Econ154b
Spring 2005

Suggested Solutions to Problem Set 3

Question 1

(a) 
\[ S^d = Y - C^d - G = Y - (3600 - 2000r + 0.1Y) - 1200 = 0.9Y - 4800 + 2000r = 600 + 2000r \]

(b) To graph the desired saving and desired investment curves, remember to solve the desired saving and desired investment equations for \( r \), which yields: 
\[ r = -0.3 + \frac{1}{2000} S^d \]
\[ r = 0.3 + \frac{1}{4000} I^d \]. Graphically,

To find the equilibrium interest rate we can use the goods market equilibrium condition that \( S^d = I^d \):

\[ S^d = I^d \]
\[ \Leftrightarrow 600 + 2000r = 1200 - 4000r \]
\[ \Leftrightarrow 6000r = 600 \]
\[ \Leftrightarrow r = 0.10 \]

We now verify that at this interest rate, the demand for goods \( C^d + I^d + G \) is equal to the supply of goods \( Y \):

\[ C^d + I^d + G = (3600 - 2000r + 0.1Y) + (1200 - 4000r) + 1200 = 6000 - 6000r + 0.1Y = \]
\[ Y = 6000 - 6000 \times 0.10 + 0.1 \times 6000 = 6000, \]

which is equal to \( Y = 6000. \)

(c)

For \( G = 1440, \) desired savings becomes

\[ S^d = Y - C^d - G = Y - (3600 - 2000r + 0.1Y) - 1440 = 0.9Y - 5040 + 2000r = 360 + 2000r \]

Solving this equation for \( r, \) we obtain: \( r = -0.18 + \frac{1}{2000} S^d. \) Clearly, this represent an upward (parallel) shift of the \( S^d \) curve. The \( I^d \) curve does not move. The interest rate will therefore rise. To find the new equilibrium interest rate, we solve again the goods market equilibrium condition that \( S^d = I^d: \)

\[ S^d = I^d \]
\[ \Leftrightarrow 360 + 2000r = 1200 - 4000r \]
\[ \Leftrightarrow 6000r = 840 \]
\[ \Leftrightarrow r = 0.14. \]

The new equilibrium interest rate is thus 0.14. We verify that at this interest rate the good market clears:

\[ C^d + I^d + G = (3600 - 2000r + 0.1Y) + (1200 - 4000r) + 1440 = 6240 - 6000r + 0.1Y = 6240 - 6000 \times 0.14 + 0.1 \times 6000 = 6000, \]

which is equal to \( Y = 6000. \)

(d) Now, when \( G=1200, \)


and setting \( S^d = I^d, \) we get

\[ S^d = I^d \]
\[ \Leftrightarrow 720 + 2000r = 1200 - 4000r \]
\[ \Leftrightarrow 6000r = 480 \]
\[ \Leftrightarrow r = 0.08. \]
Similarly, when $G=1440$,

$$S^d = \bar{Y} - C^d - G = \bar{Y} - (3600 - 2000r + 0.1(\bar{Y} - T)) - G = \bar{Y} - (3600 - 2000r + 0.1(\bar{Y} - 0.9\bar{Y} - 3600 + 2000r - 0.9G) = 5400 - 3600 + 2000r - 1296 = 504 + 2000r,$$

and setting $S^d = I^d$, we get

$$S^d = I^d$$

$$\Leftrightarrow 504 + 2000r = 1200 - 4000r$$

$$\Leftrightarrow 6000r = 696$$

$$\Leftrightarrow r = 0.1160.$$

Thus, in this case the interest rate increase from 0.08 to 0.116.

Question 2

(a) The tax-adjusted user cost of capital is:

$$\frac{uc}{1 - r} = \frac{(r+d)p}{1 - r} = \frac{(1.2) \times 1}{1 - 0.5} = 0.6$$

For the desired future capital stock, set $MPK^f = \frac{uc}{1 - r}$, and solve for $K^f$:

$$20 - 0.02K^f = 0.6 \Leftrightarrow K^f = 970.$$ 

Finally, since $K^f - K = I - dK$, $I = K^f - K + dK = 970 - 900 + (.2 \times 900) = 250$.

(b) Here, we do the same thing as in (a), but for general $r$:

$$\frac{uc}{1 - r} = \frac{(r+d)p}{1 - r} = \frac{(r+2) \times 1}{1 - 0.5} = 0.4 + 2r$$

Again, we set $MPK^f = \frac{uc}{1 - r}$, and solve for $K^f$:

$$20 - 0.02K^f = 0.4 + 2r \Leftrightarrow K^f = 980 - 100r.$$ 

And finally, $I = K^f - K + dK = 980 - 100r - 900 + (.2 \times 900) = 260 - 100r.$
(c) \( S^d = \bar{Y} - C^d - G = 1000 - (100 + 0.5\bar{Y} - 200r) - 200 = 200 + 200r \)

(d) Set \( S^d = I^d \) and solve for \( r \):
\[
S^d = I^d
\]
\[
\iff 200 + 200r = 260 - 100r
\]
\[
\iff 300r = 60
\]
\[
\iff r = 0.20
\]

We verify that at this interest rate, the demand for goods \( C^d + I^d + G \) is equal to the supply of goods \( \bar{Y} \):
\[
C^d + I^d + G = (100 + 0.5\bar{Y} - 200r) + (260 - 100r) + 200 = 560 - 300r + 0.5\bar{Y} =
\]
\[
= 560 - 300 \times 0.20 + 0.5 \times 1000 = 1000,
\]

which is equal to \( \bar{Y} = 1000 \).

