From SRAS to the Phillips curve

The SRAS curve in year $t$:

$$P_t = P^e_t (1 + \beta) (1 - a u_t + \tau)$$

$$\Rightarrow \quad \frac{P_t}{P_{t-1}} = \frac{P^e_t}{P^e_{t-1}} (1 + \beta) (1 - a u_t + \tau)$$

$$\frac{P_t}{P_{t-1}} = 1 + \frac{P_t - P_{t-1}}{P_{t-1}} = 1 + \left( \frac{\tau}{P_{t-1}} \right)$$

actual inflation rate in year $t$

$$\frac{P^e_t}{P^e_{t-1}} = 1 + \frac{P^e_t - P^e_{t-1}}{P^e_{t-1}} = 1 + \left( \frac{\tau^e}{P^e_{t-1}} \right)$$

expected inflation rate in year $t$
So now rewrite the SRAS curve:

\[ 1 + \Pi_t = \left(1 + \Pi_t^e\right) (1 + \beta) (1 - au + \varepsilon), \quad \text{or} \]

\[ \frac{1 + \Pi_t}{\left(1 + \Pi_t^e\right) (1 + \beta)} = 1 - au + \varepsilon \]

But the left-hand side is approximately equal to:

\[ 1 + \Pi_t - \Pi_t^e - \beta \quad \text{if} \quad \Pi_t, \Pi_t^e, \text{and} \beta \text{are close to 0}. \]

So:

\[ 1 + \Pi_t - \Pi_t^e - \beta = 1 - au + \varepsilon \]

\[ \Rightarrow \Pi_t - \Pi_t^e = \beta + \varepsilon - au_t \]

This is the expectations-augmented Phillips curve.
The natural rate of unemployment again.

In a long-run equilibrium, expectations about prices (and inflation) are correct: \( \Pi_t = \Pi_t^e \).

Remember too that when expectations about prices are correct, the unemployment rate is equal to the natural rate of unemployment: \( u_t = u_n \).

From the Phillips curve, if \( \Pi_t = \Pi_t^e \), then:

\[
0 = \beta + \tau - \alpha u_n \implies u_n = \frac{\beta + \tau}{\alpha}
\]

(when \( \beta \) is close to 0, this is approximately the same expression for \( u_n \) as the one on p. 6 of Lecture Notes #5).
Another way to write the Phillips curve using the solution \( u_n = \frac{\beta + \tau}{\alpha} \), the Phillips curve can also be written:

\[
\begin{align*}
\Pi_t - \Pi_t^e &= -a(u_t - u_n) \\
\text{unanticipated inflation} & \quad \text{deviation of the unemployment rate from the natural rate of unemployment}
\end{align*}
\]
Unemployment and Inflation in the 1960's

Low and stable inflation: \( \Pi_t^e \approx 0 \).

This led to a stable tradeoff between inflation and the unemployment rate which the government (the Federal Reserve) could try to exploit:

\[ \Pi_t = -a(u_t - u_n) \]
Figure 12.1 The Phillips curve and the U.S. economy during the 1960s
But in the 1970’s, all *! (^? breaks loose!

The apparent stable tradeoff vanishes.

Moral: If the government tries to exploit the tradeoff, actors in the economy figure this out and adjust their expectations about inflation.

Surprises to the inflation rate can have real effects (on unemployment and output), but predictable or systematic movements in the inflation rate cannot.
Figure 12.2  Inflation and unemployment in the United States, 1970–2002
Figure 12.7 The expectations-augmented Phillips curve in the United States, 1970–2002
In practice, last year’s inflation rate serves as a good predictor (or forecast) of this year’s inflation rate: $\Pi_t^e = \Pi_{t-1}$.

In this case, the Phillips curve becomes:

$$\Pi_t - \Pi_{t-1} = -a(u_t - u_n).$$

- If $u_t > u_n$, $\Pi_t < \Pi_{t-1}$ (inflation decreases).
- If $u_t < u_n$, $\Pi_t > \Pi_{t-1}$ (inflation increases).
- If $u_t = u_n$, $\Pi_t = \Pi_{t-1}$ (inflation does not change).

For this reason, the natural rate of unemployment ($u_n$) is sometimes called the non-accelerating inflation rate of unemployment, or NAIRU.
Figure 12.9  Actual and natural unemployment rates in the United States