SOCIAL SECURITY AND INSTITUTIONS FOR INTERGENERATIONAL, INTRAGENERATIONAL, AND INTERNATIONAL RISK-SHARING

BY

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Social security and institutions for intergenerational, intragenerational, and international risk-sharing*

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Abstract

Social security system old-age insurance systems are devices for the sharing of income risks of elderly people with others. Risks can be shared intergenerationally (with the young of the same country), intragenerationally (with other elderly of the same country), or internationally (with foreigners).

Barriers to individuals themselves sharing their risks intergenerationally, intragenerationally, or internationally are described. Optimal design of government-sponsored social security systems is considered in light of these barriers. Alternative benefits and contributions formulas for pay-as-you-go social security systems are defined and compared with existing and proposed formulas in terms of their ability to fulfill the government’s role in promoting risk-sharing. Benefits for each retired person may be tied to that person’s lifetime income without causing (as with the U.S. benefits formula today) aggregate benefits for all elderly today to be tied to their past aggregate income.

1 Introduction

The central function of social security old-age insurance is risk management, the management of the risk that some random turn of events will leave elderly people impoverished. The words “security” and “insurance” are there in the names of these programs to make this risk-management function clear. While

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social security also serves other important purposes, to force some people to save for retirement who might not have the intelligence or self-control to do so on their own, and to alter the wealth distribution of society, the risk-management function for the elderly is still the central defining function of the system.\footnote{Of course, social security reform can have important purposes beyond that of fulfilling the risk-management function better. Some authors (Feldstein, 1996; Feldstein and Samwick, 1997) have argued that there are striking benefits to encouraging saving as might be done by forcing people to invest their social security contributions, see also Mitchell and Moore (1997). Social security incentives may offset distortions already created by our tax system. Reforms of social security might reduce labor-market distortions resulting in a higher level of labor input to production, Kotlikoff (1996a).} Some of the other functions of social security old-age insurance can be handled by other programs, programs to encourage saving, general welfare programs, and progressive income taxes.

Risk management means, of course, risk-sharing. The function of insurance is to allow people to escape risks that are concentrated on them and share these risks over large numbers of people. Management of risks does not make the risks disappear, instead they are spread over many people so that their effects are less painful. A theory of social security old-age insurance therefore must be a theory of the sharing of risks, and we must have at the foundation of this theory an understanding of who takes on the risk formerly borne by the elderly, and how this spreading of risks is ultimately achieved. With such a theory, we will know whether any old-age insurance proposal is genuinely insurance, and not just something that sounds superficially like insurance. We must also know why we should consider the elderly as a group for social insurance, and we must understand why there is a legitimate role for the government in the providing of such insurance.

With risk management its central function, it would appear that the theoretical issues involved in social security design are fundamentally in the realm of theoretical finance. In this paper, I will try to adapt some lessons from theoretical finance to understanding issues of social security design. And yet the problem of designing social security systems raises issues that are fundamentally different from the risk-management issues that have dominated most of the literature on theoretical finance. The finance literature generally presumes that the fundamental theoretical problem is the individual's problem of optimizing a portfolio of investment assets. But, in designing social security, the fundamental problem is the government's problem of deciding on institutions on behalf of individuals, some of whom are currently capable of taking actions to manage their risks and some of whom are not. These institutions must promote the management of risks not only among people who are currently actively managing their own risks but also among people who are still minors or not even yet born, and also many adults today who are, because of limits on their financial management ability, or limits of

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existing financial institutions, not capable of fully managing their own risks.

The problem of optimal design of social security old-age insurance systems also differs from the characteristic problem of theoretical finance in that it need not take as given the individual wealth positions or the investment and hedging vehicles that are available. The nature of the problem is that the government can create any tax of wealth or create any investment-hedging institution. The problem of design of social security systems is thus more comprehensive, fundamental and open to major initiative than is the problem of individual optimal portfolio management. It is true that public discussion of reform of the social security system in the U.S. has often taken the form of discussing little more than whether the Social Security Trust Fund should invest in U.S. stocks as well as bonds, but the limited nature of this discussion is probably due to the limited ability of immediate national political debate to comprehend the variety of alternatives that are possible. If we assume that the focus of public debate in this country can be shifted eventually, or that the state of public debate in other countries may not face the constraints that we have here, then we will not want to take such a narrow perspective on the options for social security system design.

The assumption I will make is that social security old-age insurance systems are to be designed with a social-welfare function that gives weight to the risks of all people alive in the country today as well as to all subsequent generations. Individuals can be assumed to manage their own individual risks, with varying degrees of success and sophistication, and, also, with varying dates of birth and thus varying time horizons within which they may take actions to manage risks. Thus, the design of a social security system is inherently a problem of defining rules of the game so that limited individual risk management will promote social welfare.

There are three major groups with which elderly income risks can be shared: risks can be shared with the young people in the same country with other elderly people in the same country and with people in other countries. The existence of these three groups who might bear individual risk of the elderly motivate the title of this paper: there is the possibility of intergenerational, intragenerational, and international sharing of the risks of the elderly.

In the theoretical literature on social security in economics journals, the intergenerational risk-sharing has naturally garnered the most attention. If the elderly should have low income, then the young will step forward to help them. This form of risk-sharing is the most natural to think of, since it is a national embodiment of ancient family traditions and obligations. In the absence of strong and predictable family bonds, it is impossible for the elderly to procure such risk-sharing in free markets, since they cannot deal when they are young with the unborn (who will be young adults when they
are old) and so there is a fundamental role for the government in securing this risk-sharing.

Intragenerational risk-sharing, despite the relatively meager theoretical attention given to it, also appears to be a very important motivation for social security as we see it. Some elderly will by chance have higher income than others, and they can pool this risk among themselves. The intragenerational risk-sharing may seem to have a less secure footing in giving a role for government intervention, since people could in principle secure such risk-sharing for themselves by diversifying and hedging. In fact, however, there are various reasons why the government has a legitimate role in creating intragenerational risk-sharing. The government can take on management of risks that are revealed before the people are old enough to make risk management contracts, can promote risk-sharing for which institutions do not exist, and take on the management of risks for individuals who are not competent to manage their own affairs fully.

International risk-sharing also tends to be overlooked in discussions of social security. Individuals, in deciding on their own risk-management procedures, are operating in a global economy where they may share, using existing markets, risks with others alive at the same time, even in other countries. When governments in individual countries also create social-security systems that permit risk-sharing across generations in the country, then inherently there is an option of sharing risks between individuals in one country with unborn individuals (as well as minors and individuals incapable of managing their own affairs) of another country, even though there are no institutions to allow them to do this directly. They can share risks with unborn individuals in another country by trading with other people who are alive because of the intergenerational risk-management devices created by social security systems within individual countries.

In this paper, I will take a few sequential steps towards understanding better the intergenerational, intragenerational, and international risk-sharing functions that governments have in designing social security systems. Before doing this, I will give first a brief discussion of the nature of the unfunded pension liability as a “transition cost,” since the problem of this unfunded liability is often confused with the real risk-management issues. I will then present some simple models in which the government will be seen as setting the rules according to which individuals optimize, but at the same time doing this with concern for the broader social-welfare situation. One point that will emerge from this discussion is that the optimal social security system should define benefits in such a way that people are allowed to consume a share of national income available for generations alive at the same time. Actual and proposed social security systems usually do not do this national income-sharing at all right. For example, a proposal to have people invest their social
security contributions in individual accounts invested in the stock market, and to be entitled only to their portfolio outcomes, is just a nonstarter as a national risk-management institution for the elderly. It is not insurance at all. In the conclusion, I will consider what recommendations for reform of social security seem to emerge from this analysis, at least tentatively. I will suggest a benefits rule for a social security system that is ultimately a risk-sharing scheme, and then ask how various proposed social security reforms measure up in terms of this optimal risk management. I will discuss the proposals to the U.S. Congress of the 1994–1996 Advisory Council on Social Security from the standpoint of risk management, proposals that alter the risks to the social security system by investing in stocks, but proposals that did not stress how they will fulfill the insurance function better or to whom the risks are spread.

At various points in this paper I will make reference to a proposal I have made in my 1993 book *Macro Markets: Creating Institutions for Managing Society’s Largest Economic Risks*, that markets should be created to allow trading of claims in shares of national incomes or other important income aggregates. These markets would resemble stock markets, except that the shares would be shares in countries (or other aggregations) rather than companies, and the dividends paid would be shares of these incomes. The markets might also take the form of markets for government debt indexed to national income. The macro markets would be vastly bigger, in terms of underlying value available to trade, than the stock markets we have today. I make references to these macro markets in this paper since these markets would be fundamentally involved with the very risk-management problem that is central to the social security system design problem, and because the potential of these markets for improved risk management is great. The creation of the international macro markets might even solve most of the risk-management problem that social security is designed to deal with, and produce a greater welfare improvement than would the optimum social security system without these markets. However, I recognize that while there is an impetus now for social security reform, these macro markets are still rather far from acceptance, and so I will try to bracket these discussions. The analysis of social security reform offered here is intended to offer insights into the issues that we face today.