(d) For \( \tau = .40 \),
\[
\frac{\mu_c}{1-\tau} = \frac{(r+\tau)p_x}{1-\tau} = \frac{(r+.2)\times1}{1-.4} = 0.33 + 1.67r
\]

Again, we set \( MPK^f = \frac{\mu_c}{1-\tau} \), and solve for \( K^f \):
\[
20 - 0.02K^f = 0.33 + 1.67r \iff K^f = 983.5 - 83.5r
\]

And finally, \( I = K^f - K + dK = 983.5 - 83.5r - 900 + (.2 \times 900) = 263.5 - 83.5r. \)

Thus, we see that the \( I^d \) curve becomes less steep and its intercept increases. The desired savings curve, on the other hand, does not shift (since consumption here depends on total -not disposable- income). Hence, we find the equilibrium interest rate by:
\[
S^d = I^d
\]
\[
\iff 200 + 200r = 263.5 - 83.5r
\]
\[
\iff 283.5r = 63.5
\]
\[
\iff r = 0.2240.
\]

Finally, equilibrium consumption and investment are:
\[ C^* = 100 + 0.5(1000) - 200(0.2240) = 555.2 \]
\[ I^* = 263.5 - 83.5(0.2240) = 244.796 \]

Question 3

(a) \( PVLR = y + \frac{y'}{1+r} + a = 90 + \frac{110}{1.10} + 20 = 210. \)

(b)
\[
\begin{align*}
& c + \frac{c'}{1+r} = PVLR \\
& c + \frac{c'}{1.10} = 210 \\
& \text{When } c = 0, c' = 231; \text{ this is the vertical intercept of the budget line; when } c' = 0, \\
& c = 210; \text{ this is the horizontal intercept of the budget line; the budget line is the line that} \\
& \text{connects these two points.}
\end{align*}
\]

(c) Since \( c = c' \), we can just solve for consumption (in either period) from the budget constraint:
\[
\begin{align*}
& c + \frac{c'}{1.10} = 210 \\
& \Leftrightarrow 1.10c + c = 210 \times 1.10 \\
& \Leftrightarrow 2.1c = 231 \\
& \Leftrightarrow c = 110
\end{align*}
\]

Saving is: \( s = y - c = 90 - 110 = -20. \)

(d) Since \( y \) increases by 11, the new PVLR is 221. Thus, using our results from (c):
\[
\begin{align*}
& 2.10c = 221 \times 1.10 = 243.1 \\
& c = 115.76 \\
& s = y - c = 101 - 115.76 = -14.76
\end{align*}
\]

So part of the temporary increase in income is consumed and part is saved.

(e) Now \( y' \) increases by 11, so PVLR rises by \( \frac{11}{1.10} = 10. \) New PVLR is thus 220. Therefore,
2. \(10c = 220 \times 1.10 = 242\)
\(c = 115.24\)
\(s = y - c = 90 - 115.24 = -25.24.\)

Thus, a rise in future income leads to an increase in current consumption but a decrease in saving.

(f) A rise in initial wealth has the same effect on the PVLR (and thus on consumption) as an increase in current income by the same amount, so \(c = 115.76\) as in part (d), and
\(s = y - c = 90 - 115.76 = -25.76.\)
Thus, an increase in wealth increases current consumption and decreases saving.

Question 4

The difference in interest rates between borrowing and lending means there is a kink in the budget constraint at the no-lending, no-borrowing point, as shown below. Borrowing is zero when \(c = y + a\).

If current consumption is less than \(y + a\), the person is a saver (lender), and the budget line has slope \(-(1 + r_i)\). If current consumption is greater than \(y + a\), the person is a borrower, and faces a steeper budget constraint with slope \(-(1 + r_b)\), because the interest rate is higher.

An increase in either interest rate would steepen only the portion of the budget constraint for which that interest rate is relevant. An increase in the real interest rate on lending is shown as a shift in the budget line segment from \(BL_1\) to \(BL_2\) in the figure below. An increase in the real interest rate on borrowing is shown as a shift in the budget line segment from \(BL_3\) to \(BL_4\). If the indifference curve hits the budget line at the no-borrowing, no-lending point, as shown, then there will be no change in current or future consumption due to a change in either interest rate.
Finally, an increase in the consumer's initial wealth would lead to a parallel rightward shift of both segments of the budget line, as shown below: