Empirical Examples and Extensions

- Hedonics
- Brief Review of AIDS-style demand
- Bresnahan and the 1955 Price War
- Berry-Levinsohn and Pakes
Aside: Hedonics

The simplest sort of empirical work on differentiated products seeks to descriptively characterize the relationship between product characteristics and the prices and/or quantities of each firm. A simple regression of prices on product characteristics is called a “hedonic regression.” Griliches (1961). This type of regression is often used to show how the “reduced form” relationship between prices and quantities changes over time.
For example, hedonic regressions are used to correct the producer price index for computers. The data is a panel of prices, $p_{jt}$ and characteristics, $x_{jt}$, of products over time. Simplifying, say that in period $t$, price is regressed on $x$ to obtain the parameters of

$$p_{jt} = x_{jt} \beta_t + \epsilon_t.$$ 

In period $t + 1$, a new set of products is on the market. It would be a mistake to look at the unadjusted price of, say, a mainframe computer because most likely this year’s model is far better than last year’s model (i.e. the $x$’s have improved.). The hedonic helps us ask how much today’s models would have cost yesterday.
We can get a predicted price by using last year’s coefficients on this year’s characteristics

\[ \hat{p}_{jt+1} = x_{jt+1}\beta_t. \]

Alternatively, we could put year dummy variables in a pooled regression of log price on \( x \)'s across years and treat the changes in the dummy variable as percentage changes in price, adjusted for quality.
This hedonic technique does not generate an ideal utility-based price index, but Pakes (2001) discusses bounds on the difference between the hedonic and an ideal index. Consider a non-parametric regression of $p_t$ on $x_t$ generating the function $\hat{p}_t(x)$. We can then construct an index comparing $p_{i,t+1}$ to $\hat{p}_t(x_{i,t+1})$ – thus asking what this product would have cost last year. By the usual price-index logic (Paache/Lespeyres) this bounds the true index. Note that this does not deal with the welfare affects of truly new $x'$ – in that case $\hat{p}_t(x_{i,t+1})$ will not be defined.
Review: Continuous Demand Models

E.g. – Hausman’s papers using AIDS functional forms to ask questions about competition and/or price indices/ benefit of new goods. Hausman’s measures the benefits of new goods by integrating under the demand curve as price decrease from the “choke-price” to the observed level.

Note that in these models cannot discuss hypothetical goods and have to severely restrict the cross-price effects as the number of products increases. (With \( J \) products there are \( J^2 \) own and cross-price elasticities and these change as the product-choice set changes.)
Bresnahan’s Paper on the 1955 Auto Price War.

The 1955 Auto Price War, Bresnahan (1987) starts with a puzzle, which is the apparent decrease in automobile prices that occurred in 1955, which was an economic boom year, as compared to the surrounding years of 1954 and 1956, when automobile prices seemed to be higher. Bresnahan wanted to know whether differences in competition could account for the price change. One hypothesis is that collusive behavior collapsed in the face of the economics boom. The idea, then, as in the homogeneous goods literature on “testing the competition”, is to look for evidence on what sort of competitive regime best explains the data in each of years 1954-1956.
Bresnahan assumes that we observe, for each product (= car), a set of product characteristics $x_j$ as well as the market outcomes of price and quantities sold, $p_j, q_j$. We observe the data only at the overall market (aggregate) level, without any information on the actions of individual households. Such data is readily available from industry-oriented publications (such as *Automotive News* and Ward’s.)
For simplicity, Bresnahan uses the demand vertical model, so there is only one attribute per product. In the data, there are a number of different characteristics, such as size, horsepower and luxury, so he creates an index say $\delta_j = x_j \beta$. He assumes a uniform distribution of tastes for quality and derives the demand function.

$$q_j = q_j(\delta_j, \delta_{-j}, p_j, p_{-j}).$$

(1)

that we derived before.
The prices are endogenously determined by a price-setting Nash equilibrium. Note that in the automobile industry, the first-order condition has to be modified to account for multi-product firms. GM, for example, offers dozens of products in the market today. To solve for equilibrium prices, one must also specify a cost side of the model. Costs, particularly marginal costs, are not typically observed so, as in the Cournot models above, we often are forced to infer marginal costs with the help of the oligopoly first order condition. Bresnahan assumes that marginal costs are a convex function of quality. (This helps to ensure an equilibrium with positive shares.)
Bresnahan can then:

- Solve for reduced-form $p$ and $q$ (sketch on the board).
- Tack on errors (say they’re normal.)
- Estimate by MLE under two equilibrium assumptions: Nash in price and collusion.

Given the estimates under the two competing equilibrium assumptions, Bresnahn does a “non-nested” hypothesis test and finds, as conjectured, that collusive pricing fits better in 1954 and 1956, but Nash pricing fits better in 1955!
Critiques:

- The vertical model is way too restrictive. (Maybe better for computers.)
- What identifies demand elasticities when there is no within-year price variation? (“The model”).
- The error structure does not allow for unobservables.
Endogenous Prices and Aggregation in Discrete Choice Models

The idea is to extend supply and demand empirical models to markets with product differentiation. As in the supply and demand literature, want to account for the implications of consumer and firm optimization together with equilibrium pricing.