2 A preliminary on investment opportunities and unfunded pension liabilities

Much of the recent discussion of replacing a pay-as-you-go social security system with a fully or partially funded one, where the contributors’ social security contributions are invested in high-yielding investments such as stocks,
is motivated by the concern that social security participants are being cheated out of the possibility of earning higher returns on their contributions. The participants may look regretfully at how much they would have earned if they had invested their social security contributions in the stock market, and wonder why the government should be constraining them to make what appears to be a very bad investment. It is important for us first to consider this point of view, since it seems to involve some confusion of the important issues of risk management that are before us.

The most obvious point to make is that the returns in the stock market observed recently should not be assumed to continue indefinitely. The regret felt by social security contributors may be particularly poignant at this point of history, where the U.S. stock market has been climbing almost continually since 1982, and has made stunning gains in the past few years. But we should base our analysis not on the extraordinary returns earned by the stock market in recent years, but instead by the normal expected return that it earns.²

A point that is less obvious to the general public is that there is another cause to the low returns that is conceptually distinct from any issues of optimal design of social risk-management institutions. This cause is the burden of the unfunded pension liability.

A substantial reason that the return on their social security contributions is so low relative to other expected returns available is that the participants are not being allowed to invest their contributions in stocks, bonds, or anything else. They are not being allowed to do this because, in effect, they are paying for the gift that was made to the first generation of social security participants, who received much more in benefits than they put in. That there was such a gift in the United States is plain, since the Social Security Trust Fund today, after sixty years of receiving contributions from participants, amounts to only a little over one year’s contributions, essentially the rest having been disbursed, in the pay-as-you-go operations, to beneficiaries.³ The social security system has obligations to pay future retirees for every year that they live in retirement, but has the funds to pay for only about one such year.

Why was such a large gift made to the first generation to benefit from social security? In the United States, the initial designers of the social security system in 1935 had envisioned the building of a large trust fund. The 1939 amendments and subsequent changes prevented this from happening.

²At this point of history in the United States, we should probably assume that the expected return on the U.S. stock market is in fact lower than the historical average return, see Campbell and Shiller (1998). Siegel (1998) also argues that with higher market valuations today we cannot expect continuation of these earlier returns.

³The U.S. Social Security Trust Fund in 1996 contained $424 billion; the 1997 income was $447 billion, 1997 outlays were $395 billion.
While there was recognition at the time that a gift was being made to early beneficiaries, there was confusion on whether this gift would represent a burden to subsequent generations, see Miron and Weil (1998). Optimism about economic growth would mean that the initial burden may be later swamped by higher standards of living. Perhaps the initial gift in the U.S. was viewed as compensation for the suffering imposed by the Great Depression of the 1930s. However, similar gifts to first generations have been made in many other countries at different times, and countries with less exposure to the Great Depression, see World Bank (1994). In some countries (e.g., Ecuador) the gift was made gradually, as the coverage of the system across the population was broadened in steps. An explanation that so many countries have made gifts to early generations may lie instead in political realities; voters may see the immediate benefits without understanding the ultimate costs. Unfunded pension liabilities may have helped boost government’s perceived value to the electorate just as unfunded pension liabilities have in the past helped boost individual companies’ values by disguising the nature of their debt from the market. It may take a long time before the public thoroughly learns to see through such accounting tricks.

That gift made already has still to be paid for by someone (see Stiglitz, 1983 for an elaboration of this discussion). There is no escaping that, and as presently constituted the largely pay-as-you-go social security system has each subsequent generation paying in the form of lower returns on their social security contributions than they could get if they invested them on their own behalf.

Let us consider a simple two-period overlapping generations model in which there is no population growth and no economic growth, a model that will be used throughout this paper for its simplicity. In this simple model, there are always two generations alive at any given time, a young working generation and an old retired generation. There are always the same number of people in each generation, and we will stylize the social security system so that it has no trust fund at all, is strictly pay-as-you-go. Time is measured in units of generations. When the social security system is first set up, the first generation receives benefits $B$ as a gift, not having made any contributions, the gift equal to the contribution $B$ of the then young, which are just transferred to them immediately (pay-as-you-go). One generation later, the formerly young, now old, are paid back $B$, by a transfer of the contributions from the new young, in an amount equal to their contributions, and so on with each subsequent generation. Since each generation pays $B$ when they are young and receives $B$ back again a generation later, they are in effect earning zero interest on their contribution. If the interest rate (or return on a portfolio of investments) measured in time units of a generation is $r$, then each subsequent generation loses, in effect, the interest $rB$ they would have
earned on their contribution had they invested it at interest. The present value of the infinite stream of costs \( rB, rB, rB, \ldots \) discounted at interest rate \( r \) is, of course, just \( B \). Under this pay-as-you-go security system, every generation after the first is burdened by the obligation in amount \( B \), the unfunded pension liability, and each generation pays the interest on this and passes the obligation on to the next generation, so that the obligation is never changed.

The nation faces the fundamental problem that we cannot just switch to a fully funded social security system, allowing participants to invest their money in stocks or anything else, without finding a way to come up with the money, \( B \), to pay existing social security obligations to retired persons. It is remarkable that this simple and fundamental point is overlooked so much in public discourse about social security. Many commentators with political agendas seem to have an impulse to ignore this problem, even though the key motivating factor for their discussions appears to be really just the burden \( B \), magnified at present because the baby boom puts the burden even more heavily on the young today than the static-population model above implies.

Given that the pay-as-you-go social security system represents a sort of debt to older people, one might take the view that there is nothing to be done about it. We should just continue to follow such a system, though some of its characteristics could be reformed. If a consequence of this system is that people are not saving enough, the system can always be augmented with savings incentives. We can also hope that the original designers of the system were right that economic growth will continue to make the initial burden less severe.

This problem of the inherited unfunded pension liability \( B \) does sometimes appear in public discussions, and then it is typically referred to as a "transition cost" to moving to a fully-funded system. As such, it is misnamed. It is not a cost of transition, not a cost of changing the system. The existing pension obligations are merely a form of national debt. The burden \( B \), for the U.S. Old Age, Survivors and Disability Insurance (OASDI) system, has been estimated to amount to about 9 trillion dollars, although its exact value is ambiguous since the Primary Insurance Amount (PIA) formula does not clearly specify what would be owed on the basis of past contributions if the social security system were changed from this date forward (see Geanakoplos, Mitchell, and Zeldes, 1998).

An alternative to the present system that seems often to be suggested is that there should be a transition period towards a fully-funded system, a period during which either the current old are receiving less than their contracted benefits, to allow the current young to invest some of their contribution, or a period during which the current young are paying more than the usual amount, so that they can both support the current old and also make investments for themselves, or some combination of these two. This
would mean that the cost would be borne disproportionately by social security participants, apportioned according to their social security benefits or incomes, who are alive during the transition period.

There appears to be no reason why the transition cost should be visited disproportionately in this way on the social security benefits or incomes of social security participants who are alive at certain times. The costs of the policy of granting a gift to the first generation to receive benefits without contributing could be borne across all income levels in a more equitable manner, by transferring the obligation to pay existing social security obligations officially to the national debt, the federal government assuming the social security system obligations to be paid from general tax revenues, and allowing subsequent generations or social security participants to invest their contributions and enjoy the proceeds. Unfortunately, such a solution is politically difficult, since it would require acknowledging that our national debt is (when existing social security liabilities are included) well over twice what is currently defined as debt.

The Kotlikoff-Sachs plan (1997b) is the most forthright in addressing the solution by proposing that the existing unfunded pension liability should be paid out of national taxes (originally, they proposed a national sales tax, but then modified this to a sort of value-added tax). The Moynihan plan goes part way towards such a solution, by allowing part of the social security contributions to be diverted to a personal savings account, and making up part of the lost social security revenues to pay benefits by raising the amount of taxes that are subject to the payroll tax, thereby causing the unfunded liability to be shared a bit more equitably across income groups in our society (Pear, 1998, p. A26). But his plan, constrained as it is by his view of current political reality, does not go very far with this, initially allocating only 1 percentage point of the 12.4 percentage points on payroll income now levied for social security contributions.

While the U.S. 1994–1996 Advisory Council on Social Security, all three factions, recommended to Congress "partial advance funding for social security," two of the three factions proposed what appear to be only small changes in the advance funding of the system. The "Maintenance of Benefits" (MB) faction, while not being precise about how much advance funding should be changed, describe their changes as modest. The "Individual Accounts" faction proposed that only 1.6% of covered payroll would be allocated to be invested in the proposed individual accounts, the rest being used to continue the present essentially pay-as-you-go system. Currently, the social security system collects 12.4% of the covered payroll. Only the "Personal Security Account" faction proposed as much as a halfway transition, with 5%, rather than just 1.6%, of covered payroll going to the accounts, and, acknowledging the problem of paying existing obligations, with the existing obligations
borne by current young by the levying of a temporary 1.52% extra payroll tax over the next 72 years.\textsuperscript{4} The question of how the burden of the gift to the first generation should be borne by society is an important one but not one that will be the subject of the discussion that follows in this paper. The question that remains is how the pay-as-you-go system can be reformed to improve risk-sharing, to which we now turn.

