods. Moreover, during the dot-com period, even so-called objective research firms with no investment-banking business, such as Sanford and Bernstein,

issued recommendations every bit as optimistic as investment banks.

Our theory has several key assumptions. First, advisors are good-intentioned: they want to disclose the truth (or would incur a dishonesty cost otherwise) and they want to maximize the welfare of their advisees. Second, only some advisors understand the new technology (the tech-savvys); others do not and can only make a downward-biased recommendation (the old-fogies). We think that this assumption is only relevant during times of technological innovations. Hence, we emphasize our interest in explaining why so many bubbles arise during these times. We are happy to concede that our model may not apply to other types of bubbles such as the tulip and south-sea bubbles.<sup>1</sup> Third, there are two types of investors: smart and naive. While smart investors recognize the heterogeneity in advisors, naive ones mistakenly take whatever is said at face value.

We consider an economy with a single asset, which we call the new technology stock. There are three dates, 0, 1 and 2. At date 0, advisors are randomly matched with investors (the advisees). Advisors also observe the terminal payoff (which is realized at date 2) and can then send signals about this payoff to their advisees. A tech-savvy can send whatever signal he wants, while an old-fogie, who does not understand the new technology, is limited to a downward biased signal. Investor type is unknown to the advisor and the advisor type is unknown to the investor. The advisor-investor relationship is similar to that of a parent to a child, in which the smart kid is not sure whether dad is cool and the cool dad tries to impress his kid because he wants the kid to listen to him in the future.

At date 1, these advisors are randomly matched with a new set of investors. These investors can invest in a separate risky project requiring an initial fixed cost. Advisors again receive information about this risky project which pays off at date 2. A tech-savvy can then send whatever signal he wants, while an old-fogie is again constrained to a downward biased signal. An investor has access to the track record of his advisor, namely the signal (or recommendation) that was sent by him at date 0. A smart investor can then use this information to update his belief about his advisor's type.

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<sup>1</sup>However, some of our seminar participants argue that the tulip bubble was about a new technology to grow tulips and the south-sea bubble was about new financial technology or innovation.

To put this simple model into some context, think of the advisor at date 0 as a sell-side analyst covering technology stocks, but (counter-factually) with only good intentions. Date 1 captures the future career opportunities of this analyst, i.e. sell-side analysts typically become advisors to hedge funds or corporations later in their career. The key assumption of the model is that what the advisor says at date 0 can be used against or for him at date 1.

Lets first consider the equilibrium at date 1. Because of uncertainty about advisor type, smart investors may end up making investments when they should not because they are not sure whether a negative signal (e.g. a signal value less than the fixed cost of investing) is truly negative or if it just came from an old-fogie. We solve for a Bayesian-Nash equilibrium in the reporting strategies of the advisors and the investment policies of the advisees. In this equilibrium, tech-savvy advisors downward bias their signals over the set of states when it is not efficient for the advisee to invest. By downward biasing their signals over these states, the tech-savvy advisor allows the smart advisee to deduce that a certain set of negative signals cannot be coming from a tech-savvy advisor. And hence, it allows the advisee to avoid at least some inefficient investments. But this comes at a dishonesty cost to the tech-savvy advisor and this dishonesty cost is paid per advisee.

As a result, the tech-savvy advisor has an incentive to establish a better reputation at date 0 through his recommendation about the technology stock because smart investors use his date 0 recommendation to update on his type. The better is his reputation among smart investors at date 1, the more easily he can avoid dishonesty costs in getting his advisees to make efficient investments. As a result, he now will try to inflate his forecasts to signal to smart investors that he is tech-savvy. We show that such a Bayesian-Nash equilibrium exists at date 0. While smart advisees properly deflate this upward bias, naive investors, unfortunately, take what he says at face value.

We show that a price bubble can arise as a result. It is important to note that the assumption about heterogeneity in advisor types (tech-savvies versus old-fogies) does not bias the results in our favor. Indeed, this assumption, if anything, leads to a downward bias in prices since naive investors takes whatever old-fogies say at full value.

In other words, the “good-intentions of tech-savvies” effect has to be strong enough to overcome this baseline downward bias. It is not clear *ex ante* that this need be the case. However, we show that such a technology price bubble does exist for a wide range of parameter values—namely when an advisor has many more advisees at date 1 than at date 0. (This condition is a realistic one since many sell-side analysts during the dot-com period were young and had a long career ahead of them.) When this condition holds, the tech-savvy advisors have a strong incentive to build a better reputation by trying to distinguish themselves from old-fogies with optimistic recommendations.

To develop intuition for the price bias, let's consider two polar cases. First, suppose that there are only smart investors in the economy. In equilibrium, tech-savvy advisors will tend to upwardly bias their forecasts so as to distinguish themselves from old-fogies. However, smart investors understand this and in equilibrium, will adjust their beliefs accordingly. So price will be an unbiased signal of fundamentals. Next, suppose that there are only naive investors in the economy. In equilibrium, tech-savvy advisors will honestly disclose their signals since they do not worry about naive investors inferring their type.

When both types of investors are present in the economy, the price will be upwardly biased on average. Tech-savvy investors will bias their messages upward (the extent of this bias increases as the fraction of smart investors increases). While smart investors can de-bias these messages, naive investors are unable to do so. Since price is a weighted average of the two groups' of investors' beliefs (because investors are assumed to be risk-averse), it will be upwardly biased on average.

Our theory yields testable implications. First, the upward bias in price is largest when there are both sets of investors in the economy. One can test this implication by using data on the composition of holdings of a stock. Our theory predicts that the price-to-fundamental ratio during bubble periods ought to be highest for stocks held by both institutional and individual investors. The second prediction, again focusing on the internet period, is that the tech-savvies (which we take to be the optimistic sell-side analysts) should issue more optimistic recommendations on stocks in which the investors are mostly institutional and issue less optimistic recommendations on stocks

in which the investors are mostly individuals.

Finally, we discuss robustness issues and extensions. Notably, we show that our main results survive when we loosen two assumptions. First, we allow old-fogies to send biased messages at a cost just like tech-savvies. Second, we allow an investor at  $t = 0$  to observe the recommendations of other advisors as well. While the extension is more general than the featured model, its solution requires additional assumptions regarding off-equilibrium beliefs. While we try to specify reasonable ones, the solution is nonetheless more fragile than that of our benchmark model in that the solution would be different depending on different off-equilibrium beliefs. Moreover, the solution of the benchmark model has an added advantage that it is more intuitive and less cumbersome than that of the extension.

Our theory is related to the literature on costly signaling (see, e.g., Kreps (1990), Fundenberg and Tirole (1991)). A key theme of this paper that is in earlier work is that concerns about reputation can affect the actions of agents in trying to shape this reputation (Holmstrom and Ricart i Costa (1986) and Holmstrom (1999)) and may lead them to do perverse things such as saying the expected thing which may lead to information loss (Scharfstein and Stein (1990), Ottaviani and Sorensen (2005)) or adopt a standard of conformist behavior (Bernheim (1994)) or making politically correct statements so as to not look racist (Morris (2001)). More specifically, our model, similar to Morris (2001) but unlike the others, emphasizes the perverse reputational incentives of a “good-intentioned” advisor—the good-intentioned tech-savvy advisor in our model engages in costly signaling at date 0 so as to better help future investors. This is different from career-concerns based models such as Scharfstein and Stein (1990) in which advisors do not know their type and engage in signal jamming to achieve a better reputation for their own sake. Moreover, unlike all these papers, we focus on the interaction of sophisticated agents (tech-savvies and smart investors) and naive agents (old-fogies and naive investors) and our model is more geared toward looking at asset prices.

Finally, our paper complements recent interesting work by Hirshleifer and Teoh (2003) on the disclosure strategies of firms when some of their investors have limited

attention. Like us, they emphasize the importance of introducing boundedly rational agents in understanding the effect of disclosures on asset prices. Unlike us, their focus is on how the presentation of information may lead to different results with inattentive investors and the incentives of managers to potentially manipulate earnings to fool inattentive investors.

As we mentioned at the beginning of the introduction, there is a large literature on asset price bubbles ranging from the rational bubbles literature (see, e.g., Blanchard and Watson (1982), Santos and Woodford (1997), Allen and Gorton (1993), Allen, Morris, and Postlewaite (1993)) to the more recent literature featuring short-sales constraints and heterogeneous beliefs (see, e.g., Miller (1977), Chen, Hong and Stein (2002), Harrison and Kreps (1978) and Scheinkman and Xiong (2003)). The emphasis of our model on the communication process between advisors and investors is novel relative to these studies.

Our paper is organized as follows. We present the model in Section 2. We consider comparative static exercises and discuss related empirical implications in Section 3. We consider robustness and extensions in Section 4. And we conclude in Section 5 with a re-interpretation of the events of the internet period. Proofs are in the Appendix in Section 6.

## 2 Model

### 2.1 Set-up

We consider an economy with a single traded asset, which we call the new technology or tech stock. There are three dates, denoted by  $t = 0, 1, 2$ . The stock pays a liquidating dividend at  $t = 2$  given by

$$v = \theta + \epsilon, \tag{1}$$

where  $\theta$  is uniformly distributed on the interval  $[0, 1]$  and  $\epsilon$  is normally distributed with a mean of zero and a variance of  $\sigma^2$ .

There are two types of advisors in the economy: those that are tech-savvy (with a

mass of  $\pi_0 \in [0, 1]$  in the population) and those that are old-fogies (with a remaining mass of  $1 - \pi_0$ ). Tech-savvy advisors observe  $\theta$  (i.e. they understand the new technology) and send a report to investors at  $t = 0$ , denoted by  $s_0^{TS}$ . They have good intentions in that they want to tell the truth. Tech-savvies incur a dishonesty cost if they report a signal different from the truth. This cost is given by

$$c(s_0^{TS} - \theta)^2, \quad (2)$$

where  $c > 0$ . But tech-savvies also want to maximize the welfare of their advisees at  $t = 1$ . As we shall see, he may want incur some dishonesty cost and strategically bias his report upwards to improve the welfare of his future clients.<sup>2</sup> In contrast, old-fogies, while also good-intentioned, do not understand the new technology and can only send a report that is a downward biased version of the truth. We assume that they send a signal at  $t = 0$  given by

$$s_0^{OF} = a\theta, \quad (3)$$

where  $a \in [0, 1]$ . So the report sent by the old-fogies will always be a fraction of the true value. Advisor type is unknown to investors.

There are also two types of investors at  $t = 0$ : smart ones (with a mass of  $\rho \in [0, 1]$  in the population) and naive ones (with a remaining mass of  $1 - \rho$ ). Each investor is randomly matched with one advisor and has only access to the report from this advisor.<sup>3</sup> Smart investors are aware of the existence of old-fogies and take into account the optimal reporting strategy of tech-savvy advisors in inferring the advisor type through the messages sent. Naive investors are not aware of the heterogeneity in advisors and simply take the messages sent to them at their full value.<sup>4</sup> We assume

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<sup>2</sup> Advisors do not care about the welfare of their advisees at  $t = 0$  when we assume that investors trading the stock at  $t = 0$  have risk aversion coefficients close to zero, so that their trading is a zero-sum game. In this instance, the aggregate welfare of investors is unaffected by the advisors' reports. When advisors do not care about the distribution of wealth among their naive and sophisticated advisees nor between their advisees and those investors not advised by them, they will tradeoff between disclosing the truth at  $t = 0$  (to avoid the dishonesty cost) and to maximize the welfare of their future advisees.

