Economic policy

In this section of the course, we will study the effects of financing a given stream of government consumption. We will focus on fiscal policy (not on monetary policy) and in particular the aspects related to funding an arbitrary sequence of government expenditure.

The government’s expenditure plan will be treated as a given, and based on this the analysis will focus on the financing of such plan, under a general equilibrium framework. Given a sequence \( \{g_t\}_{t=0}^{\infty} \) of government expenditures, what can our micro foundations approach to macroeconomics tell us about the best way to provide funding for it?

There are two sub-questions involved:

1. **Q**: If it is "technologically" possible to implement lump-sum taxation, does the timing of these taxes matter? If so, how?
   
   **A**: The *Ricardian Equivalence* result tells us that timing of lump-sum taxes does not matter. This holds unconditionally in the dynastic model, but in the overlapping generations set up it depends on preferences.

2. **Q**: If lump-sum taxes are not enforceable, what kinds of distortionary taxes are best? What can we say about timing of taxation in this case?
   
   **A**: The answer to this issue is much less straightforward than the previous one, of course. The most widely mentioned distortionary taxes in the literature are levies on factor remunerations. We will analyze the case of proportional taxes on labor income \( (\tau^l_t) \), and on capital income \( (\tau^k_t) \).

   A sequence of proportional taxes \( \{\tau^l_t, \tau^k_t\}_{t=0}^{\infty} \) has to chosen so as to optimize some measure of welfare (that is, to pick the best allocation for some given ranking of outcomes). But, besides the issue of how to properly assess welfare, an important issue arising is that of the "time-consistency" of a proposed taxing sequence (the literature on this topic, as so many others, was originated by Prescott).

   Usually, models predict that the best distortionary taxing policy is to fully tax initial capital. Intuitively, since capital that has been invested is a "sunk" cost, and can not escape the taxing, then this is the least distortionary tax. Provided, of course, that the government could credibly commit to implement this tax only once. However, in the usual stationary model, at \( t = 1 \) the government’s problem is identical to that at \( t = 0 \) (only the initial capital, and maybe the history of shocks will differ). Hence, whatever was optimal at \( t = 0 \) will again be optimal at \( t = 1 \). So a "promise" on the part of the government fully tax capital at \( t = 0 \) only and never again could not be rationally believed - we say that it would not be time-consistent.
1 Ricardian equivalence

We will analyze the welfare effects of timing in lump-sum taxation by adding the presence of a government to our usual macro model. This presence takes the form of a given expenditure sequence $\{g_t\}_{t=0}^\infty$ and an ability to levy taxes on the consumer’s income. We begin with the dynastic model, and then analyze the overlapping generations setup.

In both cases, what we are interested in is finding the sequence of debt $\{B_t\}_{t=0}^\infty$ (one-period loans from the private sector to the government) and lump-sum taxes $\{\tau_t\}_{t=0}^\infty$ such that the following budget constraint is satisfied at every $t$:

$$g_t + B_{t-1} = q_t \cdot B_t + \tau_t \quad \forall t \quad \text{(GBC)}$$

(GBC) requires that sources and uses of funds be equalized in every period. Funds are used to finance expenditures $g_t$, and to repay $B_{t-1}$ (bonds issued at $t-1$ that must be redeemed at $t$). Sources are lump-sum tax collection $\tau_t$ and new government borrowing $B_t$. $q_t$ is the price of these bonds - the amount of ”money” (in this case, notice that the numeraire is $g_t$, which will turn out to be consumption goods) that the government gets for each unit of bonds $B_t$ issued. This price is just the inverse of the (gross) return on these bonds. We will assume that the initial bond position is null: $B_{-1} = 0$.

1.1 Dynastic model

Preferences will be assumed strongly monotone, so that consumers will exhaust their budget constraints at every period. Consumption goods are provided by an exogenous, deterministic endowment process. The problem will be formulated sequentially; since there is one state of the world for each $t$, just one asset per period is enough for complete markets to obtain. Agents will be allowed to hold positions in one-period loans; that is, they will be able to borrow or lend for one period at each $t$.