3 Intergenerational risk-sharing

Many economists considering the theory of social security systems have stressed the issue of purely intergenerational risk-sharing, excluding international risk-sharing: Fischer (1983), Merton (1983), Gale (1990), Diamond (1997), Demange and Laroque (1997a,b), Bohn (1998a,b). This intergenerational risk-management case is the obvious simple case to consider, as there is in this theory only one government and only one social-welfare function to consider. To set forth the risk-management issues, I will take the unconventional step for the overlapping generations model of assuming that each generation’s felicity (one-period utility) is of the mean-variance form. The mean-variance utility function is very well-known in financial discussions because of the simplicity of its implications for optimal portfolios, and I want to use it here to facilitate the discussion.\textsuperscript{5} As I have set it up here for simplicity, there is no concavity for riskless intergenerational transfers, i.e., there is no preference for riskless consumption-smoothing through time. The model will be used only for consideration of risk management.

As in the preceding section, the two-period overlapping generations model has no population growth (so that all generations have the same number of people), no technical progress, and the intertemporal discount rate is zero. Moreover, I will assume initially that all individuals who are alive will behave in a rational manner, that the only barrier to optimization is not being born. The utility function of individual \( j \), is given by:

\[
U_j = u_{yj} + u_{oj}
\]  

(1)

where \( u \) denotes felicity, \( y \) denotes young, and \( o \) denotes old. The mean-variance felicity function, the component of the utility function for each gen-

\textsuperscript{4}They project that the extra 1.52% payroll tax would still result in a shortfall of meeting existing obligations between 2000 and 2034 and excess revenues thereafter, and that “the shortfall would be met by issuing bonds to the public for the next 40 years (totaling an estimated 1.9 trillion in 2034, in 1995 dollars) and that these bonds would be fully repaid by the excess of tax revenues in the later period,” Advisory Council, p. 21, http://www.ssa.gov/policy/adcouncil/findings.htm.

\textsuperscript{5}When overlapping generations models are log-linearized, as in Bohn (1998b), then an analogous simplification is also being made. Geanakoplos and Shubik (1990) provided a model similar to the one here.
eration, is:
\[ u_{ij} = \bar{c}_{ij} - \frac{\gamma_j}{2} \text{var}(c_{ij}), \quad i = y, o. \] (2)

where \( c_{ij} \) is consumption of the \( j \)th individual of age \( i \), \( \gamma_j \) is the risk aversion parameter for the individual, a bar over a variable is its expectation, and var denotes variance.

3.1 *Management of exogenous income risks*

Let us first assume, as the simplest first case, that the \( j \)th individual earns exogenously determined random income \( y_{ij} \) when young, and \( y_{ij} \) when old. By exogenous, I mean that the model will not represent any decisions that the individual can make that might change these incomes. Both of these incomes are initially uncertain, from the standpoint of time \( t = -\infty \), and as time passes, with each succeeding generation, the uncertainty about the income of any particular generation is gradually resolved, so that eventually each generation knows its income in the time period in which it is earned.\(^6\)

Let us also assume that all people of the same age have identical risk aversions and incomes, depending only on their age, so that we can drop the \( j \) subscript for now. Without government intervention each generation consumes its own income, \( c_y = y_y \) and \( c_o = y_o \). Without government help there can be no risk-sharing because it is not possible to trade with the young before they are born, and by the time they are born all uncertainty about the period’s income has been resolved. This fact creates a role for government intervention in the markets. There is of course some potential role for partial risk-sharing between generations without government intervention, if for example we assumed that some of the risks about income when old were not revealed until after the young adults can make risk-managing deals. But such opportunities for intergenerational risk-sharing will certainly be limited, and especially since macro markets directly designed for such risk-sharing do not yet exist.

The optimal strategy for the government for risk management, from the viewpoint of \( t = -\infty \), is to pool the incomes of the young and old and divide the incomes according to the coefficients of risk aversion. The government chooses the fraction \( \alpha \) of income uncertainty to be borne by the young to minimize:

\[ W = \frac{\gamma_y}{2} \alpha^2 + \frac{\gamma_o}{2} (1 - \alpha)^2 \] (3)

The optimal \( \alpha \) is given by:

\[ \alpha = \frac{\gamma_o}{\gamma_y + \gamma_o} \] (4)

\(^6\)Very roughly, we might consider uncertainty about time of death as a sort of income shock. More satisfactory discussions of this uncertainty are in Storesletten et al. (1998) and in Demange and Laroque (1997a,b).
Achieving this pooling of incomes means transferring \((1 - \alpha)\) of the young’s income to the old, and \(\alpha\) times the old’s income to the young, and possibly transferring a fixed real amount between the generations also, not determined in this model. After these transfers are made, all individuals’ incomes (and consumptions) will be perfectly correlated with national income, and thus with each other’s income and consumption, in accordance with the optimality conditions shown in Breedon (1979). Of course, in practice, we would not expect to see any social-insurance system to go to the extreme of such complete pooling of incomes.

Part of the difficulty faced by designers of government social security in managing intergenerational risk-sharing is that strategies like the above to further risk management will also tend to carry with them effects on the preexisting distribution of income. The government has no sure way of knowing which people face more uncertainty about income, and which individuals face higher expected value of income. People who might know that their income has a high expected value and low variance may have lowered expected utility, conditional on this knowledge, relative to others, by an income-pooling scheme like that described above. It is not possible to create a social-insurance system that does not have such effects on the preexisting distribution of income since the government does not have the knowledge to prevent such effects. A government-imposed risk-sharing system is necessarily a blunt tool to deal with individual risks.

Note that this system of transfers does not need to entail any expected creation of an unfunded pension liability, and there are no “transition costs” to adopting it. If, however, adoption of the system of transfers is delayed until some of the information about the original income uncertainty is resolved, then the transfer will no longer be a pure risk-management device that all parties would rationally agree to, but would be in part a confiscation of existing wealth.

The personal income-tax system, the corporate-profits tax system, and social safety net that are already in place provide some of this intergenerational risk-sharing, as well as intragenerational risk-sharing. If the investments do well for the old, then there will be more tax revenue, from both the personal income taxes and the corporate profits taxes, which will enable the government to allow the young to enjoy greater services or lower taxes. If the investments turn out badly, then both the young and old will suffer. The suffering of the old will be mitigated via the lower taxes and social safety net, while the young will suffer from lower government tax receipts creating either higher taxes for the young or lower government services.

One might say that part of the reason that we base individual taxes on income, rather than impose a lump-sum tax on everyone, is for just such risk-management purposes, not only intergenerationally, but also intragener-
ationally. Thinking ahead to those who are not yet born, the tax and social safety net system that we have may be regarded as a risk-management device against the risk of being born with little ability or in a socially disadvantaged position. While we are accustomed to describing progressive income tax systems as motivated by a sense of justice in society, this sense of justice may be given a Rawlsian interpretation, as here, and a system of justice becomes a system of risk management (Harsanyi (1953) and Rawls (1971)).

The amount of redistribution, here determined by $\alpha$, would perhaps be chosen differently for intergenerational risk management than for intragenerational risk management for the young. One reason why we may need an old-age security system, and not just an income tax and social safety net system that treats the elderly no differently than the young, is that we may feel that risk aversion is higher for the elderly. This higher risk aversion of the elderly may be a result of different circumstances of the elderly rather than different tastes; for example, the very elderly will have much greater difficulty living as street people or beggars. There is also some experimental evidence indicating that the elderly are more risk averse, see Barsky et al. (1997). Rather than model such circumstances formally, we merely represent their circumstances as a different risk aversion. If we make such a representation of their utility, we will wish to provide more income security to the retired people than we do to society at large, and this then is a justification for old-age insurance as apart from general social insurance. The same notion that utility is effectively age-related could also be used to justify other proposals for age-related transfers for remedying income inequality, such as the proposed government transfers to the very young of Haveman (1988) and Ackerman and Alstott (1999), motivated by the observation that very young people have important opportunities that could give them a head start in life.

Unfortunately, from the standpoint of this model, the social security system that we have in the United States does not handle this risk management even approximately right. The real benefits that are paid to retired persons are indexed to their own lifetime income before they were retired, the PIA formula basing their benefit on the average value of 35 years of their social security income adjusted to the present by a wage index, and the real buying power of this income is preserved over the span of their retirement by indexation to the consumer price index. From a risk-management viewpoint, the benefits are tied to the wrong generation's income.\(^7\)

\(^7\)Since people are not born only once a generation as in the overlapping generations model, but are born fairly evenly over the years, the wage index used at time of retirement causes benefits to respond somewhat to younger generations' income, but the wage index averages over young and old workers' incomes and no further adjustments tied to wage indexes are made after retirement.
3.2 Choice of risky versus riskless investments

The analysis above led us to a simple pay-as-you-go social security system that differs essentially from the one we have in the United States today only in the benefits payout formula. But the model that led to that conclusion assumed away the choice of physical investments. Since much of the discussion of design of social security systems presumes that there is underinvestment in risky assets like shares in corporations, let us now turn to a simple model that stresses this choice.