<sup>3</sup>We allow an advisor to advise multiple investors. The exact number of advisees is not important, because we implicitly assume that advisors do not care about the welfare of their advisees at  $t = 0$  (see footnote 2).

<sup>4</sup>This assumption fits with empirical evidence by Malmendier and Shantikumar (2004) about the inability of individual investors to see through incentives of sell-side analysts.



that both smart and naive investors maximize mean-variance preferences given by:

$$E_i(W_i) - \frac{1}{T}Var_i(W_i), \quad (4)$$

where  $T$  is an individual investor's risk tolerance and  $W_i$  is terminal wealth of investor  $i$  at  $t = 2$ . For most of our analysis, we will assume that  $T$  is close to infinity. Investors establish positions in the asset based only on the reports of their advisors at  $t = 0$ .

At  $t = 1$ , the advisors are matched with a new set of investors. For simplicity, we assume that these investors are risk neutral. Each of these investors has an opportunity to invest in a different risky (new technology) project. The fixed cost of the project is  $I$ , which is a constant between 0 and 1. The payoff of the project is  $f$ , which is uniformly distributed on the interval  $[0, 1]$ .  $f$  is independent of  $\theta$ . Tech-savvy advisors observe  $f$  and send a report to investors at  $t = 1$ , denoted by  $s_1^{TS}$ . They continue to want to maximize the welfare of their new advisees and incur a dishonesty cost of

$$c(s_1^{TS} - f)^2, \quad (5)$$

where  $c > 0$  if their report differs from the truth. Again, old-fogies, while also good-intentioned, do not understand this new technology and send a signal at  $t = 0$  given by

$$s_1^{OF} = af, \quad (6)$$

where  $a \in [0, 1)$ . We impose a parameter restriction that  $a \geq I$ . This restriction ensures that at least for some states of the economy an old-fogie advisor would recommend investors to invest in the project. In addition, we assume that an advisor is now randomly matched with  $n$  advisees. Advisor type is unknown to investors.

At  $t = 1$ , investors will again rely on the single advisor that they are matched with in deciding on whether to make an investment. Investors only get information about the past report (at  $t = 0$ ) of the advisor that they are matched with. Again, there are two types of investors. The smart investors use this past information to update their belief about the type of their advisor at  $t = 1$ , which we denote by  $\pi_1$ . Naive investors again just take whatever their advisors tell them at face value.

The idea of the  $t = 1$  set-up is that it is a reduced-form meant to capture a stream of future advising engagements in an advisor's career. More specifically, one can think of the advisor as a sell-side analyst. He makes recommendations at  $t = 0$  regarding the technology stock. Later in his career ( $t = 1$ ), he becomes an advisor to hedge funds or corporations on other projects. They have information regarding his track record. The parameter  $n$  captures the number of such future advising engagements. For simplicity, we have assumed that each advisor gets the same number of advisees at  $t = 1$ . More realistically, advisors with better reputation, i.e. higher  $\pi_1$ 's, get a larger  $n$  number of advisees. This would only help to strengthen our results.

## 2.2 Equilibrium at $t = 1$

We begin by deriving a Bayesian-Nash equilibrium for the reporting strategy of the advisors and the investment policies of investors. Note that  $\pi_1$ , the probability that the advisor type is tech-savvy assigned by smart investors, can only take three values depending on the report sent at  $t = 0$ : 0,  $\pi_L$  (a constant) and 1. We take this as given in this sub-section. We will show that this is indeed the outcome from the game at  $t = 0$  in the next sub-section.

### 2.2.1 Smart Investors Have Perfect Information About Advisor Type: $\pi_1 = 1$ or $\pi_1 = 0$

We begin our analysis with the case in which a smart investor knows for sure whether his advisor is tech-savvy or an old-fogie. We will look for an equilibrium in which the tech-savvy advisor tells the truth and investors follow the efficient investment rule of investing when their beliefs about  $f$  are greater than  $I$ , the fixed cost of investing.

**Proposition 1** *Suppose that  $\pi_1 = 1$  or  $\pi_1 = 0$ . A Bayesian-Nash equilibrium at  $t = 1$  consists of the following profiles. The tech-savvy advisor truthfully reports his information, i.e.  $s_1^{TS} = f$ . The old-fogie reports  $s_1^{OF} = af$  by assumption. A smart investor is able to deduce  $f$  from the message sent by his advisor, denoted by  $s_1$ , and invests if  $f \geq I$  and does not invest when  $f < I$ . A naive investor invests if  $s_1 \geq I$  and does not invest if  $s_1 < I$ .*

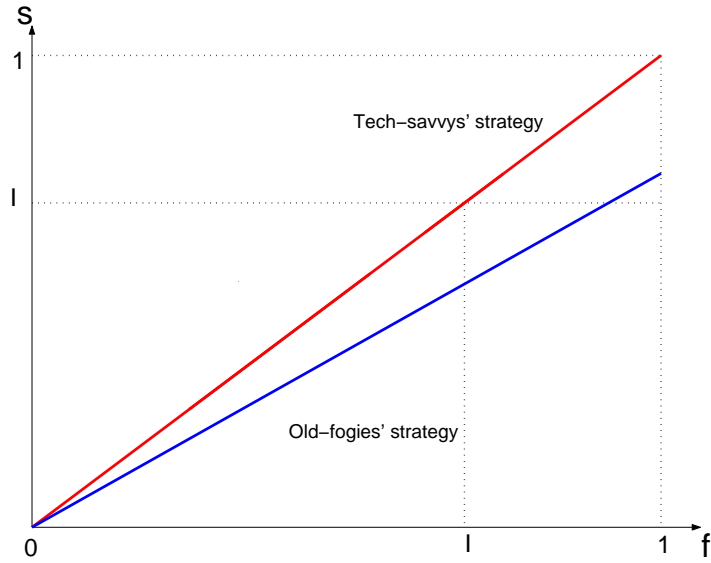


Figure 1: Advisors' strategies at  $t = 1$  with perfect information about type.

Figure 1 illustrates the reporting strategies of tech-savvy and old-fogie advisors. Lets check that this is indeed a Bayesian-Nash equilibrium. Given the reporting strategies of the two types of advisors, the smart investor can deduce  $f$  and hence sticks with the efficient investment rule: invest only if  $f \geq I$ . His expected gain is

$$E[\max(f - I, 0)] = \int_I^1 (f - I)df = \frac{1}{2}(1 - I)^2. \quad (7)$$

There is nothing to check for the naive investors since we assume that they always listen to whatever message is sent and invests only if  $s_1 \geq I$ . They make an efficient investment decision if their advisor happens to be tech-savvy but may under-invest if their advisor happens to be an old-fogie.

Given the investment strategies of the two types of investors, it is optimal for tech-savvy advisors to report the truth. He has nothing to gain from deviating from the truth because the smart investor knows his type and the naive investor listens to whatever he says. He would only incur a dishonesty cost by lying. There is nothing to check for old-fogies since we assume that they always report  $s_1^{OF} = af$ . So we have proven that the profiles described in Proposition 1 constitute a Bayesian-Nash equilibrium.

### 2.2.2 Smart Investors Have Imperfect Information About Advisor Type:

$$\pi_1 = \pi_L$$

When smart investors have imperfect information about advisor type, i.e.  $\pi_1 = \pi_L$ , then a tech-savvy advisor has an incentive to report a downward biased signal of  $f$  for realizations of  $f$  that are neither too high or too low. Since a smart investor does not have perfect information about his advisor's type, he will naturally infer that signals less than  $I$  are sometimes sent by old-fogies and will sometimes invest when he should not. A tech-savvy advisor can (dishonesty) cost efficiently alleviate this problem by severely downward biasing his messages so as to communicate to his smart advisee that certain messages could not be coming from him. But the cost of doing this is the tech-savvy advisor has to incur some dishonesty costs in equilibrium. As we will show in the next sub-section, this gives him an incentive at  $t = 0$  to incur some initial dishonesty cost so as to convince future investors of his type. We begin by formally constructing the Bayesian-Nash equilibrium of this sub-game.

**Proposition 2** *Suppose  $\pi_1 = \pi_L$ . A Bayesian-Nash equilibrium consists of the following profiles. A tech-savvy advisor's reporting strategy is given by*

$$s_1^{TS} = \begin{cases} f & \text{if } f \geq f^* \\ bI & \text{if } bI \leq f < f^* \\ f & \text{if } f < bI \end{cases} \quad (8)$$

where  $f^*$  solves the equation

$$c(f^* - bI)^2 = \rho(I - f^*), \quad (9)$$

and

$$b \equiv \frac{1}{\pi_{LL} + \frac{1}{a}(1 - \pi_{LL})} \in (a, 1), \quad (10)$$

and

$$\pi_{LL} = \frac{\pi_L}{\pi_L + (1 - \pi_L)/a} < \pi_L. \quad (11)$$

The solution is given by

$$f^* = \frac{1}{2} \sqrt{(\rho/c - 2bI)^2 + 4(\rho I/c - b^2 I^2)} - \frac{1}{2}(\rho/c - 2bI), \quad (12)$$

and  $bI < f^* < I$ . The old-fogie reports  $s_1^{OF} = af$ . After observing a signal  $s_1$  sent from his advisor, a smart investor will invest if  $s_1 > bI$  and does not invest if  $s_1 \leq bI$ . A naive investor invests if  $s_1 \geq I$  and does not invest if  $s_1 < I$ .

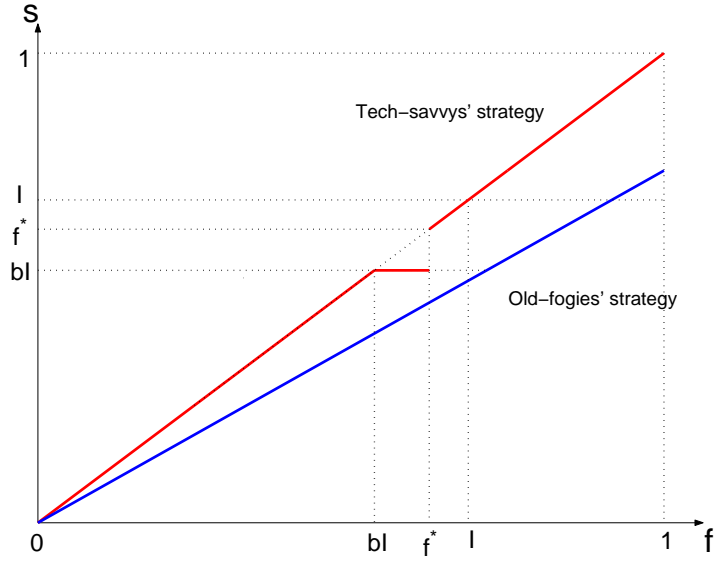


Figure 2: Advisors' strategies at  $t = 1$  with imperfect information about type.

Figure 2 illustrates tech-savvy advisors' reporting strategy. Lets verify that this is indeed a Bayesian-Nash equilibrium. Take as given the reporting strategy of the advisors and verify the optimality of the smart investor's investment policy. First, suppose that  $s_1 \geq I$ . The message could be from a tech-savvy or an old-fogie. But it does not matter to the smart investor who it came from since such a signal leads to the inference that  $f > I$  given the reporting strategies of the two types of advisors. So the investor invests.