We write down the consumer’s sequential budget constraint in the dynastic economy:

$$c_t + q_t \cdot B_t + l_t = \omega_t + B_{t-1} + l_{t-1} \cdot R_t - \tau_t$$

where $l_t$ denote the net borrowing/lending position at the end of period $t$; $\omega_t$ is the exogenously given endowment of consumption goods at $t$.

We assume that a No-Ponzi game condition holds; then we may consolidate the budget constraint to

$$\sum_{t=0}^\infty p_t \cdot c_t = \sum_{t=0}^\infty p_t \cdot \omega_t - \sum_{t=0}^\infty p_t \cdot \tau_t + \sum_{t=0}^\infty (p_{t+1} - p_t \cdot q_t) \cdot B_t$$
We can normalize $p_0 = 1$, and we also have that

$$\frac{p_t}{p_{t+1}} \equiv R_{t+1}$$

In equilibrium, government and private debt must yield the same rate of return, or otherwise asset markets would not clear:

$$q_t = \frac{1}{R_{t+1}}$$

This implies that

$$p_{t+1} - p_t \cdot q_t = 0$$

There is only one state of the world (this is a deterministic model), but there are two assets. In fact, one of them is redundant.

Replacing in the consumer’s budget constraint, we obtain

$$\sum_{t=0}^{\infty} p_t \cdot c_t = \sum_{t=0}^{\infty} p_t \cdot \omega_t + \sum_{t=0}^{\infty} p_t \cdot \tau_t$$

While the government’s consolidated budget reads

$$\sum_{t=0}^{\infty} p_t \cdot g_t = \sum_{t=0}^{\infty} p_t \cdot \tau_t$$

But then if we replaced this in the consumer’s budget constraint, we would realize that in fact what is relevant for the decision making agent is not the taxing stream, but the expenditure stream $\{g_t\}_{t=0}^{\infty}$:

$$\sum_{t=0}^{\infty} p_t \cdot c_t = \sum_{t=0}^{\infty} p_t \cdot \omega_t + \sum_{t=0}^{\infty} p_t \cdot g_t \quad \text{(RE)}$$

This is the "Ricardian Equivalence": the timing of taxes is not relevant.

For a more formal statement of this equivalence, we first need to define the competitive equilibrium in this economy:

**Definition:** A competitive equilibrium is a sequence $\{p_t, c_t, (g_t), B_t, q_t, r_t, \tau_t\}_{t=0}^{\infty}$ such that:

1. Consumers’ utility is maximized, subject to their budget constraint.
2. The Government’s budget constraint is satisfied.

In the case of an endowment economy, this condition requires that

$$c_t + g_t = \omega_t$$

In the case of a production economy, it requires that

$$c_t + K_{t+1} + g_t = F(K_t, n_t)$$
4. Firms maximize profits - in the case of a production economy.

Notice that in naming the sequence \( \{ p_t, c_t, (g_t), B_t, q_t, r_t, \tau_t \}_{t=0}^{\infty} \), we have written the government’s expenditure stream \( g_t \) between parenthesis. The reason is that in fact this is given, and as such is not a decision variable that should be part of the equilibrium. It could be treated as a parameter in the problem (for example, in an endowment economy the endowment could be redefined as net of government expenditures).