Suppose there is a single risky real investment opportunity (stocks, say) that each generation can purchase when young, and that any quantity of this investment can be purchased. There is also a riskless investment (stored commodities, say) with a lower expected return. The two investment choices represent different investments for the nation in physical capital. As a caricature, we might say that investing in risky high-technology stocks might mean investing in companies that manufacture computers and software, while investing in stored commodities might mean placing barrels of nitrogen-packed dried food in the basement. (Some people do this. See, for example, Secure Future Food and Supplies, http://www.securefuture.com/.)

In this model, each generation faces a choice between risky and riskless investments to provide income when they are retired. They can make the risky investment in as much quantity as they want, and every real dollar invested in the risky project, at the expense of an investment in the riskless project, yields an expected increment in income next period (when they are old and retired) \( \mu \), where \( \mu \) is an excess income, relative to the expected increment in retirement income they could obtain from investing the same dollar in the low-yielding riskless investment. Call \( \sigma \) the standard deviation of the income, per dollar invested, in the risky investment project. We assume that the investment only has an impact for the one generation, so that there are no further repercussions of such an investment. At this point, let us disregard differences across individuals in risk aversion, and drop subscripts from \( \gamma \).

In the absence of any government intervention, and assuming that the income from the investment is uncorrelated with the other income that people earn, then each individual in each young generation will maximize:

\[
U = x\mu - \frac{\gamma x^2}{2} \sigma^2
\]

with respect to the amount invested in the risky investment \( x \). The optimal investment is then:

\[
x = \frac{\mu}{\gamma \sigma^2}.
\]

This is the individual optimum, but it is clearly not the social optimum since it fails to take into account the potential risk-sharing with the young.
If a benevolent social-planner government were to take charge of the investment and allocation process, the planner would then optimally make the investment with consideration of the entire income, of young and old combined, and then, since the utility function is symmetric between the two generations, split the income equally between the two generations. The utility of an individual in either generation, assuming the equal split, would then be:

$$U = \frac{x\mu}{2} - \frac{\gamma x^2}{8}\sigma^2. \tag{7}$$

If the government chooses $x$ to maximize this utility, then the optimal $x$ is:

$$x = \frac{2\mu}{\gamma \sigma^2}. \tag{8}$$

The government would make $x$ twice as large as would have occurred if the decision had been left to the market. The utility of both generations would be increased as a result of the government intervention. Only the government can achieve this, and individuals cannot do this on their own, since the risk-sharing contract between young and old cannot be created privately. The next generation’s young are not born yet or are not yet of age to sign such a risk-sharing contract when this generation’s young must make the investment. We have what appears to be an air-tight case for government intervention to increase the amount of investment in our society.

If the government forces people to contribute to the social security system and invests the proceeds in a trust fund of risky stocks it would be, to some extent, taking command of the $x$ described above, and pushing it to a higher level than it would have had if people were left to their own devices. (I say only to some extent, since people who already own more in stocks than the government invests could offset the effect of the government and invest less in stocks on their own, as Bohn (1998a) stresses, but in practice this effect is likely to be limited as most people own little stocks. Gustman and Steinmeier (1998) find that pension wealth does not effectively displace other saving.) Thus, this simple model appears to justify the government’s building up a larger social security trust fund invested in stocks.

In this simple model, however, the government does not need to take command of $x$. All it need do to achieve the same effect is put in place a pay-as-you-go social security system that subtracts from the social security benefit paid from the young to the old half of the excess income, above income from the riskless investment, from the risky investments. Any income in excess of the amount they would earn from only riskless income would be taxed at 50% (under the assumption we have made that all people have the same risk aversion and there are the same number of persons in each generation), and any income short of the amount they would earn from only
the riskless investment would be made up at 50%. Since the model assumes that all living people optimize, then since people know when young that such a social security plan is in place, they will, in deciding the amount \( x \) to invest when they are young, face exactly the same maximization problem that the government faced just above, and so they will choose the same \( x \), and the outcome will be the same as if the government had taken command of the investment process. In effect, all that this model suggests need be done is to put in place an appropriately means-tested pay-as-you-go risk-sharing social security system.\(^8\) (I shall mention some cautions about this conclusion in the penultimate section of this paper.)

### 3.3 Allowing for correlation with other income

An important factor that was assumed away in the above simple analysis is a possible correlation \( \rho \) between the uncertain intergenerational risky investment outcome and the income \( y \) of the new young generation at the time that the investment income is realized. For a social planner who will choose the optimal investment and share the proceeds equally between the two generations, the increment to total utility from investing \( x \) in the risky investment is:

\[
U = \frac{x \mu}{2} - \frac{\gamma}{8} (x^2 \sigma^2 + 2x \rho \sigma_y)
\]

where \( \sigma_y \) is the standard deviation of the income of the younger generation at the time the risky investment pays out. If the government chooses \( x \) to maximize this utility, then the optimal \( x \) is:

\[
x = \frac{2 \mu - \gamma \rho \sigma \sigma_y}{\gamma \sigma^2}.
\]

Note that if \( \rho \) is positive, the socially optimal investment will be less than was derived above. Moreover, it is even possible that the socially optimal investment could be less than the amount that would be produced without any intergenerational risk-sharing. Thus, consideration of intergenerational risk-sharing could mean that we are investing too much, not too little, in risky investments. One is tempted to say, though, that the assumption that \( \rho \) is not too large seems reasonable, and so that the theory still suggests that the level of risky investments that people will undertake without governmental incentives is below the social optimum. Unfortunately, to decide

\(^8\)The existing income-tax system would have similar incentive effects for investment, by allowing risk-sharing. The personal and corporate income-tax systems tend to have negative incentive effects on the level of saving, and the corporate income-tax system has a negative incentive effect on corporate versus noncorporate investment. Presumably these effects are of little relevance from the standpoint of a mandatory social security system. The incentive effect of these taxes on high-expected-value high-risk corporate investments versus alternative corporate investments is still to encourage the risky investments.
how much the government should invest in the risky investment (or to derive
the optimal risk-sharing arrangement that encourages the private sector to
do so), we need to know the relevant correlations and standard deviations.
These are not so easily estimated. The relevant correlation $\rho$ is the correla-
tion, over long, intergenerational, time intervals, between returns on the next
new major risky investments that the country must make and the remaining
components of national income. Presumably, these new major risky invest-
ments are of a kind that this country has not made before, since incentives
to push investments to higher levels have not been there before. There is no
guarantee that we have any way to estimate this correlation with historical
data.

Fortunately, the government does not need to estimate the parameters in
the above expression if it wants to provide market incentives for the optimal
risk-sharing arrangement. If all individuals alive are behaving rationally,
then all that the government needs to do is to split the risk between the
generations, by deciding that the two generations alive at one time will share
the national income (equally in the stylized model).

Of course, there are changes in the model that, if made, would destroy this
result. For example, we could suppose that the risk-aversion parameter for
the old is different than for the young, or that the utilities are not additively
separable between generations. Any justification for government involvement
in making investment decisions would have to hinge on such properties of the
model, and in this sense once the government's risk-sharing initiatives are
fulfilled, any justification for government investing in high-risk high-return
assets for social security is not so obvious as some advocates suggest.

4 Intrigenerational risk-sharing

Intragenerational risk-sharing is the pooling of risks among individuals in a
single generation. Social security systems have important functions in such
intragenerational pooling. If a retired individual turns out, for reasons of bad
investments or bad health or the like, to have a low income in retirement, the
system can help, even without any sharing of income risks with people outside
the generation of retired persons. Two important issues must be discussed
about such intragenerational risk-pooling: the issue of moral hazard and the
issue of prior income inequality.

The moral-hazard issue arises when we make benefits relate to individu-


\[^{9}\text{Moral hazard did not arise in the previous section, where we assumed that all people in a generation are identical and where the transfers made could be based on aggregates, rather than individual incomes. In practice, of course, intergenerational risk-sharing is not likely to be done entirely in terms of generation aggregates.}\]
will have no incentive to work. Even with partial insurance, there will be some drops in labor-force participation, some shifts to the informal sector, and some tendency for early retirement. These consequences of public risk-management systems such as social security systems have proven to be very important in many countries, see James (1997), World Bank (1994). We shall see that there is a justification for a special social security program pooling risks among the old, and this will not optimally be just a social-welfare system that disregards age if moral hazard differs between young and old, or if risk aversions differ between young and old, or if variances of income differ between young and old.