Next, lets suppose that  $s_1 \in (f^*, I)$ . For a signal sent in this region of the support, the signal again could be from a tech-savvy or an old-fogie. But this time, the smart investor cares who it came from. Let  $\pi_{LL}$  be the posterior probability that a signal in

this region came from a tech-savvy advisor, i.e.

$$\pi_{LL} = Pr\{\text{tech} - \text{savvy} | s_1 \in (f^*, I)\} \quad (13)$$

Then by Bayes Theorem, we have that

$$\pi_{LL} = \frac{\lambda(s_1 | \text{tech} - \text{savvy})\pi_L}{\lambda(s_1 | \text{tech} - \text{savvy})\pi_L + \lambda(s_1 | \text{old} - \text{fogie})(1 - \pi_L)}, \quad (14)$$

where  $\lambda$  denotes a probability density function. Given the tech-savvy advisor's reporting strategy, the probability distribution of his signal is

$$\lambda(s_1 | \text{tech} - \text{savvy}) = \begin{cases} 1 & \text{if } f \in [0, bI) \\ 0 & \text{if } f \in (bI, f^*) \\ 1 & \text{if } f \in [f^*, 1] \end{cases} \quad (15)$$

and  $Pr(s_1 = bI | \text{tech} - \text{savvy}) = f^* - bI$ . Note also that since the signal from an old-fogie has uniform distribution in interval  $[0, a]$ ,

$$\lambda(s_1 | \text{old} - \text{fogie}) = 1/a, \quad \forall s_1 \in [0, a]. \quad (16)$$

Thus, if  $s_1 \in (f^*, I)$ , then

$$\pi_{LL} = \frac{\pi_L}{\pi_L + (1 - \pi_L)/a} < \pi_L \quad (17)$$

Given such a signal, the smart investor's inference of the project fundamental is

$$E[f | s_1] = \pi_{LL}s_1 + (1 - \pi_{LL})s_1/a. \quad (18)$$

Thus, the investor finds it optimal to invest in the project if

$$s_1 > \frac{I}{\pi_{LL} + \frac{1}{a}(1 - \pi_{LL})} = bI. \quad (19)$$

Now suppose that  $s_1 \in (bI, f^*]$ . Then the smart investor deduces that the signal must be from an old-fogie since the reporting strategy of the tech-savvy would never send a signal in this region:

$$E[f | s_1] = s_1/a > \frac{bI}{a} = \frac{I}{1 - (1 - a)\pi_{LL}} > I. \quad (20)$$

Hence the smart investor invests.

If  $s_1 = bI$ , then the signal must be from a tech-savvy advisor since the reporting strategy puts non-trivial mass at the point  $bI$ . The logic follows. Note that

$$Pr\{s_1 = bI|\text{tech} - \text{savvy}\} = f^* - bI, \quad (21)$$

and

$$Pr\{s_1 = bI|\text{old} - \text{fogie}\} = Pr\{s_1 \in (bI - \epsilon, bI + \epsilon)|\text{old} - \text{fogie}\}_{\epsilon \rightarrow 0} = \frac{2\epsilon}{a} \Big|_{\epsilon \rightarrow 0} = 0. \quad (22)$$

Thus, the conditional probability that such a signal comes from a tech-savvy is

$$\begin{aligned} & Pr\{\text{tech} - \text{savvy}|s_1 = bI\} \\ &= \frac{Pr(s_1 = bI|\text{tech} - \text{savvy})\pi_L}{Pr(s_1 = bI|\text{tech} - \text{savvy})\pi_L + Pr(s_1 = bI|\text{old} - \text{fogie})(1 - \pi_L)} = 1 \end{aligned} \quad (23)$$

Given such an inference, the investor would not invest.

If  $s_1 < bI$ , then the analysis is similar to the case in which  $s_1 \in (f^*, I)$ .  $\pi_{LL}$  is again the posterior probability that a signal in this region came from a tech-savvy advisor. Given such a signal, the smart investor's inference of the project fundamental is

$$E[f|s_1] = \frac{s_1}{b}. \quad (24)$$

In this case,  $s_1 < bI$ , so the smart investors do not invest. Hence, we have shown that the postulated investment policy of the smart investors is indeed optimal given the proposed reporting strategy of the tech-savvy advisors and that of the old-fogies.

We will now show that the tech-savvy advisor's reporting strategy is optimal given the investors' investment policies. Suppose that  $f \geq I$ , so that it is efficient for investors to invest. Then it is optimal for the tech-savvy advisor to tell the truth. Suppose that  $f \in (f^*, I)$ . In this case, investment is not efficient. Now the proposed strategy of smart investors is to invest anyways. If the advisor tells the truth, then the cost to the sophisticated investors is  $n\rho(I - f)$ . If the advisor biases his message, then he would have to deflate it to  $bI$  to keep the smart investor from investing according to the proposed investment rule. It is important to note that there is no point of deflating

the signal to a level higher than  $bI$ , since smart investors would still choose to invest. The dishonesty cost of deflate the message is

$$nc(f - bI)^2. \quad (25)$$

Note that if  $f > f^*$ , then

$$nc(f - bI)^2 > nc(f^* - bI)^2 = nc(I^* - f) > n\rho(I - f) \quad (26)$$

using the definition of  $f^*$  given in the proposition above. As a result, there is no incentive to under-report.

Suppose that  $f \in [bI, f^*]$ . If the advisor tells the truth, then the cost to sophisticated investors is again  $n\rho(I - f)$ . If the advisor reports  $bI$ , then the cost is  $nc(f - bI)^2$ . Note that if  $f \in [bI, f^*]$ ,

$$nc(f - bI)^2 \leq nc(f^* - bI)^2 = n\rho(I - f^*) \leq n\rho(I - f), \quad (27)$$

again from the definition of  $f^*$ . Thus, it makes sense to report  $bI$ .

Finally, if  $f < bI$ , then the advisor simply tells the truth since investors will not invest anyways.

### 2.2.3 The Gain from Improved Reputation for a Tech-Savvy Advisor

The completeness of information regarding advisor type (or advisor reputation from the perspective of a smart advisor) will affect the investment strategy of smart investors but has no effect on naive investors. If a tech-savvy advisor has a perfect reputation, a smart investor's welfare is given in equation (7). Thus, the value function for a tech-savvy advisor is

$$V_1 = n\rho \int_I^1 (f - I)df, \quad (28)$$

which is proportional to the number of advisees he has and the provability that they are smart.

If the smart advisor has an imperfect reputation, the expected investment profit to a smart investor is

$$\int_{f^*}^1 (f - I)df, \quad (29)$$



since the investor will end up investing if the signal is above  $f^*$ . In addition, he has to deflate his message and pay a dishonesty cost if the fundamental variable is between  $bI$  and  $f^*$ . The expected cost to him is

$$\int_{bI}^{f^*} c(f - bI)^2 df. \quad (30)$$

He is concerned with his own dishonesty cost and the smart investors' gain from investment, which is given by

$$V_2 = n\rho \int_{f^*}^1 (f - I)df - \int_{bI}^{f^*} nc(f - bI)^2 df. \quad (31)$$

Hence, the incremental gain from a perfect reputation is

$$V_1 - V_2 = n\rho \int_{f^*}^I (I - f)df + \int_{bI}^{f^*} nc(f - bI)^2 df. \quad (32)$$

The first term represents a gain from preventing smart investors from taking inferior projects, and the second term represents a gain from avoiding the dishonesty cost. We derive some simple comparative statics for this gain from a better reputation.

**Proposition 3** *The gain from a better reputation to a tech-savvy advisor,  $V_1 - V_2$ , increases with the number of advisees ( $n$ ) and the fraction of smart investors in the population ( $\rho$ ), and decreases with the fraction of tech-savvy advisors ( $\pi_0$ ) and the degree of old-fogie-ness ( $a$ ).*

The proof is in the Appendix. But the intuition for these results are simple. First, as the number of advisees  $n$  increases, the tech-savvy advisor has to incur more dishonesty costs. So a one-time initial increase in reputation early on can be used by future generations of advisees—the greater the number of advisees, the bigger the gain to achieving a better reputation early on. Similarly, having a better reputation only matters if there are smart investors around to use it. Hence, the gain to a better reputation increases with  $\rho$ . On the other hand, the better the initial reputation ( $\pi_0$ ), the less valuable is the gain to a better reputation and so  $V_1 - V_2$  decreases in  $\pi_0$ . Finally, the larger is  $a$ , the more old-fogies are like tech-savvies and the less the inefficiency in investment by investors and the less valuable is the gain to a better reputation.

## 2.3 Equilibrium at $t = 0$

### 2.3.1 Tech-Savvy Advisor's Reporting Strategy

As we discussed earlier, a good reputation among smart investors is potentially useful to a savvy advisor with good intention since it reduces the possibility of inefficient investment by smart investors and also reduces the advisor's dishonesty cost in period 2 when he tries to minimize this inefficient investment by biasing his reports.

In this sub-section, we will construct the Bayesian-Nash equilibrium in which the tech-savvy advisors biases his reports in an attempt to build a better reputation.

**Theorem 1** *A Bayesian Nash equilibrium at  $t = 0$  consists of the following profiles. The reporting strategy of a tech-savvy advisor is*

$$s_0^{TS} = \begin{cases} \theta & \text{if } \theta \geq a \\ a & \text{if } \theta^* < \theta < a \\ \theta & \text{if } \theta \leq \theta^* \end{cases} \quad (33)$$

where  $\theta^* \in [0, a)$  is a constant determined by

$$\theta^* = \begin{cases} a - \sqrt{\frac{V_1 - V_2}{c}}, & \text{if } a - \sqrt{\frac{V_1 - V_2}{c}} > 0 \\ 0, & \text{otherwise} \end{cases} . \quad (34)$$

An old-fogie reports  $s_0^{OF}$ . After observing a signal  $s_0$  sent, a smart investor infers the advisor's type according to the following rule: if  $s_0 \geq a$ , the advisor is tech-savvy for sure ( $\pi_1 = 1$ ); if  $\theta^* < s_0 < a$ , the advisor is old-fogie for sure ( $\pi_1 = 0$ ); if  $s_0 \leq \theta^*$ , the advisor's type remains unclear, and his reputation as a tech-savvy advisor is

$$\pi_L = \frac{\pi_0}{\pi_0 + (1 - \pi_0)/a}, \quad (35)$$

which is lower than the advisor's initial reputation ( $\pi_0$ ).