Notwithstanding the way government expenditures are presented in the definition, equipped with a competitive equilibrium we are now ready to state the following:

**Theorem 1 (Ricardian equivalence in the dynastic model)** Let the sequence \( \{ p_t, c_t, g_t, B_t, q_t, r_t, \tau_t \}_{t=0}^{\infty} \) be an equilibrium. Then \( \{ p_t, c_t, g_t, \hat{B}_t, q_t, r_t, \hat{\tau}_t \}_{t=0}^{\infty} \), is also an equilibrium if

\[
\sum_{t=0}^{\infty} p_t \cdot \hat{\tau}_t = \sum_{t=0}^{\infty} p_t \cdot \tau_t
\]

and the sequence \( \{ \hat{B}_t \}_{t=0}^{\infty} \) is picked to satisfy the government’s budget constraint:

\[
\hat{B}_t \cdot q_t - \hat{B}_{t-1} - \hat{\tau}_t = B_t \cdot q_t - B_{t-1} - \tau_t
\]

**Proof.** The proof of this theorem is immediate from (RE). The new tax-borrowing mix chosen by the government does not alter the actual budget constraint faced by the consumer; hence, since his maximization problem remains completely unaltered, the optimizing choices remain the same. ■

1.2 Overlapping generations

The overlapping generations scheme seems more suitable to highlight the inter-generational transfer aspects that may be involved when government picks a tax-borrowing mix to finance expenditures. Changes in timing will alter the budget constraints faced by consumers if different generations are involved in the change. The actual effect of this on present and future generations' well-being will depend on the extent to which the current generation values the welfare of its offspring.

Hence, a limited version of the Ricardian Equivalence result holds in this case. In order to analyze it, we will have to make use of a competitive equilibrium. In this case, this will take the form of a sequence

\[
\{ c_t(t), c_t(t+1), g_t, l_t, B_t, \tau_t(t), \tau_t(t+1), r_t, q_t \}_{t=0}^{\infty}
\]
such that the conditions for it to be an equilibrium are satisfied. Then we can state the following:

**Theorem 2 (Ricardian equivalence in the overlapping generations model)**

Let the sequence \( \{c_t(t), c_t(t+1), g_t, l_t, B_t, \tau_t(t), \tau_t(t+1), r_t, q_t\}_{t=0}^{\infty} \) be an equilibrium. Then so is \( \{c_t(t), c_t(t+1), g_t, l_t, \hat{B}_t, \hat{\tau}_t(t), \hat{\tau}_t(t+1), r_t, q_t\}_{t=0}^{\infty} \) where

\[
\hat{\tau}_t(t) + \frac{\hat{\tau}_t(t+1)}{r_{t+1}} = \tau_t(t) + \frac{\tau_t(t+1)}{r_{t+1}} \quad \forall t
\]

and

\[
q_t \cdot \hat{B}_t - \hat{B}_{t-1} + \hat{\tau}_t(t) + \hat{\tau}_{t-1}(t) = q_t \cdot B_t - B_{t-1} + \tau_t(t) + \tau_{t-1}(t) = g_t \quad \forall t
\]

**Proof.** You may notice that the theorem states the equivalence for the case where the present value of taxation is not changed for any generation. The argument, therefore, will be of the same nature as the dynastic case. First recall that the consumer’s sequential budget constraints are

\[
c_t(t) + q_t \cdot B_t = \omega_t(t) - \tau_t(t)
\]

\[
c_t(t+1) = B_t + \omega_t(t+1) - \tau_t(t+1)
\]

The chart shows that the change in taxes implies no alteration to the actual budget constraint:

The point \( c^* \) will be chosen, regardless of whether the endowment is \( \omega \) or \( \omega' \). The slope of the budget line is \(-r_{t+1}\).

Next, let us suppose that government chooses between taxing young at 0 or old at 0, and that the old care about the utility enjoyed by the young at 0. Will the choice have an impact in total welfare?
We assume that the utility of the old at 0 is a function of the utility of their offspring:

\[ u_{-1} (c_{-1}(0), u_0 [c_0(0), c_0(1)]) \]

Government’s budget constraint requires that:

\[ \tau_{-1}(0) + \tau_0(0) = g_0 \]

And the private budgets for are:

\[
\begin{align*}
    c_{-1}(0) &= \omega_{-1}(0) - \tau_{-1}(0) - b_{-1} \\
    s + c_0(0) &= \omega_0(0) - \tau_0(0) + b_{-1} \\
    c_0(1) &= s \cdot r_1 + \omega_0(1)
\end{align*}
\]

where \( b_{-1} \geq 0 \) is a bequest that the old leave behind to their descendants, and \( r_1 \) is the return on savings between periods \( t \) and \( t + 1 \).