Let us consider a very simple model that illustrates risk management with moral hazard. Suppose that individual $j$ in this generation has an income variance, conditional on information available before birth, equal to $\sigma_i^2$, where $\sigma_i$ is the standard deviation of income in the $i$th period of life, $i = y$ when young and $i = o$ when old, and that each individuals income is uncorrelated with all other individuals’ incomes. Suppose that there are many people, and so there is virtually no aggregate income variance (let us say for this example that there is none at all). Incomes are unrelated across ages, young and old, and so we can consider the problem separately for the two ages. The government must choose how much of income to pool. It selects a parameter $\lambda_i$ between zero and one, pools the fraction $\lambda_i$ of all incomes at a time among people in this generation, and shares this pooled income equally among all individuals. The after-pooling income variance is $(1 - \lambda_i)^2 \sigma_i^2$.

Suppose now that there is an efficiency loss $\alpha_i \lambda_i$ caused by the moral hazard where $\alpha_i$ is a parameter showing how much people in that age group are vulnerable to moral hazard. Then, substituting into the above utility function for an individual, the increment to utility related to governmental risk-sharing for an individual is:

$$U_i = -\alpha_i \lambda_i - \frac{\gamma_i}{2} (1 - \lambda_i)^2 \sigma_i^2, 0 < \lambda_i < 1, \ i = y, o$$  \hspace{1cm} (11)

and the optimal fraction of income to share is found to be:

$$\lambda_i = 1 - \frac{\alpha_i}{\gamma_i \sigma_i^2}, \ i = y, o$$  \hspace{1cm} (12)

The $\lambda_o$ may be higher than the $\lambda_y$ if the $\gamma_o$ is higher, since the old are more risk averse, if the $\alpha_o$ is lower, the old have less opportunities to shirk since they are less likely to be working, and if $\sigma_i$ is higher, they may be more dependent on risky investments like stocks, and their labor income is more uncertain because of uncertain health and ability to work. Any of these differences would justify a higher $\lambda$ for the old, and this would make a justification for a social security system that is different from the income-tax-social-safety-net system that is not tied to age.
Prior income inequality is another issue that becomes difficult when thinking about intragenerational risk management for social security. The above model would imply that the social security benefit received when old has nothing to do with income when young. Indeed, in practice, our social security systems are part of the mechanisms that reduce intragenerational income inequality, and this model would appear to recommend, if $\alpha_e = 0$, carrying this leveling of retirement incomes to completion. This elimination of income inequality could be given a risk-management interpretation, and a model of optimal risk management might lead us to the conclusion that prior income inequality should be eliminated by social security. But, in fact, the social security system has no mandate to eliminate such inequality.

We will have to assume that the extent to which the social security system redistributes income between existing rich and poor is given exogenously. In practice, this means that the benefits paid on social security must be related in part to income while young. In the formula given in the conclusion below, benefits therefore depend on income when young, as does the present system, although the dependence will be of such a form that it makes it possible for people whose income is at a certain level relative to the average to share old-income risk among themselves and with others at the same level, and does not guarantee to anyone a repeat of their young income in old age.

A potential problem with the intragenerational risk-sharing justification for a program that treats the elderly differently from the young is that it might appear that individuals can do the risk-sharing for their old age themselves without any government help. To the extent that elderly people are living off investments, then they can completely pool their risks just by all investing in a fully diversified portfolio.

In fact, there are many practical limitations to the ability of the elderly to diversify their income by themselves, and so indeed, there does remain a role for the government in providing intragenerational risk-sharing specially designed for the elderly. First of all, the risks may become realized before they are old, and before they are even capable of making risk-management arrangements for themselves. The problem that one cannot make contracts when one is unborn can arise here too, just as it did for intergenerational risk management. Second, many people of retirement age still work, and suffer great variability in their retirement income as their health and abilities become more uncertain. Private insurance for general individual income risk does not exist; presumably because of the selection bias problem, the bad risks tend to come to the companies that might offer it. Disability insurance does function on a private basis, since the insurance covers only income losses from verifiable causes, and since medical exams can be used to exclude certain bad risks. Even so, selection problems inhibit insurance and cause their premiums to be high enough that few people buy it on their own, certainly
few of the elderly. Third, many people just do not have the foresight to diversify their portfolios wisely or take out disability insurance policies.

5 International risk-sharing

Even though there is not a global government that would share the risks of one generation of the world with another, the basic social security risk management problem is still an international one. Indeed, corporate investments on which some investment of trust funds is based are inherently, and increasingly, international. Moreover, if the public is someday able to trade in claims on national incomes with macro markets, as I have argued they should, then people will naturally be hedging their own national income risks and investing in other national incomes.

To gain some sense of the potential importance of international risk-sharing, I have done some calculations that may give some indication of the relative magnitude of the intergenerational variance when compared with the international variance of national incomes (as proxied for by gross domestic products). I used the Penn World Table data on populations (POP) and real per capita gross domestic products (RGDPCH), from the National Bureau of Economic Research web site, for the 51 countries for which there was complete data, data for every year from 1950 to 1992. The combined gross domestic products of these 51 countries averaged, over the 43 years, 78% of the total gross domestic product (GDP) of all 151 countries. Most of the countries for which data were incomplete were small countries. For the 51 countries, the average moving standard deviation of the 10-year percentage change in real per capita GDP was 18.0%, and the 20-year percentage change in real per capita GDP was 25.2%. These numbers give some clue as to the variability of intergenerational income within a country. In contrast, the standard deviation of the 10-year change in world per capita real GDP (taking these 51 countries as the world) is only 9.5%, and the standard deviation of the 20-year change in world per capita GDP (again for these 51 countries) is 15.6%.10 From these numbers, it appears that the standard deviation of the world change in real per capita GDP over long time intervals is about half that of the within-country change, and so the variance would be about a quarter as large. This would suggest that perhaps most of the utility gain that might be achieved by intergenerational risk-sharing could be achieved

10 All these estimated standard deviations are probably biased downwards as measures of the actual standard deviation, since in computing the variance the denominator was the usual one, the number of observations years (33 or 23) minus one, while, because of the overlap of the time intervals, there were less than these numbers of independent observations. The bias should not affect the comparisons between country and world standard deviations.
instead by international risk-sharing.

While these numbers are suggestive, it has to be admitted that the data are not available to get precise estimates of the extent of within-country intergenerational or world intergenerational income variance. At the very least, these data are consistent with the plausible notion that international risk-sharing is as important as the within-country intergenerational risk-sharing.

5.1 An international overlapping generations model

What should the government do to promote optimal risk-sharing internationally? Let us consider an overlapping generations model of international diversification of risks, a model that is in its essential elements a variation on the capital asset pricing model (CAPM) in finance, but where the highlighted problem is the government’s role in dealing with the problem that certain agents, the young, are incapable of choosing portfolios before they are born or mature. We will see that, in one model, if we can assume that each generation will hedge optimally on international markets when young its income when old, then all the government need to do is create a system to make transfers between the two generations of their own country based on their after-international-hedging incomes.

Let us adopt the two-period overlapping generations model used above for J countries, with all individuals identical within each country. For this example, I will concentrate here on uncertainty about income when old, and assume that random income is independent across generations. The model will set the uncertainty about income when young to zero (this simplification is not essential to the basic results). I will also assume initially that all uncertainty, represented by the J × J variance matrix of old income conditional on information when young Σₒ, is completely resolved for everyone between the time when they are young and the time when old.

We suppose that individuals in country j are able effectively to sell off their income when old for a price, p_j. Call the J × 1 column vector of shares of an individual in country j in per capita income of each other country q_j, and call yᵢⱼ income while young and yₒᵢⱼ income while old, before individuals’ hedging, and yₒ the J element column vector whose jth element is yₒᵢⱼ. Then felicity when old of an individual in country j from the standpoint of their youth is given by:

\[ uₒᵢⱼ = \bar{y}_ₒᵢⱼ + q'_j (\bar{y}_ₒ - p) - \frac{γ_j}{2} (Σₒᵢⱼ + 2q_j Σₒ,j + q'_j Σₒq_j) \]  

(13)

Each individual maximizes this utility while young, purchasing risk-management contracts (shares) by dealing with the young of other countries, and so differentiates this expression with respect to q_j and sets the result to zero.

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We find that the demand is given by:

\[ q_j = \Sigma_o^{-1}(\bar{y}_o - p) / \gamma_j + e_j \]  

(14)

where \( e_j \) is an elementary vector, a \( J \)-element column vector which is zero except in the \( j \)th place, where it is 1. Assembling these vectors into a \( J \times J \) matrix \( Q \), we have:

\[ Q = \Sigma_o^{-1}(\bar{y}_o - p)\iota\Gamma^{-1} - I \]  

(15)

where \( \iota \) is a \( J \) element column vector of ones and \( \Gamma \) is a \( J \times J \) diagonal matrix with \( \gamma_j \) as the \( j \)th diagonal element.