Figure 3 illustrates tech-savvy advisors' reporting strategy described in Theorem 1. We will now check that this is an equilibrium. Take as given smart investors' learning rule and verify the optimality of a tech-savvy advisor's reporting strategy. First, suppose that  $\theta > a$ . Since the fundamental value  $\theta$  is greater than  $a$ , reporting

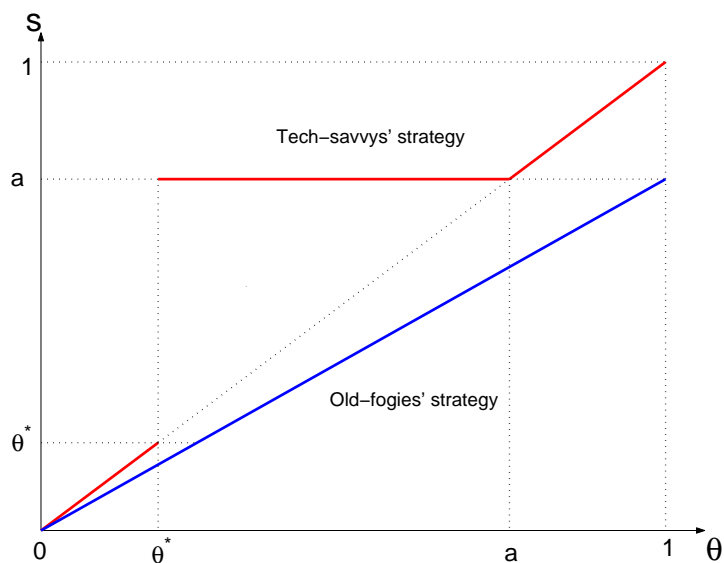


Figure 3: Advisors' strategies at  $t = 0$ .

the truth reveals the tech-savvy advisor's type since an old-fogie would never send such a signal. Now suppose that  $\theta \in [\theta^*, a]$ . If  $\theta$  is below  $a$ , the advisor can try to distinguish himself from an old-fogie by inflating his signal to  $a$ . However, this will incur a dishonesty cost of  $c(\theta - a)^2$ . Also note that a savvy advisor would never partially inflate his report to a level below  $a$ , since it would hurt his reputation given smart investors' learning rule. Since the dishonesty cost increases quadratically with the degree of report inflation, as the fundamental value  $\theta$  decreases, the cost of inflating the report increases. As  $\theta$  drops to a threshold level given by  $\theta^*$ , the advisor will become indifferent between separating himself from an old-fogie by paying a dishonesty cost or to maintain his initial reputation.  $\theta^*$  is exactly determined by equation (34).

Finally, suppose that the fundamental value  $\theta$  is below  $\theta^*$ . In this case, it is too costly for the advisor to signal his type by inflating his message to  $a$ . We also note that partially inflating the signal would not improve the advisor's reputation at all. Thus, the advisor chooses to send a truthful signal.

Now we verify the optimality of the smart investors' inference rule, given tech-savvy advisors' reporting strategy. If  $s_0 \geq a$ , the signal must be from a tech-savvy since old-fogies would never report such a signal. Thus,  $\pi_1 = 1$ . If  $s_0 \in (\theta^*, a)$ , the

signal must be from an old-fogie, since tech-savvies would never report signals in this region. If  $s_0 \leq \theta^*$ , the signal could be from either a tech-savvy or an old-fogie, and the probability it is from a tech-savvy is given by the Bayes rule:

$$\begin{aligned} Pr[\text{tech} - \text{savvy} | s_0 \leq \theta^*] &= \frac{\lambda(s_0 | \text{tech} - \text{savvy})\pi_0}{\lambda(s_0 | \text{tech} - \text{savvy})\pi_0 + \lambda(s_0 | \text{old} - \text{fogie})(1 - \pi_0)} \\ &= \frac{\pi_0}{\pi_0 + (1 - \pi_0)/a}, \end{aligned} \quad (36)$$

which is exactly  $\pi_L$  defined in equation (35). We have also just verified our claim earlier that  $\pi_1$  can only take on one of three values—0, 1, and  $\pi_L$ .

The cut-off value  $\theta^*$  captures the degree to which the tech-savvy advisor biases his report. The lower is  $\theta^*$ , the greater the bias. The bias is maximal when  $\theta^* = 0$  since this means that for the interval  $[0, a)$ , the tech-savvy advisor reports  $a$ . So a measure of the upward bias in the tech-savvy's report is  $a - \theta^*$ .

**Proposition 4** *The upward bias of the tech-savvy advisor's reporting strategy (given by  $a - \theta^*$ ) increases with the number of advisees at  $t = 1$  ( $n$ ) and the fraction of smart investors ( $\rho$ ), and decreases with the fraction of tech-savvy advisors ( $\pi_0$ ).*

Proposition 4 shows that message inflation is more severe in period 1 if there is a bigger fraction of smart investors. The intuition for these comparative statics is similar to that of  $V_1 - V_2$  since the upward bias in the initial report is driven by the need to gain a better reputation.

### 2.3.2 Asset Price at $t = 0$

We now derive the equilibrium price of the tech stock at  $t = 0$ . An individual investor, who observes a signal  $s_{i,0}$  from his advisor and takes the asset price  $p$  as given, chooses his asset holding  $x_i$  according to

$$\max_{x_i} x_i [E_i(\theta | s_{i,0}) - p] - \frac{1}{T} x_i^2 \text{Var}_i(\theta + \epsilon | s_{i,0}). \quad (37)$$

By solving the first order condition, we have that

$$x_i = \frac{T (E_i(\theta | s_{i,0}) - p)}{2 \text{Var}_i(\theta + \epsilon | s_{i,0})}. \quad (38)$$

We are assuming that investors' aggregate risk tolerance is  $T$ .

For simplicity, we assume that there is a flip of a coin at the beginning of the economy (with  $\pi_0$ , the advisor to all investors will be tech-savvy) and with probability  $1 - \pi_0$ , the advisor will be an old-fogie.). All investors in the economy (both smart and naive) receive the same signal, which we will simply denote by  $s$ . Based on the signal given by the advisor,  $s$ , the two types of investors will determine their beliefs of the asset fundamental. To simplify notation, we denote

$$\hat{\theta}^s(s) = E^s(\theta|s), \quad \Sigma^s(s) = Var^s(\theta + \epsilon|s) \quad (39)$$

as the mean and variance of smart investors' belief. Their net asset holding is

$$x_s = \frac{\rho T (\hat{\theta}^s - p)}{2 \Sigma^s}. \quad (40)$$

We denote

$$\hat{\theta}^n(s) = E^n(\theta|s), \quad \Sigma^n(s) = Var^n(\theta + \epsilon|s) \quad (41)$$

as the mean and variance of naive investors' belief. Their net asset holding is

$$x_n = \frac{(1 - \rho) T (\hat{\theta}^n - p)}{2 \Sigma^n}. \quad (42)$$

The market clearing condition is

$$\rho x_s + (1 - \rho) x_n = \bar{x}, \quad (43)$$

where  $\bar{x} > 0$  is the net asset supply. By solving the market clearing condition, we obtain the equilibrium asset price:

$$p = \frac{\rho \Sigma^n}{\rho \Sigma^n + (1 - \rho) \Sigma^s} \hat{\theta}^s + \frac{(1 - \rho) \Sigma^s}{\rho \Sigma^n + (1 - \rho) \Sigma^s} \hat{\theta}^n - \frac{2}{T} \frac{\bar{x} \Sigma^s \Sigma^n}{\rho \Sigma^n + (1 - \rho) \Sigma^s}. \quad (44)$$

We will focus on the case that investors are close to risk-neutral, or equivalently their risk tolerance  $T \rightarrow \infty$ . In this case, the trading among investors is simply a wealth transfer game and it does not affect the aggregate welfare of investors.

In the risk-neutral case, the asset price is a weighted average of smart and naive investors' beliefs:

$$p(s) = \frac{\rho \Sigma^n(s)}{\rho \Sigma^n(s) + (1 - \rho) \Sigma^s(s)} \hat{\theta}^s(s) + \frac{(1 - \rho) \Sigma^s(s)}{\rho \Sigma^n(s) + (1 - \rho) \Sigma^s(s)} \hat{\theta}^n(s), \quad (45)$$

where the weights depend on the proportion of the two groups ( $\rho$ ) and their belief variances,  $\Sigma^s$  and  $\Sigma^n$ . The asset price depends on the signal sent by the advisor.

Except for the case in which  $s < \theta^*$ ,  $\Sigma^n(s) = \Sigma^s(s)$ . Even when  $s < \theta^*$ ,  $\Sigma^n(s)$  would be close to  $\Sigma^s(s)$  as long as the variance of  $\epsilon$  is much larger than the variance of  $\theta$ . Thus, it is reasonable to assume that

$$p(s) = \rho \hat{\theta}^s(s) + (1 - \rho) \hat{\theta}^n(s). \quad (46)$$

This price function greatly simplifies our discussion.

For each possible value of  $\theta$ , the signal in the market depends on the type of the advisor. The advisor's type is random with a probability of  $\pi_0$  as a tech-savvy and a probability of  $1 - \pi_0$  as an old-fogie. The asset price after taking expectation over the realization of the advisor's type is

$$\begin{aligned} E[p(s)|\theta] &= \pi_0[\rho \hat{\theta}^s(s^{TS}(\theta)) + (1 - \rho) \hat{\theta}^n(s^{TS}(\theta))] \\ &\quad + (1 - \pi_0)[\rho \hat{\theta}^s(s^{OF}(\theta)) + (1 - \rho) \hat{\theta}^n(s^{OF}(\theta))] \\ &= \rho[\pi_0 \hat{\theta}^s(s^{TS}(\theta)) + (1 - \pi_0) \hat{\theta}^s(s^{OF}(\theta))] \\ &\quad + (1 - \rho)[\pi_0 \hat{\theta}^n(s^{TS}(\theta)) + (1 - \pi_0) \hat{\theta}^n(s^{OF}(\theta))] \end{aligned} \quad (47)$$

In an efficient market with risk-neutral investors, the price of an asset should be an unbiased estimator of the asset's fundamental value:

$$E[p] = E[v]. \quad (48)$$

In our model, the unconditional mean of the asset fundamental is

$$E[v] = E[\theta] = 1/2. \quad (49)$$

Thus, the deviation of the unconditional mean of the price represents a bubble.

We evaluate the mean of the price by taking average of equation (47) over all possible values of  $\theta$ . We note that the first bracket of equation (47), which represents the contribution of smart investors' belief in prices, is unbiased:

$$\begin{aligned}
& E\{\pi_0 \hat{\theta}^s(s^{TS}(\theta)) + (1 - \pi_0) \hat{\theta}^s(s^{OF}(\theta))\} \\
&= \pi_0 E\{E^s[\theta | s^{TS}(\theta)]\} + (1 - \pi_0) E\{E^s[\theta | s^{OF}(\theta)]\} \\
&= \pi_0 E[\theta] + (1 - \pi_0) E[\theta] \\
&= E[\theta] = 1/2.
\end{aligned} \tag{50}$$

By substituting the last equation into equation (47), we obtain

$$\begin{aligned}
E[p(s)] - E[v] &= (1 - \rho) E[\pi_0 \hat{\theta}^n(s^{TS}(\theta)) + (1 - \pi_0) \hat{\theta}^n(s^{OF}(\theta)) - \theta] \\
&= (1 - \rho) E[\pi_0 s^{TS}(\theta) + (1 - \pi_0) s^{OF}(\theta) - \theta]
\end{aligned} \tag{51}$$

According to our discussion earlier,

$$s^{TS}(\theta) = \theta + (a - \theta) I_{\theta \in (\theta^*, a)}, \quad s^{OF}(\theta) = a\theta, \tag{52}$$

where  $I$  is an indicator function. Thus,

$$\begin{aligned}
E[p(s)] - E[v] &= (1 - \rho) E[\pi_0 \theta + \pi_0 (a - \theta) I_{\theta \in (\theta^*, a)} + (1 - \pi_0) a\theta - \theta] \\
&= (1 - \rho) \{ \pi_0 E[(a - \theta) I_{\theta \in (\theta^*, a)}] - (1 - a)(1 - \pi_0) E[\theta] \} \\
&= \frac{1}{2} (1 - \rho) [\pi_0 (a - \theta^*)^2 - (1 - \pi_0)(1 - a)].
\end{aligned} \tag{53}$$

The average price bias is proportional to the fraction of naive investors  $(1 - \rho)$ . The average price bias also depends on the tradeoff between the message inflation from tech-savvy advisors,  $\pi_0 (a - \theta^*)^2 / 2$ , and the message deflation from old-fogie advisors,  $(1 - \pi_0)(1 - a) / 2$ . As a result, we are able to establish the following theorem regarding the existence of a technology bubble.