First application: autos, again, with a focus on “realistic” own- and cross-price elasticities.
Data are at least market-level observations on prices, quantities and characteristics of products. Might also have data on characteristics of purchasing consumers and on firm production (but this is rare.) Consumers modeled in a discrete choice framework. The utility of consumer $i$ for product $j$ depends on observed and unobserved (by us) characteristics, $(y, \nu)$, of the consumer and observed and unobserved (by us) characteristics of the product, $(x, \xi)$. Each firm produces a given set of products whose production cost depends on a vector of cost shifters $w$; presumably, $w$ includes the product characteristics, $x$. The equilibrium is Nash in prices.
To derive demand, we assume that choices are all products in the market and an outside good. The utility of a choice is determined by a parametric form for the interaction between consumer characteristics and product attributes (Lancaster, McFadden, etc.) The demand function is then derived by explicitly aggregating over the choices of consumers with different characteristics.
Endogenous Prices

As in traditional homogeneous goods models, the econometric endogeneity of prices follows from the presence of unobserved characteristics.

In autos unobserved characteristics include style, dealer quality, tradition. How to explain: Chevy Cavalier, Mazda Miata, BMW 300’s.

Assuming no unobserved product characteristic also leads to overfitting problem.
Unobserved characteristics are usually ignored in discrete choice literature. Micro studies sometimes add product-specific intercept, but often have too many choices. Also, product intercept is co-linear with $p$, $x$.

Solution: Allow for unobserved characteristic, $\xi_j$

$$u_{ij} = x_j\beta - \alpha p_j + \xi_j + \epsilon_{ij}, \quad (2)$$

$$= \delta_j + \epsilon_{ij}, \quad (3)$$

where $\delta_j = x_j\beta - \alpha p_j + \xi_j$. 
Problem: $\xi$ enters $s_j$ in a highly non-linear fashion and is correlated with (at least) prices. Can solve for $\xi$ given parameters of model. However, one $\xi$ for every observation, need more restrictions. Following traditional demand literature, we assume that $\xi$ is uncorrelated with some “demand shifters”; here these are product characteristics. Could use other restrictions (endogenous characteristics?)
Inverting the Market Share Function

For the true model: \( s_j = s_j(\delta, \theta) \).

Invert to find mean levels of utility:

\[
\delta = s^{-1}(s, \theta) \tag{4}
\]

Now could estimate demand parameters by IV from, e.g.,:

\[
\delta_j = x_j \beta - \alpha p_j + \xi_j, \tag{5}
\]

Alternatively, can think of solving for \( \xi_j \) directly.
Instruments

- Cost shifters
- Changes in Rival’s $x’s$ over time/markets (sub-market?)
- Panel Data structures: (e.g. same $\xi$ for different product in different markets)

Intuition: Elasticities are “identified” by changes in the choice set over time (including changes in price), holding preferences constant (or controlling for changes in preferences from observed data.)
Logit Example.

Shares:

\[ s_j(\delta) = \frac{e^{\delta_j}}{1 + \sum_r e^{\delta_r}}, \]  \hspace{1cm} (6)

Share of outside good:

\[ s_0(\delta) = \frac{1}{1 + \sum_r e^{\delta_r}}. \]  \hspace{1cm} (7)

\[ \Rightarrow \ln(s_j) - \ln(s_o) = \delta_j = x_j \beta - \alpha p_j + \xi_j. \]  \hspace{1cm} (8)

Berry (1994) also gives analytic inverse for vertical model, nested logit. More complicated models require a numerical inverse.
In the logit case, estimate

\[ y_j \equiv \ln(s_j) - \ln(s_o) = \delta_j = x_j \beta - \alpha p_j + \xi_j. \]  

(9)

by 2SLS \((p_j \text{ is endogenous})\). Instruments are other-firms \(x's\), cost shifters – or else panel data assumptions.

Recall that the mean utility level, \(\delta_j\), determines all behavior including market share and cross-price effects. Markups for price-setting firms vary only with market share.
Functional Form

A well-known solution to problems with logit: Interact Product and Consumer Characteristics. Random coefficients example:

\[ u_{ij} = x_j \beta_i - \alpha p_j + \epsilon_{ij}. \]  \hspace{1cm} (10)

More generally,

\[ u_{ij} = \delta_j + \mu(x_j, p_j, \nu_i, \theta) + \epsilon_{ij}. \]  \hspace{1cm} (11)

To get aggregate demand have to compute a complicated integral. Aggregation problem solved by simulation, as in Pakes (1986).
Details on the BLP Utility Functions

“Cobb-Douglas”

\[ U(\nu_i, p_j, x_j, \xi_j; \theta) = (y_i - p_j)^\alpha G(x_j, \xi_j, \nu_i)e^{\epsilon(i,j)}, \tag{12} \]

Taking logs

\[ u_{ij} = \alpha \log(y_i - p_j) + x_j \beta_i + \xi_j + \epsilon_{ij}. \tag{13} \]