Market equilibrium requires (assuming for simplicity that all countries have the same population) that \( Q\iota = 0 \) and so, solving for price, we find that the equilibrium price is:

\[ p = \bar{y}_o - \Sigma_o\iota(\iota'\Gamma^{-1}\iota)^{-1} \]  

(16)

Substituting this expression for \( p \) into the equation for \( Q \), we find that \( Q = -M \) where \( M \) is the idempotent matrix \( I - \iota(\iota'\Gamma^{-1}\iota)^{-1}\iota'\Gamma^{-1} \). Then, after-hedging income (consumption) \( c_o \) is given by \( c_o = C'y_o + M'\bar{y}_o - M'\Sigma_o\iota(\iota'\Gamma^{-1}\iota)^{-1} \), where \( C = I - M \). It follows that the post-hedging variance of consumption is \( C'\Sigma_o C \) and the post-hedging mean of income is \( \bar{y}_o - M'\Sigma_o\iota(\iota'\Gamma^{-1}\iota)^{-1} \). Finally, the maximized utility of the representative individual in country \( j \) is:

\[ U_j = y_{yj} + \bar{y}_{oj} - M_j'\Sigma_o\iota(\iota'\Gamma^{-1}\iota)^{-1} - \frac{\Gamma_j}{2}C_j'\Sigma_o C_j. \]  

(17)

This utility level is not at the highest level attainable since risk-sharing with the (unborn) younger generation was not possible. Suppose the government undertakes such risk-sharing by arranging a pay-as-you-go social security system that takes an amount \( b_j y_o \) from the young and gives it to the currently old. Moreover, suppose the government undertakes international risk-sharing by purchasing an amount \( q_j \) of claims on national incomes on behalf of the older generation. The combined utilities of the older and younger generations, consistent with the government budget constraint, is:

\[ U_{cj} = y_{yj} - \frac{\gamma}{2}b_j'\Sigma_o b_j + \bar{y}_{oj} + q_j'(\bar{y}_o - p) \]

\[ - \frac{\gamma}{2}(\Sigma_{o j j} + 2(q_j' + b_j')\Sigma_o j + (q_j' + b_j')\Sigma_o (q_j + b_j)). \]  

(18)

Maximizing this utility with regard both to \( b_j \) and \( q_j \), we find that the optimal values are:

\[ b_j = -\frac{\Sigma_o^{-1}(\bar{y}_o - p)}{\gamma_j} \]  

(19)
or

\[ q_j = 2\Sigma_\alpha^{-1}(\bar{y}_o - p)/\gamma_j - e_j. \]  

(20)

We see, then, the optimal pay-as-you-go social security payment from young to old has a rather complicated form: it depends on the variance matrix of world national incomes, on the expected incomes in all countries, and on the price of claims on these incomes.

Even though the optimal pay-as-you-go social security system has this complicated form, in equilibrium, if all countries are optimizing in this manner, we can substitute out for price and derive a simpler expression for the optimal pay-as-you-go social security system. If all countries make these arrangements, then the equilibrium price is:

\[ p = \bar{y}_o - \frac{\Sigma_\alpha(l'T^{-1}l)^{-1}}{2} \]  

(21)

and \( Q = -M \) as before. It follows in this equilibrium that each person’s income is perfectly correlated with world income, and scaled by the inverse of the risk-aversion parameter for that country. In contrast to the case above where all risk management is handled by individuals, so that only older-generations move with world income, in this example government intervention is able to make both young and old incomes correlate perfectly with world income.

The government intervention in the markets produces a higher level of mean-variance utility for everyone. But what was the essential element of this government intervention? Could the government have achieved the same utility in a simpler way? The answer in this example is, if everyone is investing optimally, yes. The simpler strategy to achieve the same total welfare is for the government merely to make the transfer of \( b^t_y \) from the young to the old, without itself taking any part in the shares of foreign-country risks, so long as there are private markets that allow old individuals to buy and sell shares in incomes. If the government announces a rule that the transfer will be made from young to old according to this rule, then the government’s social security rule will create an incentive for people, when young, to hedge the risky income after government transfers from the young to the old. Then the optimization problem for the young has the same solution for \( q_j \) as above. Thus, this model offers a justification for a government creating a pay-as-you-go social security system whose intergenerational transfers are tied to world income, but does not justify any governmental investment or risk management itself.

If everyone is optimizing, the transfer rule that the government would follow is very simple. Substituting into equation (21) for price given by
equation (19), we find that the vector $b_j$ has the form:

$$b_j = \frac{-\mu'(\Gamma^{-1}\mu)^{-1}}{2\gamma_j}$$

and in the case where all risk aversions are the same, this means just that $b_j = -\mu/(2J)$, i.e., the government merely transfers half of its share of world income from old to young (in exchange for a fixed real amount) without regard for the covariance matrix $\Gamma$ or the price vector $p$. Thus, this rule is very simple, but only under the assumption that all prices are set optimally, and all countries are following this optimal rule. There is an analogous result in the CAPM that, if everyone else is behaving optimally, investors need only hold the market portfolio and the riskless asset, and there too the assumption that others are behaving optimally is required.

There is also another governmental transfer rule that is even simpler. It allows the government to achieve the optimal sharing of risks for its citizens even without the assumption that all countries are optimizing, relying instead only on the assumption that its own citizens are behaving optimally. The government need only specify that a fraction (a half in this model since there are as many old as young and since old and young here have the same risk aversion) of after-hedging income of the old is transferred to the young (possibly in exchange for a fixed real amount). The utility then faced by the old is:

$$U_{o,j} = \frac{1}{2}q_j'(\bar{y}_o - p) - \frac{\gamma}{8}(\Sigma_{o,j} + 2q_j'\Sigma_{o,j} + q_j'\Sigma_{o}q_j).$$

One finds that the quantity vector $q_j$ that maximizes this expression is the same as (20). Moreover, if all countries do this, then we achieve the same perfect world-sharing of income. Thus, there is no need for governments to undertake any involvement in world risk management. It is critical to note that for this method of achieving optimal risk management to work, the social security benefit must be tied to after-hedging income, i.e., it must be means-tested using all income including investments income.

While this skeletal model implies no government actions to hedge internationally, there are variations on this theme that would justify it. As mentioned above, the result that no government investment in hedging vehicles is needed after the risk-sharing is established could be overturned by changing the model so that risk aversions are different between young and old or by dropping the assumption of additively separable utility. Also, if information about incomes when old is revealed two periods earlier, even before the old were born, then there is no chance for these people to do any hedging of these risks before they are born. In this case, for true optimality, the government would have to undertake international hedging investments for people before they were born.
6 Limitations on the ability to manage risks

In several places above it was assumed that individuals would manage their income risks by taking the appropriate hedging or investment positions in financial markets, and this assumption simplifies the role of the government in risk-sharing. I have already alluded to the fact that people's ability to do this is imperfect. Let us now consider in more detail two reasons why their ability to hedge their risks is imperfect, and why there might thus be a government role in investing and hedging. The two reasons are that appropriate hedging markets do not exist, and that most people are not now prepared, psychologically or intellectually, to do such hedging.

6.1 Nonexistence of appropriate markets

The macro markets defined above, the markets for claims on major income aggregates such as national incomes, do not exist, yet. Any effort to use existing securities markets to hedge lifetime income risks must necessarily rely on some assumed correlation between the returns on these securities and the returns on claims on individual incomes.

Baxter and Jermann (1997) argue that for the United States the correlation between returns on securities and innovations in the present value of labor income is very high. Part of their argument is based on a theoretical presumption that a Cobb-Douglas production function, competitive factor markets is reasonable, and a multiplicative random technology shock term. With these assumptions, despite any movements in the random technology shock term, the labor- and capital-share in incomes is constant, and hence total labor income and total capital income must be perfectly correlated. They conclude that returns to capital and labor must also be perfectly correlated. This conclusion actually requires an assumption, not stated in the Baxter/Jermann paper that there is no randomness to the rate of creation or economic depreciation of either capital or labor, for otherwise there would be variations in the return between existing capital or labor, even though the total flows to all capital and all labor are perfectly correlated. For intergenerational purposes, depreciation of existing capital ought to be very important: by most assumptions most capital depreciates to nothing over decades.

Bohn (1998a,b) adopts for his basic model the Baxter/Jermann production function with a multiplicative technology term whose log is a random walk and assumes no stochastic depreciation for his model of the effects of the social security system. Thus, he has built into his model the perfect correlation between returns to human and physical capital. In Bohn (1998a) he considers extensions of the model including stochastic depreciation which he calls random salvage value for capital, which breaks down this perfect
correlation. With such models, the public’s ability to manage risks (without the macro markets I have described above) is reduced.

Baxter and Jermann (1997) use data on labor income and capital income, and estimate a vector autoregressive model for the two for four countries, U.S., UK, Germany, and Japan. They conclude that innovations in the present value of capital income are highly positively correlated with innovations in the present value of labor income, and therefore that claims on existing capital can be used quite effectively to hedge labor income risk. They reach the conclusion that people should be very short the stock markets of their own countries in order to hedge their own country risk.

While the conclusion that people should be short their own country’s stock market appears very plausible, and likely right, it appears that they may be overestimating the correlation between income returns and stock-market returns. The first problem is their assumption that returns to existing capital and labor can be inferred accurately using data on aggregate flows to capital and labor. It may be better to use stock-market returns data to learn about returns to investing in existing stocks. Second, the authors chose a cointegrated model for log labor and capital income, with cointegrating vector equal to [-1, 1]. Thus, they are embodying the assumption of a constant labor share.