**Theorem 2** *When  $n$  is large enough,  $\theta^*$  approaches zero. So a technology price bubble arises when  $\pi_0 a^2 - (1 - \pi_0)(1 - a) > 0$  (this condition is easily satisfied for a wide range of values for  $\pi_0$  and  $a$ ).*

### 3 Comparative Statics and Empirical Implications

In this section, we consider some comparative static exercises to develop intuition for the equilibrium. Then we use one set of these exercises to develop some testable implications of our model.

We will consider the effects of changing the fraction of smart investors ( $\rho$ ), the fraction of tech-savvy advisors ( $\pi_0$ ) and the degree of old-fogie-ness ( $a$ ) on the average price bias and the bias in the report of the tech-savvy advisor's  $t = 0$  recommendation ( $a - \theta^*$ ). Figure 1 presents the results of changing  $\rho$ . For these results, the other parameters are set the following values:

$$a = 0.8, \pi_0 = 0.7, I = 0.8, n = 4000, \text{ and } c = 1. \quad (54)$$

Notice that the average price bias is non-linear in  $\rho$  (taking on negative values when  $\rho = 0$  and is zero when  $\rho = 1$ ). It peaks at a value for  $\rho$  of around 0.3. On the other hand,  $a - \theta^*$  increases with  $\rho$ , starting at zero for  $\rho = 0$  and reaching a peak of  $a$  at  $\rho = 1$ . The intuition is simple. When  $\rho = 1$ , tech-savvy advisors bias their reports to the greatest extent possible but there is no price bias since all investors are smart and understand the strategy of tech-savvy advisors to build reputation. When  $\rho = 0$ , tech-savvy advisors do not bias their reports at all since all investors are naive and will simply listen to what they say. However, the price bias is negative since naive investors also take what old-fogies say at full value and hence price will tend to be below fundamental value. The key point to note here is that the effect of interest in this paper is strong enough to overcome the baseline effect that naive investors also listen to old-fogies.

Figure 2 presents the results for changes in  $\pi_0$ . Now the value of  $\rho$  is set at 0.3 for this exercise and  $\pi_0$  is allowed to change. Notice that the price bias is also non-linear in  $\pi_0$ , while the recommendation bias of tech-savvy advisors decreases with the fraction of tech-savvy advisors in the economy. The intuition for these results are again simple. When all advisors are tech-savvy, there is no price bias since advisors have no need to build reputation. When there is a close to zero fraction of tech-savvy advisors in the economy, the few tech-savvy advisors will bias their recommendations to the greatest



extent possible, but have no effect on prices since there are so few of them. Indeed, when there are only old-fogies, the price bias is again negative since naive investors take what they say at full value.

Figure 3 presents the results for changes in  $a$ , the degree of old-fogie-ness of the old-fogies. The patterns are similar to that of  $\pi_0$ . If  $a = 1$ , then there is no difference in between old-fogies and tech-savvies, and so tech-savvies have no incentive to bias their recommendations and there is no price bias. When  $a = 0$ , tech-savvies bias their recommendations to the greatest extent possible, but have no effect since the old-fogies are so pessimistic.

While each of these three sets of exercises offer some hope of being testable, we think that the first set involving  $\rho$  is the most realistic. The first prediction involves the relationship between price bias and  $\rho$ . Lets consider the internet period for our analysis since this was clearly a period of excitement about a genuinely new technology. More pragmatically, there is also a lot of data during this period. Lets take the price bias to be the market-to-book ratio of a stock. Our model predicts that in the cross-section, this ratio is non-linear in the heterogeneity of institutional (which we assume is smart) and individual (which we assume is naive) investors' holdings in the stock: smaller when there are stock holders are only retail or only institutional and larger when there is a mix of both. This strikes us as being a genuinely testable implication.

The second prediction, again focusing on the internet period, is that the tech-savvies (which we take to be the optimistic sell-side analysts) should issue more optimistic recommendations on stocks in which the investors are mostly institutional and issue less optimistic recommendations on stocks in which the investors are mostly individuals. We have less confidence in this prediction because sell-side analysts' incentives are known to be influenced by investment banking and trading commissions. In other words, they are not purely good-intentioned as the advisors in our model. Nonetheless, controlling to the greatest extent possible for these off-setting incentives, it might still be interesting to see whether this second prediction is true.

And third, our model can deliver an “anti-bubble” in that valuations might be depressed artificially after bubble periods. If investors after a bubble period become

skeptical of advisors who are too upbeat, then prices may then be downward biased.

## 4 Robustness and Extensions

### 4.1 Other Equilibria

In this paper, we construct an equilibrium at  $t = 0$  in which a tech-savvy advisor biases his message to  $a$  (or slightly above  $a$ ) because there is no way an old-fogie would deliver such a message (i.e. an old-fogie's message support is on the interval  $[0, a]$  by assumption). Alternatively, one could attempt to construct an equilibrium in which the tech-savvy advisor biases his message to some other value, say  $a_0$ , where  $a_0 < a$ . This might be advantageous because the advisor incurs a lower dishonesty cost. Suppose that the tech-savvy advisor commits to only saying  $a_0$  for realizations of fundamental value  $\theta$  around  $a_0$ . If smart investors know that only tech-savvy advisors say  $a_0$  with a high probability, then tech-savvy advisors may be able to more cost efficiently signal their type. However, such an equilibrium requires more public information or coordination between tech-savvy advisors and smart investors than biasing the message to  $a$ . Biasing to slightly above  $a$  is a natural strategy to signal to smart investors since it arises naturally out of the non-overlapping support of old-fogies and smart investors at  $a$ —old fogies would never say  $a$ , directly from knowledge of their support. The strategy of biasing to  $a_0$  would require more public knowledge (an announcement pre-game that  $a_0$  is what smart investors bias to). As such, we focus on the equilibrium centered on  $a$ , though we acknowledge the possibility of other equilibria requiring more pre-game coordination.

### 4.2 Alternative Assumptions

Our results depend on two assumptions. First, old-fogies always report a downward biased signal. What happens if old-fogies are allowed to send biased messages at a cost just like tech-savvies? Second, an investor at  $t = 0$  only observes the messages sent by his advisor. What if an investor can observe the recommendations of other advisors as well? In this subsection, we extend our model to relax these two assumptions.

This extension yields several new insights. Notably, old-fogies want to bias their recommendations upwards at  $t = 0$ . However, tech-savvies can still separate themselves from old-fogies by inflating their recommendations to a level that is too costly for old-fogies to mimic. Such separation is feasible when tech-savvies' true beliefs are sufficiently above those of old-fogies. Otherwise tech-savvies truthfully report their beliefs, which are mimicked by old-fogies, resulting in a pooling of the two types. Since naive investors take their advisors' signals at face value, we show that a bubble can still arise even while relaxing these two key assumptions.

But in order to solve the model in this more general setting, we need to make additional assumptions related to the off-equilibrium beliefs of market participants. While we think these assumptions are reasonable, other equilibria could also arise under alternative assumptions about their off-equilibrium beliefs, as is commonly the case with Bayesian Nash equilibria. In this sense, our solution here is more fragile than that of our benchmark model.

We adopt the same set-up as in Section 2.1. At  $t = 0$ , both tech-savvy and old-fogie advisors form their beliefs about a new technology  $\theta$ . While the tech-savvies' belief is correct ( $\hat{\theta}^{TS} = \theta$ ), the old-fogies' belief is downward biased ( $\hat{\theta}^{OF} = a\theta$ ). Both types of advisors report a signal to their advisees based on their beliefs. Extending our basic model, we assume that old-fogies, like tech-savvies, can also choose to bias their signal (recommendation) relative to their own belief, subject to a dishonesty cost:

$$c(s_0^{OF} - \hat{\theta}^{OF})^2. \quad (55)$$

In addition, we assume that at  $t = 0$  there are many advisors in the market covering the same technology stock and their signals are observable not only to their advisees, but also to other investors. Consequently, smart investors can compare signals from all advisors to infer the asset fundamental  $\theta$ . We assume that a naive continues to only use his advisor's signal and accepts it at face value.

At  $t = 1$ , each advisor is matched with a new set of investors and assists them in making further investment decisions. We view the advising activity at  $t = 1$  as an advisor's private consulting just like before. However, we now assume that both tech-

savvy and old-fogie advisors can choose to bias their signals, subject to the dishonesty cost, to maximize their advisees' welfare based on their heterogenous beliefs.

We now solve for the equilibrium. At  $t = 1$ , an advisor, with an imperfect reputation  $\pi \in (0, 1)$  (probability as a tech-savvy), needs to assist her advisees with whether to invest in a real project. As we discussed before, when a tech-savvy advisor with an imperfect reputation believes that the project's liquidation value  $f$  is below the cost  $I$ , smart investors might choose to invest in the project even if the advisor truthfully reports her pessimistic belief. This is because smart investors attribute a positive probability to the advisor being an old-fogie. The optimal communication strategy on the part of tech-savvy advisors is to downward bias his messages over some beliefs so as to communicate to his smart advisee that certain messages could not be coming from him. An old-fogie advisor, with a reputation  $\pi \in (0, 1)$ , also has the same incentive to deflate her recommendation over the same belief region.

Proposition 5 summarizes the Bayesian Nash equilibrium at  $t = 1$ , with the proof given in the Appendix.

**Proposition 5** *The equilibrium at  $t = 1$  consists of the following profiles. The reporting strategy of a tech-savvy advisor, with a reputation of  $\pi \in [0, 1]$  among his advisees as a tech-savvy, is*

$$s_1^{TS}(\hat{f}^{TS}) = \begin{cases} \hat{f}^{TS} & \text{if } \hat{f}^{TS} \geq f^* \\ dI & \text{if } dI \leq \hat{f}^{TS} < f^* \\ \hat{f}^{TS} & \text{if } \hat{f}^{TS} < dI \end{cases} \quad (56)$$

where  $\hat{f}^{TS}$  is the advisor's belief about the project's fundamental  $f$ . The parameters  $f^*$  and  $d$  are determined by the following equations:

$$n\rho(I - f^*) = nc(f^* - dI)^2 \quad (57)$$

$$\frac{1}{2}(f^* + dI) = b(\pi)I \quad (58)$$

where

$$b(\pi) = \frac{1}{\hat{\pi} + \frac{1}{a}(1 - \hat{\pi})}, \quad \text{with } \hat{\pi} = \frac{\pi}{\pi + \frac{1}{a}(1 - \pi)}. \quad (59)$$

The reporting strategy of an old-fogie advisor, with a reputation of  $\pi$ , has the same reporting strategy as that of a tech-savvy advisor with the same reputation:

$$s_1^{OF}(\hat{f}^{OF}) = \begin{cases} \hat{f}^{OF} & \text{if } \hat{f}^{OF} \geq f^* \\ dI & \text{if } dI \leq \hat{f}^{OF} < f^* \\ \hat{f}^{OF} & \text{if } \hat{f}^{OF} < dI \end{cases} \quad (60)$$

where  $\hat{f}^{OF}$  is the advisor's belief about the project's fundamental  $f$ .