Let

\[ \beta_{ik} = \bar{\beta}_k + \sigma_k \nu_{ik}, \tag{14} \]
\[ u_{ij} = \delta_j + \mu_{ij}, \text{ with} \] (15)

where
\[ \delta_j = x_j \bar{\beta} + \xi_j \] (16)

and
\[ \mu_{ij} = \alpha \log(y_i - p_j) + \sum_k \sigma_k x_{jk} \nu_{ik} + \varepsilon_{ij} \] (17)

INCOME DRAWS are assumed to be log-normal
\[ y_{it} = \exp(\bar{y}_t + \sigma \nu_{iy}), \] (18)

with parameters calculated from CPS.
Calculating the Market Share via Simulation:

Condition on $\nu_i, y_i$ – this is a logit and get closed form. Take draws on $\nu_i, y_i$ and average over the implied logit shares:

$$\sum_{i=1}^{ns} \frac{e^{\bar{\mu}_{ij}+\delta_j}}{\sum_k e^{\bar{\mu}_{ik}+\delta_k}}, \quad (19)$$

where $\bar{\mu}_{ij} = \alpha \log(y_i - p_j) + \sum_k \sigma_k x_{jk} \nu_{ik}$.

BLP then provide an algorithm (a contraction mapping) that solves for $\delta$ given the parameters and a set of simulation draws. Have to hold the simulation draws fixed as the parameters change (or change in objective function is due to change in simulation.)

Have to account for the simulation variance – not linear and gets (relatively) worse as the shares get small.
Review of the GMM estimation algorithm

- Guess a parameter
- Solve for $\delta$ and therefore $\xi$.
- Interact $\xi$ and instruments $z$ – these are the moment conditions $G(\theta)$.
- Calculate an objective function – how far is $G(\theta)$ from zero? $f(\theta) = G'AG$ for some positive definite $A$.
- Guess a new parameter and try to minimize $f$.
- Variance of $\hat{\theta}$ includes variance in data across products and simulation error as well as any sampling variance in the observed market shares.

(Can simplify the algorithm since $\delta$ in linear in some parameters.)

**More on Functional Form**

Problem with $\epsilon_{ij}$’s: effectively add new product characteristic every
time add new product. Any product has positive sales and all products are substitutes. Extension to “no $\epsilon$” model is difficult but possible.

Vertical model gets rid of $\epsilon_{ij}$’s. But, all substitution is local – only with next highest and next lowest $p$. Substitution still doesn’t depend on $x$. 
Firm Behavior with Multi-Product Firms

Marginal Cost:

\[ mc_j = w_j \gamma + \omega_j. \]  

(20)

Might also model \( \ln(mc_j) \), make \( mc \) a function of \( q \), etc.

Profits:

\[ \pi_f = \Sigma_{j \in J_f} (p_j - mc_j) M s_j(p, x, \xi; \theta), \]  

(21)
First-order Condition:

\[ s_j(p, x, \xi; \theta) + \sum_{r \in \mathcal{J}_f} (p_r - mc_r) \frac{\partial s_r(p, x, \xi; \theta)}{\partial p_j} = 0. \]  \hspace{1cm} (22)

Given the demand function, it is possible to solve this for the vector of mc’s and so for markup, \( b_j(p, x, \xi, \theta) \) and for \( \omega_j \).

In particular, write the foc as

\[ s + \Delta(p - mc) = 0, \]  \hspace{1cm} (23)

where \( \Delta \) has one on the diagonal, one’s on the off-diagonals of jointly owned products and zeros elsewhere. MC is then:

\[ mc = p + \Delta^{-1}s. \]  \hspace{1cm} (24)
Estimating from the FOC

Given the demand parameters, can think of estimating the equation,

\[ mc_j = p_j - b_j(p, x, \xi, \theta) = w_j \gamma + \omega_j. \]  \hspace{1cm} (25)

Just as in estimating demand, estimates of the parameters \( \gamma \) can be obtained from orthogonality conditions between \( \omega \) and appropriate instruments.

Can also estimate supply and demand together.
Notes:

- We do not require a unique equilibrium
- the markup depends on $\xi$, $\omega$ and so is econometrically endogenous
- other static equilibria are easy (e.g. qty-setting, collusion)

Caveats:

- Nash Pricing
- no dynamics
- no production data
- no direct data on consumers
BLP

Question of interest: substitution patterns in auto demand; policy issues such as effect of Voluntary Export Restraints (VERs).

- Data: 20 years of public data on \( p, x, q \) of approx 100 automobile models per year. Also use info on distribution of consumer income.

- Utility Function:

\[
  u_{ij} = \bar{x}_j \beta - \alpha_i p_j + \sum_k \sigma_k x_{jk} \nu_{ik} + \epsilon_{ij}. \tag{26}
\]

- Firm behavior: \( \ln(mc) = w\gamma + \omega \), multi-product firms.

Econometric issues include simulating the shares, solving for \( \xi \) and asymptotics in number of products and/or time.