Bottazzi, Pesenti, and Van Wincoop (1996) showed data on capital shares of a number of countries, and find that capital share was not as stable in many of these as it was in the Baxter and Jermann data. They also did an analysis using stock-market-returns data, and obtained a negative correlation between returns in U.S. financial markets and innovations in national income present values.

While all of the evidence on the adequacy of existing markets to permit hedging national income risks is not yet in, I would say that it appears quite risky to base a national income-hedging strategy solely on existing markets. It is thus difficult, in the absence of macro markets, for anyone to do the effective international hedging of income risks. At this time of reform of social security systems around the world, there may be a great opportunity for governments to innovate and create the macro markets, as for example by issuing government bonds indexed to their own national income, thereby making possible some of the international hedging that is optimal for international risk management.

6.2 Failures of individuals to manage risks

The models of risk management described above mostly presumed that people are evaluating risks optimally. The appeal of such models is that they allow us to understand, in simple direct terms, the nature of incentives pro-
vided by the present institutions and the outcomes if people respond as they should to these incentives. But, the notion that people will manage risks optimally may be excessively academic. It does not appear that people today are thinking in terms that are well-captured by these models, or that they are optimizing overall risk in any sense that is nearly optimal.

The above analysis, and the assumption that people are well-described as optimizing in this manner, would have to imply that people are thinking about such things as the correlation between labor income and investment returns. One model above used the parameter \( \rho \) which was the correlation between investment income and the next generation's labor income; another model used the covariance matrix \( \Sigma_o \) of incomes of countries around the world.

I did a Nexis search of newspapers and magazines in English, searching on the word "correlation" and "mutual fund," to try to find how often mutual funds say anything about the correlation of their returns to anything resembling income aggregates. The search produced many hits with mutual fund and correlation in the same story, but not a single one with the correlation between the mutual-fund returns and any proxy for aggregate national income.

People seem to be largely unaware of the importance of using financial markets to hedge existing income risks. Part of the reason that they may be so unaware of this possibility is just that there are now no markets for long-term claims on income aggregates or labor income, and so the returns in these markets or the changes in the present value of these incomes are not even observed. Since these returns are unobserved, they are less salient, less easily perceived by the general public, and hence, less likely to be hedged.

Euler equation models have long been known to imply that people apparently should have been investing more in stocks over the last century than they have, see Grossman and Shiller (1981). This problem has sometimes been referred to as "the equity premium puzzle," following an article by Mehra and Prescott (1985) that stressed it.

The public seems to have widespread misconceptions about the risk inherent in long-term bonds. The public has tended to view these as safe investments, in terms of their volatility, while stocks are risky. But, as shown by Siegel (1998), U.S. long-term bonds have actually been riskier, in terms of their real variance, than U.S. stocks.

The public seems often to use naive rules of thumb for portfolio allocation across broad aggregates, and this behavior is a problem for models that presume proper public risk management. This is analogous to a problem with optimizing models of the saving decision; people do not appear to be proper life-cycle savers (see, for example, Shefrin and Thaler, 1988; Carroll and Summers, 1991). Individual differences in savings propensities appear to
result in large variation in wealth at retirement, and roughly half of the people in the middle income deciles have virtually no personal assets at all as they approach retirement, Venti and Wise (1997). Many retired people suddenly contract their consumption at the point of retirement, as if the effects of retirement were a surprise to them, even though they should have seen them coming, Bernheim, Skinner, and Weinberg (1997). At the very least, a social security system must provide insurance against the consequences of making such egregious errors in risk management or savings decisions.

7 Summary and relation to various social security reform proposals

We have seen some models that imply that the optimal government intervention for optimal intergenerational, intragenerational, and international risk-sharing, and the optimal level of investment in risky investment projects, would amount to no more than a sort of pay-as-you-go social security system indexed to incomes of the young and old. These models suggest that there is no need for the government to mandate the building of a large trust fund of investable assets. There are also political obstacles and agency problems with the government’s taking on the responsibility for investments or for international hedging positions, and so there is something naturally attractive about confining a social security system to its essential social-insurance function, leaving out any government mandates that would direct investments.

Let us consider idealized social security contributions and benefits formulas that the models above suggest will promote better intergenerational, intragenerational, and international risk-sharing, but do not involve government participation in investing. (We may suppose that these formulas coexist with other programs encouraging individual saving and investment that are not considered here.) These formulas will involve exchanges of income risk between the young and old, thereby reducing total risk for both. These formulas can be viewed as benchmarks against which to judge risk management in the alternative proposals for reform of the system, though, of course, they cannot be used to judge other objectives (such as encouraging savings) of these proposals. The social security contributions and benefits formulas are:

\[
\text{contribution}_{yj} = \left( a - b \frac{y_0}{y_y} \right) y_{yj} \tag{24}
\]

\[
\text{benefit}_{oj} = \Phi \left( \frac{y_{yj-1}}{y_{y-1}} \right) y_y - c y_{oj} \tag{25}
\]

where \( \text{contribution}_{yj} \) is the contribution made by the \( j \)th working (young) person in the country and \( \text{benefit}_{oj} \) is the benefit to the \( j \)th retired (old) person in the country. The variable \( y_{yj} \) is the \( j \)th young individual’s after-tax
and after-hedging income from sources other than social security, \( y_y \) is per capita income of the young in the country after-tax and after-hedging (the average over \( j \) of \( y_{yj} \)), \( y_{oj} \) is the income of the \( j \)th old (retired) person after taxes and after-hedging, and \( y_o \) is the per capita income of old people in the country after-tax and after-hedging (the average over \( j \) of \( y_{oj} \)). The function of income that the currently old had when they were young, last period, as a fraction of per capita income of the young last period, \( \Phi(y_{yj-1}/y_{y-1}) \), \( \Phi > 0 \), is included to make the benefits formula depend on prior income, just as social security systems do today. As discussed above, I regard it as a constraint imposed on designers of social security that benefits must depend on contributions. Moreover, the function \( \Phi \) may also be motivated as providing a scaling to the benefit, on the assumption that people with higher income last period have higher expected income and variance of income this period. The \( \Phi \) function can also be used to make the benefits formula progressive, as it is in the United States today. Let us suppose that this is a linear function such that the average over \( j \) of \( \Phi(y_{yj-1}/y_{y-1}) \) is a given constant \( \bar{\Phi} \). (The average of \( y_{yj-1}/y_{y-1} \) is one by construction.) Thus, the total benefits paid to all retirees is unaffected by the aggregate level of income of the young last period. All parameters, \( a, b, c, \) and \( \Phi \) are strictly greater than zero.

The parameters can be set so that benefits equal contributions so the system is pay-as-you-go just as our present system, though it differs from the present system in that it is a risk-sharing system. This equality is achieved by making \( a = \bar{\Phi} \) and \( bn = c \) where \( n \) is the number of young working people per retired person (inverse of the dependency ratio), so that \( n \) is a little below 5 in the United States today. (We would expect \( n \) to decline in most countries in coming decades, see World Bank (1994), Tables A.1 p. 343 and A.2, p. 349.)

Let us consider a rough example of the parameters of these benefits and contributions formulas that would represent a modest adjustment of the social security system in place in the United States today, a step in the right direction. These values are not intended as a serious proposal, but just an indication of what might be done, and what might even have some chance of political support. Let us set \( a = 0.25 \) and \( b = 0.10 \), so that if \( y_o = y_y \) the tax rate on income of the young is 0.15, or not much more than the 12.4% paid by employee and employer together today. With these parameter values, however, the tax rate rises and falls reflecting relative incomes of the young and old, and offering a little income risk-sharing to both of them. Assuming a steady-state value for \( n \), the number of working per retired, of 3, then we would make \( \Phi \) equal 0.75 and \( c \) equal 0.30. The average retiree with no other income would then receive 75% of the average income of the young, but would, if there is other income, give up 30% of that income. If we set \( \Phi(y_{yj-1}/y_{y-1}) \) equal to \( 0.375 + 0.375y_{yj-1}/y_{y-1} \) then we would have a
progressive system, which guarantees for someone with no income at all, none when young and none from non-social-security sources when old, a retirement income of 37.5% of the average income of young people.

With these values for the benefits formula, retired persons give up 30% of their income from sources other than social security. This percent is not far from the present formula in the United States, where workers aged 65-69 who began collecting social security benefits at age 65 see their benefits cut by one-third of their salary and wages above the social security "cap" of $14,500. However, the above formulas involve no cap, the above formulas apply to all income, not just salary and wage income, and the benefits reduction is not limited to people under 70. It is important that the formulas apply to all income and all retired persons, since their purpose is sharing of income risks, including everyone’s income and including the risks to nonlabor income.

The risk-sharing function served by the benefits formula’s negative relation to all retired persons’ total income is fundamental to the functioning of social security as insurance. Unfortunately, in today’s controversy over the social security cap in the United States, risk-sharing appears almost never to be mentioned, and the debate over the cap seems to be almost entirely over questions of fairness, disincentives to work and the work ethic, and on the effects on the government budget.

I have left out of these functions the limit on income that is subject to the social security tax in the present U.S. social security system. In the present system, contributions are made as a fraction of income only up to a certain limit, $65,400 in 1997. I left the limit out of the idealized formulas, since I see no reason not to extend the risk-sharing to more wealthy people, but of course, the formulas could be altered to incorporate these thresholds. Realistically, if we are politically constrained to only modest adjustments in the present system, we would have to expect that the benefits and contributions formulas above would apply only to incomes up to such limits.

I have also left out of the above formulas any discussion of the fact that n changes through time, due to changing birth and death rates. Recognizing these changes, we may continue to use the existing trust fund to build up assets in times when there are many young, and spend down in times when there are few young.

These benefits and contributions formulas may look unfamiliar and unlike what we would expect of insurance premia and payments. We are accustomed to thinking that true insurance must guarantee a benefit whose value is fixed, and that the premium must be fixed. But, in fact, if the social security system is to function as true insurance, minimizing the effects on people of risks by spreading these risks over many people, then it is essential that the formulas have a form that swaps own income for a broader aggregate of income, as accomplished by these formulas. The concept that the best
insurance program for the elderly is one in which their benefits are fixed in real terms, since that entails no risks at all for them is valid, in a sense, but this ignores the effects of the program on the nonelderly. Proponents of fixed real benefits sometimes speak as if these fixed real benefits could be provided at no expense to others, as if providing such benefits is just a matter of conceiving the right program. In fact, providing a fixed real income to the elderly is not free, and not an efficient system from the standpoint of risk management. The above formulas are designed for better risk-sharing, reducing the concentration of the risks on the young that the present system entails.

These benefits and contributions formulas are very simple embodiments of some of the principles revealed by the analysis above, but there are also more complicated formulas that should also be considered. Governments may want themselves to hedge some of the risks that are revealed for their people before they are born, as described above. This means that governments may want to take long-term international hedging positions, and their hedging would then naturally be reflected in the benefits and contributions formulas themselves. Moreover, given the fact that many people, even after they are born, are still apparently incapable of doing hedging of any sort themselves, the government may want ideally to undertake some international hedging for them, and reflect the result in the benefits formulas. The contributions and benefits formulas above could be changed so that the young and old in the country are not swapping income with each other, but are both swapping income with the rest of the world, therefore directly promoting income risk-sharing with the rest of the world. Note that in this case the contributions and benefits can no longer be expected to equal each other, even approximately, since both contributors and beneficiaries are now net receivers of world income, whose future values are uncertain. The social security administration would then have to take hedging positions in foreign markets to make feasible their application of such formulas. It would ideally issue bonds indexed to own-country national incomes and invest abroad, ideally in such bonds of other countries. The government adoption of such international risk-sharing on behalf of their citizens might possibly be more acceptable in some small less-developed countries, where the magnitude of their own-country risk is more severe, where the advantages of issuance of government bonds tied to own-country national income would be more prominent, and where the foolishness of investing solely in one's own country would be more obvious.

Another variation on these benefits and payouts formulas might also be suggested by labor-supply distortions considerations: the dependence of the benefits and contributions on incomes of the old or young might be split into two parts, the individual's own income and the income of the aggregate age cohort. To the extent that benefits and contributions are tied to the
aggregate cohort income, rather than individual income, the labor-market distortions are diminished. Moreover, the formulas could give different weight to labor and capital income, reflecting the different distortionary taxation issues between the two.

Let us finally review how well the existing social security system in the United States and some of the proposed alternative systems in this country live up to these benefits and contributions formulas or their variations. The issue is not whether they are in full accordance with the risk-sharing formulas, but how much they contribute toward such risk-sharing, even approximately.

The existing U.S. social security system is not in accordance with the above formulas for any parameter values. The existing social security system fulfills little of the intergenerational risk-sharing function, and by tying benefits to the old to the CPI index without regard to the incomes of the young, it might make the intergenerational risk-management situation even worse than it would be if there were no social security system. It certainly does nothing at all to promote international risk-sharing, and the way benefits are calculated there is no offset in contributions from the young if there is high hedging income received from abroad by the old. Since under present law contributions are more tied to incomes than are benefits, the benefits formula is progressive, and there is an intragenerational risk-sharing function. However, since the social security contribution is cut off at relatively low income levels, the intragenerational risk-sharing is limited.\footnote{Of course, the Survivors Insurance and Disability Insurance components of the U.S. Social Security System provide additional intragenerational risk-sharing. Kotlikoff and Sachs (1997b) point out that the current Social Security System old age insurance also generates new intragenerational risks because of the way benefits are affected by changes in family structure.}

The 1994-96 Advisory Council on Social Security was called primarily to propose how to “fix” the U.S. Social Security System, which appears to have meant that they should deal primarily with the problem that the Social Security Trust Fund was projected to be exhausted by the year 2029. They were not asked to study whether the social security function was well designed as an instrument of social insurance, and so it is not surprising that they made virtually no attempt to think about theoretical risk-management issues. It is also not surprising that they did not come up with proposals that involved major new transfers among people, which effective risk management would require.

The Council (1997, p. 8; http://www.ssa.gov/policy/adcouncil/findings.htm/#overview) divided into three factions, but all factions agreed that the social security benefits should not be means tested: “Moreover, means-testing would send the wrong signal to young people and wage earners generally. The message would be ‘if you are a saver and build up income to supplement
social security, you will be penalized by having your social security benefits reduced." This message is both unfair to those who work and save and creates the wrong incentives." Apparently, the Council was aware of the distortionary effects of such means testing but unaware of its risk-management advantages.

No faction of the Council proposed anything like the risk-sharing benefits formulas described above; certainly none proposed that benefits to the elderly would have anything to do with the incomes of the young. Thus, there appears to be no intergenerational risk-sharing in any of these proposals. All three factions proposed that some part of the social security contributions would be invested in the stock market, and none of them addressed the issue of sharing the risks of these investments intergenerationally.

The Maintenance of Benefits (MB) faction specified that it would "maintain the present social security benefit and tax structure essentially as is" (Advisory Council, 1997, p. 14), without specifying explicitly how this will be guaranteed to be feasible when part of the trust fund (about 40% according to their plan) is invested in stocks and thus subject to market shocks. Since they say that theirs is a defined benefit plan, it is clear that they do not intend the elderly retired persons to bear any of this stock-market risk. There are also words in their statement that may be interpreted as suggesting that in the event of a major stock-market downturn, the young would bear the full brunt of the loss. While they show little awareness in their report of the risk of such poor performance of the stock market, they did mention risks that might shift social security out of actuarial balance, risks of changed life expectancies and of other costs. For these risks, they make it clear that the contributions tax rate will be adjusted, and so it would appear that they intend the stock-market risk to be borne exclusively by the young with the burden allocated among them with respect to their social security income (Bohn, 1998b, models the social security budget constraint in essentially these terms when analyzing the effects of investing the Trust Fund in stocks).

The other two factions of the Council, the Independent Accounts (IA) faction and the Personal Security Account (PSA) faction both specified moving the social security system in the direction of a defined contribution plan, wherein people would enjoy the returns on the funds invested in their own accounts. Their plans thus move in the direction of less intragenerational risk-

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12 a We include in our proposal an increase in the contribution rate of 0.8% for the employee with matching rate for the employer, to go into effect about 50 years from now. This increase of a combined 1.6 percent of payroll may not be needed when the year 2045 is reached, but we build it into law now so that it can be used if needed at that time to maintain a stable ratio between the trust funds and the next year's outgo .... " see Robert M. Ball, et al., "Social Security for the 21st Century: A Strategy to Maintain Benefits and Strengthen America's Family Protection Plan," http://www.ssa.gov/policy.adcouncil/ball1.htm.
sharing than the present system entails. Beneficiaries will have less money to live on if their own investments do badly.

None of the three plans mentions international risk-sharing, and in fact they occasionally quote statistics on historical returns that do not even specify which country was used, perhaps reflecting the longstanding popular assumption that the United States is the only plausible investment.\textsuperscript{13} If the IA or PSA investments are made substantially in the U.S. stock markets, not diversifying internationally, then beneficiaries will see some substantial risk to their retirement incomes that is wholly inconsistent with the notion of social insurance and effective risk management.

In evaluating any social security proposals, it must be borne in mind that any proposal that has a serious chance of passing must be a sort of political compromise, politically acceptable not only now but also in the distant future. There are many other factors to consider beyond the risk-management issues considered here, such as existing tax law distortions, the need to promote savings, and the need to appeal to popular notions of fairness. It is only hoped that this paper will have helped promote discussion, by bringing up more clearly the true insurance functions of social security which have been confused and blurred in most public discussions.

\textsuperscript{13}Some other plans do a little better on risk-sharing than these plans do. The Kotlikoff-Sachs plan specifies that the social security contributions would be invested internationally.
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