After observing the signal, a naive investor invests if and only if the signal is above  $I$ . A smart investor does not invest if the signal is equal to or below  $dI$  and invests otherwise.

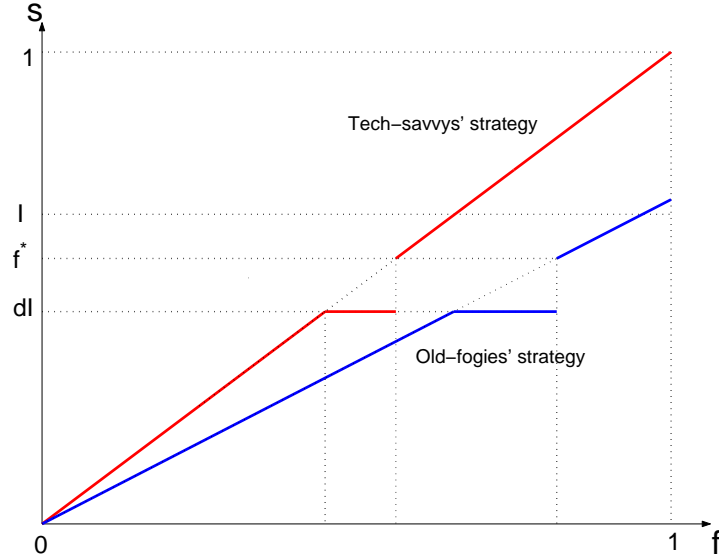


Figure 4: Advisors' strategies at  $t = 1$  with imperfect information about type.

Proposition 5 shows that tech-savvy and old-fogie advisors, with the same reputation and the same level of project belief, will choose to report the same signal, as illustrated in Figure 4. The proof of Proposition 5 is standard. We verify the optimality of advisors' reporting strategy by taking investors' learning rules as given, and then verify the optimality of investors' learning rules by taking advisors' reporting strategies as given. Since, in equilibrium, advisors do not report signals in region  $(dI, f^*)$ , we

need to specify an off-equilibrium belief for smart investors if an advisor chooses to send a signal  $s_1 \in (dI, f^*)$ . In the proof, we assume that conditioned on advisor type, an investor believes that his advisor's belief is uniformly distributed on  $(s_1, f^*)$ .<sup>5</sup>

According to Proposition 5, an imperfect reputation ( $\pi < 1$ ) creates inefficiencies for both types of advisors, i.e., both tech-savvy and old-fogie advisors need to incur dishonesty cost to avoid inefficient investment by their smart advisees if the advisors' beliefs are in  $(dI, f^*)$ . Furthermore, the smart advisee makes an inefficient investment if the advisor's belief is in  $(f^*, I)$ . Thus, for a benevolent old-fogie advisor with a reputation  $\pi$ , the expected inefficiency, dishonesty cost incurred by herself together with the investment loss by her advisees, is

$$K^{TS}(\pi) = \int_{f^*}^I n\rho(I - \hat{f})d\hat{f} + \int_{dI}^{f^*} nc(\hat{f} - dI)^2d\hat{f}. \quad (61)$$

For a benevolent old-fogie advisor with a reputation  $\pi$ , the expected inefficiency is

$$K^{OF}(\pi) = \int_{f^*}^I n\rho(I - \hat{f})d\hat{f}/a + \int_{dI}^{f^*} nc(\hat{f} - dI)^2d\hat{f}/a. \quad (62)$$

Note that

$$K^{OF}(\pi) = K^{TS}(\pi)/a. \quad (63)$$

Based on the expressions for  $K^{TS}$  and  $K^{OF}$ , we can directly verify that they are monotonically decreasing with the advisor's reputation  $\pi$ , as stated in the following proposition.

**Proposition 6** *Both  $K^{TS}(\pi)$  and  $K^{OF}(\pi)$  decrease with  $\pi$ , and are zero when  $\pi = 1$ .*

Proposition 6 shows that both tech-savvy and old-fogie advisors can benefit from a good reputation. Thus, at  $t = 0$ , tech-savvy advisors have incentives to separate themselves from old-fogie advisors by reporting an optimistic signal. At the same time,

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<sup>5</sup>This assumption is reasonable because according to the derivation of the advisors' optimal reporting strategies, an advisor (independent of the type) never wants to inflate her signal, but might want to deflate her signal if her belief is below  $f^*$ . Thus, a signal in  $(dI, f^*)$  could come from an advisor who chooses to deflate her signal, but does not deflate it enough to the optimal level  $dI$ .

old-fogie advisors also have incentives to mix with tech-savvy advisors by inflating their signals too. Due to these incentives from both types, the equilibrium has two outcomes: (1) a separating outcome in which tech-savvy advisors report an extremely optimistic signal that is too costly for old-fogie advisors to match (this happens when the fundamental is sufficiently high); and (2) a pooling outcome, in which tech-savvy advisors truthfully report their belief and old-fogie advisors match such a recommendation (when the fundamental is not too high).

The following proposition summarizes the equilibrium, with the proof given in the Appendix.

**Proposition 7** *Under certain sufficient conditions:*

$$1 - a \leq \sqrt{K^{OF}(0)/c} \quad (64)$$

$$[\sqrt{K^{OF}(0)} - \sqrt{K^{TS}(\hat{\pi})}]/a < \sqrt{K^{OF}(0) - K^{OF}(\hat{\pi})} < \sqrt{c}(1 - a)/a \quad (65)$$

where  $\hat{\pi} = \frac{\pi_0}{\pi_0 + (1 - \pi_0)/a}$ , we have the following Bayesian Nash equilibrium at  $t = 0$ .

Given a tech-savvy advisor's belief  $\hat{\theta}^{TS}$ , which is equal to the true value  $\theta$ , her reporting strategy is

$$s_0^{TS}(\hat{\theta}^{TS}) = \begin{cases} a\hat{\theta}^{TS} + z & \text{if } \hat{\theta}^{TS} \geq \theta^* \\ \hat{\theta}^{TS} & \text{if } \hat{\theta}^{TS} < \theta^*, \end{cases} \quad (66)$$

where

$$z = \sqrt{K^{OF}(0)/c}, \quad (67)$$

and  $\theta^* \in (0, 1)$  is defined as

$$\theta^* = \frac{a}{1 - a} \sqrt{[K^{OF}(0) - K^{OF}(\hat{\pi})]/c}. \quad (68)$$

Given an old-fogie advisor's belief  $\hat{\theta}^{OF}$ , which is equal to  $a\theta$ , her reporting strategy is

$$s_0^{OF}(\hat{\theta}^{OF}) = \begin{cases} \hat{\theta}^{OF} & \text{if } \hat{\theta}^{OF} \geq a\theta^* \\ \hat{\theta}^{OF}/a & \text{if } \hat{\theta}^{OF} < a\theta^*. \end{cases} \quad (69)$$

Naive investors always take the signal from their advisors at the face value. If  $\theta < \theta^*$ , their beliefs are correct; if  $\theta \geq \theta^*$ , their belief could be either upward biased when matched with a tech-savvy advisor or downward biased when matched with an old-fogie advisor.

Smart investors are always able to correctly figure out the value of  $\theta$  by comparing signals available in the market. Furthermore, when  $\theta > \theta^*$ , there is a separating outcome, tech-savvy advisors inflate their signal to  $a\theta + z$ , while old-fogie advisors report  $a\theta$ . Thus, smart investors are able to identify the type of their advisors. When  $\theta < \theta^*$ , there is a pooling outcome, i.e., all advisors send the same signal equal to  $\theta$ . Thus, smart investors attribute probability  $\hat{\pi} = \frac{\pi_0}{\pi_0 + (1-\pi_0)/a}$  to their advisors as tech-savvy.

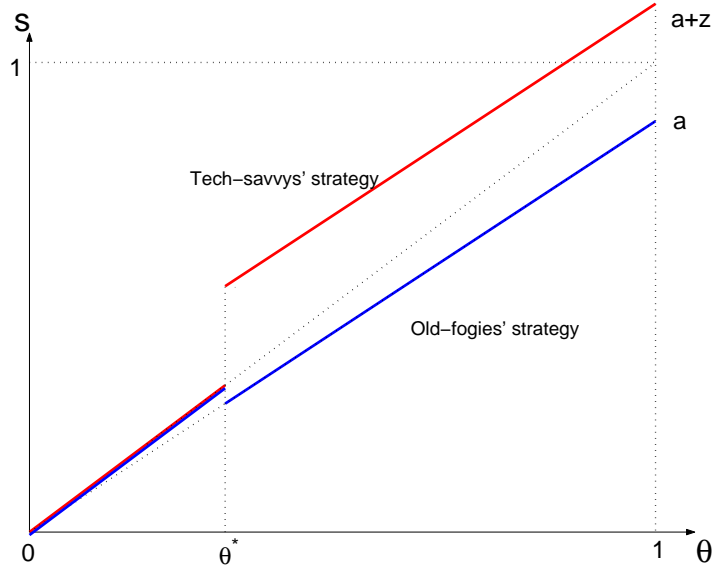


Figure 5: Advisors' strategies at  $t = 0$ .

Proposition 7 shows that, as illustrated in Figure 5, when the technology fundamental  $\theta$  is sufficiently high ( $\theta \geq \theta^*$ ), tech-savvy advisors are able to separate themselves from old-fogie advisors by inflating their signal. It is too costly for old-fogie advisors to match tech-savvies' signal because their belief is substantially below tech-savvies'. When  $\theta$  is small ( $\theta < \theta^*$ ), tech-savvy and old-fogie advisors' beliefs are close enough that it becomes too costly for tech-savvy advisors to separate themselves, and conse-



quently there is a pooling equilibrium, in which old-fogies inflate their signal to match tech-savvies' truthful report.

The proof of Proposition 7 is standard. We again need to specify certain assumptions for investors' learning rules when they receive off-equilibrium signals. These assumptions are similar in spirit to those used to derive the equilibrium at  $t = 1$ .

According to our discussion earlier, the asset price at  $t = 0$  is a weighted average of smart and naive investors. Since smart investors can always figure out the exact value of  $\theta$  by comparing signals, biases in price can only be driven by biased beliefs of naive investors. When  $\theta \in (0, \theta^*)$ , naive investors' belief is unbiased. Thus, the asset price is also unbiased. When  $\theta \in (\theta^*, 1)$ , the difference between the asset price and asset fundamental is

$$\begin{aligned} p(\theta) - \theta &= (1 - \rho)[\pi_0(a\theta + z - \theta) - (1 - \pi_0)(1 - a)\theta] \\ &= (1 - \rho)[\pi_0 z - (1 - a)\theta]. \end{aligned} \tag{70}$$

In the first equation above,  $(1 - \rho)$  is the fraction of naive investors in population, among whom a fraction of  $\pi$  is matched with and the rest  $1 - \pi_0$  is matched with old-fogie advisors. The term  $(a\theta + z - \theta)$  represents the upward bias in naive investors' belief when they are matched with tech-savvy advisors and the term  $(1 - a)\theta$  represents the downward bias in naive investors' belief when they are matched with old-fogie advisors. It is direct to see from equation (70) that the bias in price is always positive under the following sufficient condition in Proposition 8.

**Proposition 8** *If  $\pi_0 z > (1 - a)$ , the asset price is always upward biased when  $\theta \geq \theta^*$ .*

## 5 Conclusion

We conclude by re-interpreting the events of the internet period in light of our model. In the aftermath of the internet bubble, many point to the role of biased advisors in manipulating the expectations of naive investors. We agree with the focus on the role of advisors but point out that there is something deeper in the communication process between advisors and investors that lead to an upward bias in prices during times of

excitement about new technologies even absent any explicit incentives on the part of analysts to sell stocks.

Our model suggests that the internet period was a time when there is a natural concern for investors about whether their advisors understand the new technology or not, i.e. do they have an old-fogie for an advisor or a tech-savvy guy? Investors do not want to listen to old-fogies. As a result, good-intentioned advisors have an incentive to signal that they are tech-savvy by issuing optimistic forecasts because they want to be listened to in the future. Unfortunately, naive investors do not understand the incentives of advisors to upward bias their forecasts and as a result, price ends up being upwardly biased.

This view is not totally without empirical support. In addition to the evidence cited in the introduction, it is well known that the reports issued by sell-side analysts are typically only read by institutional investors, who for the most part, do a good job of de-biasing analyst recommendations. Unfortunately, during the internet period, a lot of retail investors, through television programs and the like, took the positive, upbeat recommendations of analysts a bit too literally. Again, this is not to say that analysts were solely good-intentioned, but simply that when there are naive investors, there can be a bubble during times of technological excitement even if all analysts are good-intentioned.

## 6 Appendix

### 6.1 Proof of Proposition 3

By integrating equation (32), we have

$$V_1 - V_2 = \frac{n\rho}{2}(I - f^*)^2 + \frac{nc}{3}(f^* - bI)^3. \quad (71)$$

By substituting in equation (9), we can transform the last equation into

$$V_1 - V_2 = n\rho(I - f^*) \left[ \left( \frac{1}{2} - \frac{b}{3} \right) I - \frac{f^*}{6} \right]. \quad (72)$$

This equation directly implies that  $V_1 - V_2$  increases with  $\rho$  and decreases with  $f^*$ .

Equation (9) implies that  $f^*$  increases with  $b$ . Equation (10) implies that  $b$  increases with  $\pi_{LL}$ , which also increases with  $\pi_0$ . Thus,  $f^*$  increases with  $\pi_0$  and  $V_1 - V_2$  decreases with  $\pi_0$ . Equation (10) also implies that  $b$  increases with  $a$ , therefore  $f^*$  increases with  $a$  and  $V_1 - V_2$  decreases with  $a$ .

### 6.2 Proof of Proposition 4

Proposition 3 implies that the gain from a good reputation,  $V_1 - V_2$ , increases with  $n$  and  $\rho$ , and decreases with  $\pi_0$ . Then, equation (34) implies that  $\theta^*$  (weakly) decreases with  $n$  and  $\rho$ , and (weakly) increases with  $\pi_0$ .

### 6.3 Proof of Proposition 5

First, take as given tech-savvy and old-fogie advisors' reporting strategies, we verify the optimality of smart investors' investment rule based on the realization of  $s_1$ .

1.  $s_1 \in [a, 1]$ . This signal must be from a tech-savvy advisor and the investment project's payoff  $f = s_1 \geq a \geq I$ . Thus, it is optimal for the smart investor to invest in the project.
2.  $s_1 \in [f^*, a]$ . This signal could be from a tech-savvy or an old-fogie. Given the prior that the advisor has probability  $\pi$  to be a tech-savvy, the updated probability is  $\hat{\pi} = \frac{\pi}{\pi + (1-\pi)/a}$ . Note that for any  $s_1 < a$ , then  $\hat{\pi}$  is the probability

that a smart investor places on the signal being from a tech-savvy. Thus, the expected project payoff is

$$E\{f|s_1 \in [f^*, a]\} = \hat{\pi}s_1 + (1 - \hat{\pi})s_1/a > s_1 \geq a \geq I. \quad (73)$$

Hence the investor invests.

3.  $s_1 \in (dI, f^*)$ . Since neither tech-savvies nor old-fogies report signals in this range in equilibrium, we need to specify a certain off-equilibrium belief for the investor in order to derive her investment decision. We assume that after receiving a signal in  $(dI, f^*)$ , the investor believes that the signal could be from either tech-savvies or old-fogies with beliefs between  $s_1$  and  $f^*$ . Since it is impossible to formulate the probability of advisors to send such an off-equilibrium signal, we assume that conditioned on advisor type, the advisor's belief is uniformly distributed on  $(s_1, f^*)$ . Thus, the investor's updated belief of the project payoff is

$$\begin{aligned} E[f|s_1 \in (dI, f^*)] &= [\hat{\pi} + (1 - \hat{\pi})/a] \frac{(s_1 + f^*)}{2} \\ &> [\hat{\pi} + (1 - \hat{\pi})/a] \frac{(dI + f^*)}{2} \\ &= [\hat{\pi} + (1 - \hat{\pi})/a] b(\pi)I \\ &= I. \end{aligned} \quad (74)$$

Consequently, the investor would choose to invest.

4.  $s_1 = dI$ . Such a signal could be from either a tech-savvy and an old-fogie when they have a belief between  $dI$  and  $f^*$  and choose to under-report. Given that the prior beliefs that the advisor has a probability  $\pi$  to be a tech-savvy, the investor's updated probability is  $\hat{\pi}$ . Furthermore, since advisors' beliefs are uniformly distributed, the investor's expectation of the project payoff is

$$\begin{aligned} E[f|s_1 = dI] &= [\hat{\pi} + (1 - \hat{\pi})/a] \frac{(dI + f^*)}{2} \\ &= [\hat{\pi} + (1 - \hat{\pi})/a] b(\pi)I \\ &= I. \end{aligned} \quad (75)$$

Thus, the investor is indifferent between invest and not to invest.

5.  $s_1 \in [0, dI)$ . Such a signal could be a truthful signal from either a tech-savvy or an old-fogie. The investor's expectation of the project payoff is

$$\begin{aligned}
E[f|s_1] &= [\hat{\pi} + (1 - \hat{\pi})/a]s_1 \\
&< [\hat{\pi} + (1 - \hat{\pi})/a]dI \\
&< [\hat{\pi} + (1 - \hat{\pi})/a]b(\pi)I \\
&= I.
\end{aligned} \tag{76}$$

Thus, the investor chooses not to invest.

Next, take the smart investor's investment strategy as given, we derive both tech-savvy and old-fogie advisors' optimal reporting strategies. Both type of advisors want to maximize their advisees' investment profits and at the same time to minimize dishonest cost if they choose to mis-report their signals. Given this common objective, tech-savvy and old-fogie advisors have the same optimal reporting strategy. Thus, in the proof below, we do not differentiate them. We denote  $\hat{f}$  as a given advisor's belief about the project fundamental. Based on the realization of  $\hat{f}$ , we have the following cases.

1.  $\hat{f} > I$ . The advisor would truthfully report their belief because it is optimal to invest in the project and all investor would do so after receiving a truthful signal.
2.  $\hat{f} \in [f^*, I)$ . The advisor believes that it is inefficient to invest in the project. If she truthfully reports her belief in the signal, a naive advisee takes the signal at the face value and thus will choose not to invest. However, a smart advisee cannot distinguish whether the signal is from a tech-savvy or old-fogie advisor and will choose to invest according to her investment strategy. The advisor can avoid the inefficient investment by her smart advisees by deflating/ the signal to  $dI$ . In doing so, the advisor has to incur a dishonesty cost of

$$nc(\hat{f} - dI)^2 \geq nc(f^* - dI)^2 = n\rho(I - f^*) > n\rho(I - \hat{f}), \tag{77}$$

which is her advisees' investment losses. Since the cost is higher than the avoided investment losses, it is not optimal for the advisor to deflate her signal to  $dI$ . It

is also not optimal to deflate her signal to a level higher than  $dI$ , since a smart advisee will invest after observing such a signal. Thus, the advisor would simply report her belief.

3.  $\hat{f} \in (dI, f^*)$ . Similar to the previous case, the advisor believes that it is inefficient to invest in the project and faces a dilemma that her smart advisees would choose to invest after observing a truthful signal from her. To avoid this outcome, it is optimal for the advisor to deflate her signal to  $dI$ , because her dishonesty cost is now smaller than her smart advisees' potential investment losses:

$$nc(\hat{f} - dI)^2 < nc(f^* - dI)^2 = n\rho(I - f^*) < n\rho(I - \hat{f}). \quad (78)$$

Also note that the advisor won't deflate her signal to a level higher than  $dI$  since her smart advisees would still choose to invest after receiving such a signal. Thus, the advisor would deflate her signal to  $dI$ .

4.  $\hat{f} \in [0, dI]$ . The advisor would truthfully report her belief, because the advisor regards the project as inferior and all her advisees would avoid the project after receiving a truthful signal.

## 6.4 Proof of Proposition 7

Take advisors' reporting strategies as given, it is direct to verify the optimality of smart investors' learning rule. Hence we focus on the verification of the optimality of an individual advisor's reporting strategy, taking investors' learning rule and other advisors' reporting strategies as given.

First, we verify the optimality of a tech-savvy advisor's reporting strategy. Depending on the realization of her belief  $\hat{\theta}^{TS}$ , which is equal to  $\theta$ , we have the following cases.

1.  $\hat{\theta}^{TS} \in [\theta^*, 1]$ . The advisor knows that other tech-savvy advisors share her belief and will report signals equal to  $a\hat{\theta}^{TS} + z$ , and that old-fogie advisors will report signals equal to their belief,  $a\hat{\theta}^{TS}$ . The advisor can choose to inflate her signal to

$a\hat{\theta}^{TS} + z$ , just like other tech-savvies, and obtain a perfect reputation among smart investors. The investor could also truthfully report her belief or inflate her signal to a level lower than  $a\hat{\theta}^{TS} + z$ . (Note that there are no incentives for the advisor to deflate her signal.) Since these alternative signals do not appear in equilibrium, we need to specify an off-equilibrium belief for smart investors in order to evaluate the advisor's optimal choice. We assume that if smart investors receive a signal in region  $[\hat{\theta}^{TS}, a\hat{\theta}^{TS} + z)$ , they believe the signal could be from either tech-savvies or old-fogies, and thus assign the sender probability  $\hat{\pi} = \frac{\pi_0}{\pi_0 + (1 - \pi_0)/a}$  to be a tech-savvy. Given the investors' learning rule, the feasible choices for the advisor are to truthfully report her belief or to inflate the signal to  $a\hat{\theta}^{TS} + z$ , which give a reputation of  $\hat{\pi}$  and 1, respectively. The benefit of inflating the signal is  $K^{TS}(\hat{\pi}) - K^{TS}(1) = K^{TS}(\hat{\pi})$ . The dishonesty cost of inflating the signal is

$$c(a\hat{\theta}^{TS} + z - \hat{\theta}^{TS})^2 = c[z - (1 - a)\hat{\theta}^{TS}]^2. \quad (79)$$

When  $\hat{\theta}^{TS} > \theta^*$ , inequality conditions in (65) imply that the dishonesty cost is less than  $K^{TS}(\hat{\pi})$ . Thus, the advisor's optimal strategy is to inflate the signal to  $a\hat{\theta}^{TS} + z$ .

2.  $\hat{\theta}^{TS} \in [0, \theta^*)$ . In this region, tech-savvy and old-fogie investors' signals are pooled at the same level  $\hat{\theta}^{TS}$ . To verify the optimality of a tech-savvy advisor's strategy, we assume that, if an advisor sends a signal above or equal to  $\hat{\theta}^{TS}$ , smart investors assign her a probability  $\hat{\pi}$  as tech-savvy; and if the advisor sends a signal below  $\hat{\theta}^{TS}$ , smart investors assign her a probability zero as tech-savvy. It is then direct to see that a tech-savvy advisor's optimal choice is to truthfully report her belief.

Next, we verify the optimality of an old-fogie advisor's reporting strategy. Depending on the realization of the advisor's belief  $\hat{\theta}^{OF}$ , which is equal to  $a\theta$ , we have the following cases.

1.  $\hat{\theta}^{OF} \in [a\theta^*, a]$ . The advisor knows that other old-fogie advisors will report their belief  $\hat{\theta}^{OF}$ , while tech-savvy advisors will report  $\hat{\theta}^{OF} + z$ . We need to show that the

advisor's optimal choice is to truthfully report her belief and then be identified as old-fogie. She could choose to inflate her signal to  $\hat{\theta}^{OF} + z$  and thus be identified as tech-savvy. The dishonesty cost that she would incur is  $cz^2$ , while the benefit is  $K^{OF}(0)$ , which is exactly  $cz^2$  by construction of  $z$  in Proposition 7. Thus the advisor is indifferent between telling her belief or inflating the signal to  $\hat{\theta}^{OF} + z$ . The advisor could also inflate her signal to  $\hat{\theta}^{OF}/a$ , the minimum level to obtain a partial reputation  $\hat{\pi}$  as a tech-savvy. The dishonesty cost incurred is

$$c(\hat{\theta}^{OF}/a - \hat{\theta}^{OF})^2 = c\frac{(1-a)^2}{a^2}\hat{\theta}^{OF^2} \geq K^{OF}(0) - K^{OF}(\hat{\pi}), \quad \forall \hat{\theta}^{OF} > \theta^*. \quad (80)$$

Since  $K^{OF}(0) - K^{OF}(\hat{\pi})$  is the benefit of inflating the signal, it is not optimal for the advisor to do so when her belief is above  $\theta^*$ . Overall, truthfully reporting the belief  $\hat{\theta}^{OF}$  is an optimal choice.

2.  $\hat{\theta}^{OF} \in [0, a\theta^*]$ . The old-fogie advisor has two feasible choices. She can either obtain a reputation  $\hat{\pi}$  by inflating her signal to  $\hat{\theta}^{OF}/a$  or she can identify herself as old-fogie by reporting her true belief  $\hat{\theta}^{OF}$ . Since the cost of inflating the signal

$$c(\hat{\theta}^{OF}/a - \hat{\theta}^{OF})^2 = c\frac{(1-a)^2}{a^2}\hat{\theta}^{OF^2} \leq K^{OF}(0) - K^{OF}(\hat{\pi}), \quad \forall \hat{\theta}^{OF} \leq \theta^*. \quad (81)$$

Thus, the advisor's optimal choice is to inflate the signal.



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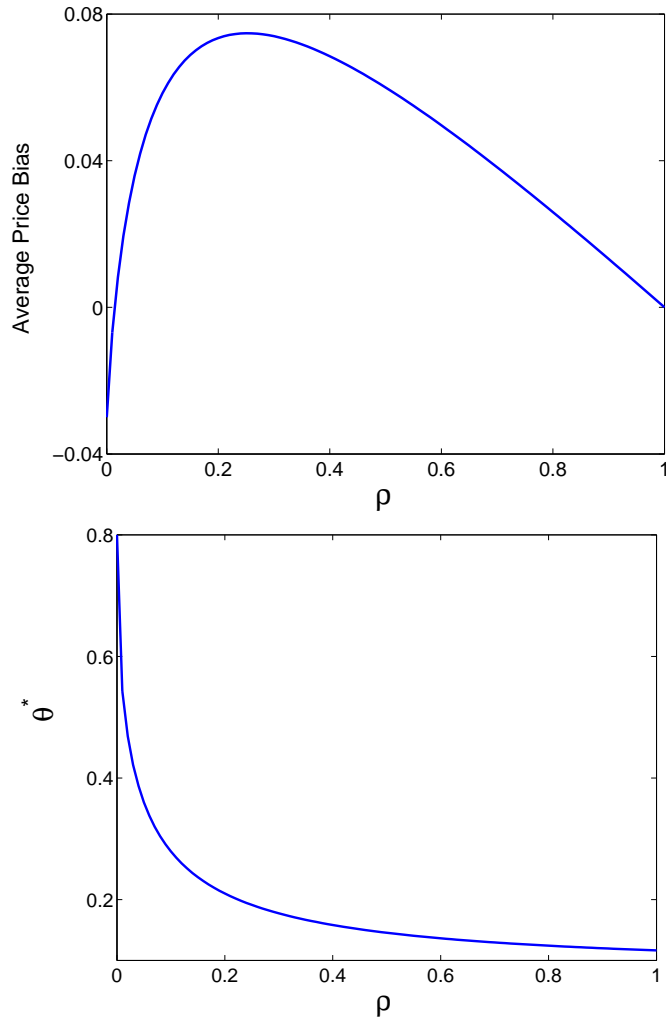


Figure 6: The effects of the fraction of sophisticated investors ( $\rho$ ) on the average price bias and tech-savvy advisors' threshold for inflating their messages ( $\theta^*$ ). The following parameters have been used in this plot:  $a = 0.8$ ;  $\pi_0 = 0.7$ ;  $I = 0.8$ ;  $n = 4000$ ;  $c = 1$ .

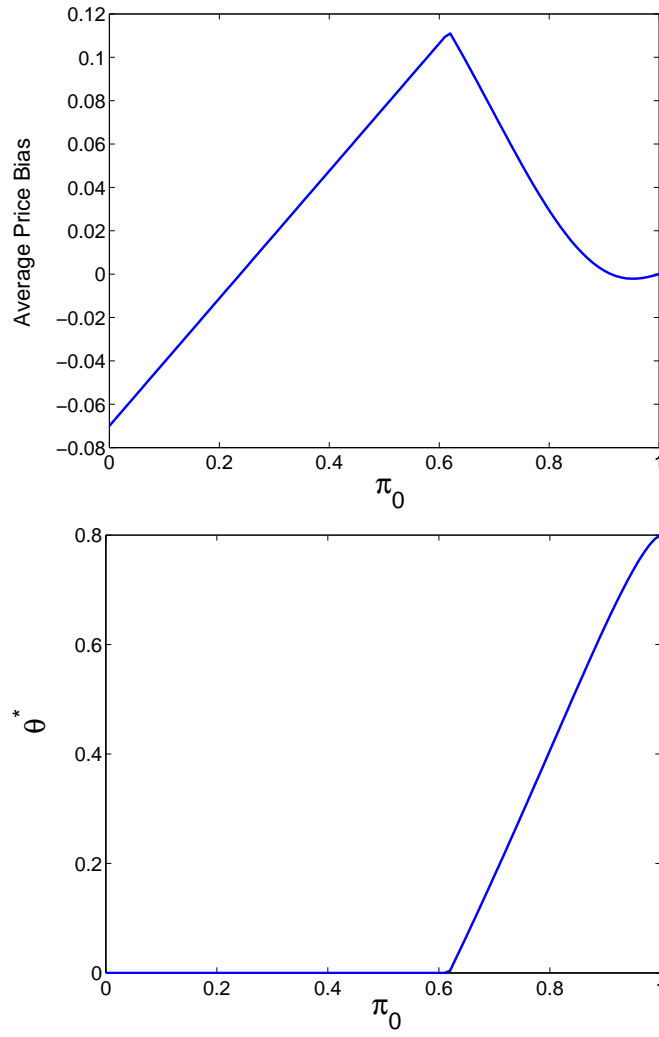


Figure 7: The effects of the fraction of tech-savvy advisors ( $\pi_0$ ) on the average price bias and tech-savvy advisors' threshold for inflating their messages ( $\theta^*$ ). The following parameters have been used in this plot:  $a = 0.8$ ;  $\rho = 0.3$ ;  $I = 0.8$ ;  $n = 4000$ ;  $c = 1$ .

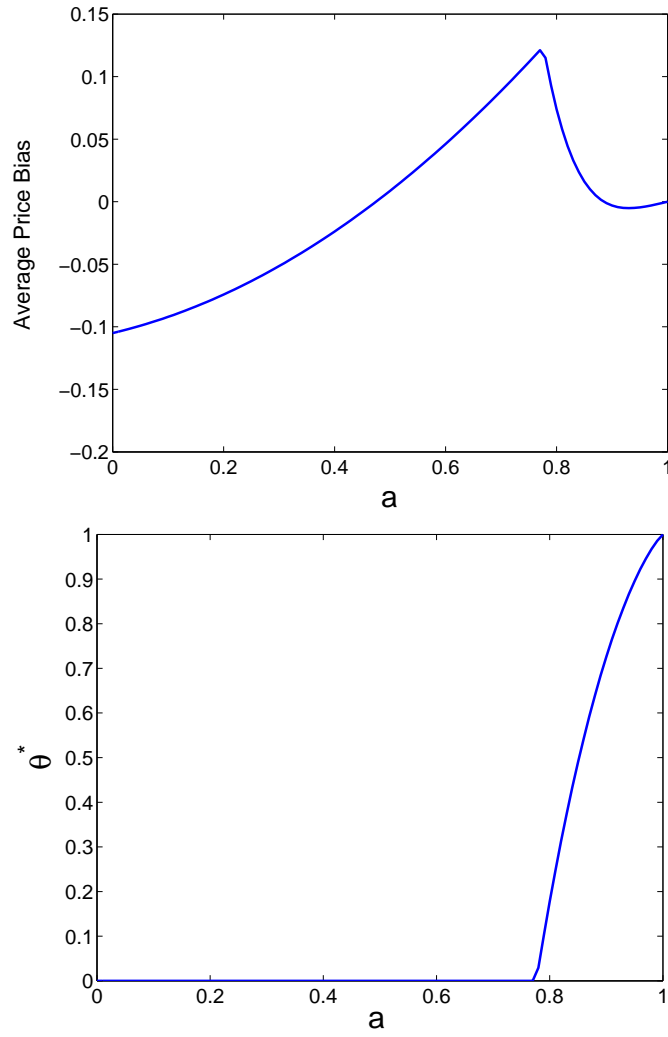


Figure 8: The effects of old-fogie advisors' bias ( $a$ ) on the average price bias and tech-savvy advisors' threshold for inflating their messages ( $\theta^*$ ). The following parameters have been used in this plot:  $\pi_0 = 0.7$ ;  $\rho = 0.3$ ;  $I = 0.8$ ;  $n = 4000$ ;  $c = 1$ .