The old solve:

\[
\max_{b_{-1}} \{ u_{-1} [\omega_{-1}(0) - \tau_{-1}(0) - b_{-1}], u_0 [\omega_0(0) - \tau_0(0) + b_{-1}, \omega_0(1)] \}
\]

s.t. \( b_{-1} \geq 0 \)

(We have used the fact that \( s = 0 \) must prevail in equilibrium in an endowment economy.)

The chart shows the trade-off faced by the government:

The slope of the straight line is \(-1\), reflecting that every unit of extra consumption given to the young must be subtracted from the old. The point \((**\)) is the optimizing choice of the old; it implies a certain bequest \( b_{-1} \geq 0 \).
government can only induce a consumption choice with \( b_{-1} \geq 0 \); therefore all points to the right of \((*)\) are not feasible. If the government chooses any taxation between \((*)\) and \((***)\), then in fact nothing changes and the old ”correct” the ”bad” choice of the government through the appropriate choice of \(b_{-1} \). However, if the government chooses a taxation mix to the left of \((***)\), then the solution to the bequest level becomes a corner solution.

Therefore, changes in taxation timing will yield changes in the consumption allocation whenever bequest constraints bind. Otherwise, they will not.

2 Optimal distortionary taxes

Next we address the second issue raised in the introduction of this section. We will use the standard neoclassical growth model in its dynastic version, and with endogenous labor supply (valued leisure). We will take as given a (constant) sequence of government expenditure \( \{g_t\}_{t=0}^{\infty} \), with \( g_t = g \ \forall t \). Lump-sum taxation will not be available any more, but rather taxes on factor remunerations: capital and labor income. The government will choose the optimal mix so as to maximize the representative agent’s utility.

This is a high dimensional problem. The decision making agent chooses a consumption–leisure-capital accumulation path given the tax rates. So the government, when choosing taxation, has to take into account how these rates induce different optimal choices on the part of the consumer.

We will assume that the individual’s preferences over consumption and labor streams are represented by a time separable utility function with discount factor \( \beta \in (0, 1) \). This function is strictly increasing in \( c_t \), and strictly decreasing in \( n_t \), and concave. A central planner seeking an optimal allocation in a deterministic economy must thus solve:

\[
\max_{\{c_t, K_{t+1}, n_t\}} \left\{ \sum_{t=0}^{\infty} \beta^t \cdot u(c_t, n_t) \right\}
\]

s.t. \( c_t + g + K_{t+1} = F(K_t, n_t) + (1 - \delta) \cdot K_t \)

We want to study the outcome from distortionary taxes, so we need to decentralize the problem. Let \( \tau_t \) denote the proportional tax on labor income, and \( \theta_t \) that on capital income. Then we have the following:

**Definition:** A competitive equilibrium is a sequence

\[
\{c_t, K_{t+1}, n_t, p_t, r_t, w_t, \theta_t, \tau_t\}_{t=0}^{\infty}
\]

such that:
1. The consumption-leisure-capital accumulation path \( \{c_t, K_{t+1}, n_t\}_{t=0}^{\infty} \) maximizes consumers’ utility subject to the budget constraint

\[
\sum_{t=0}^{\infty} p_t \cdot (c_t + K_{t+1}) = \sum_{t=0}^{\infty} p_t \cdot \left[ (1 - \tau_t) \cdot w_t \cdot n_t + R^K_t \cdot K_t \right]
\]

where \( R^K_t = 1 + (1 - \theta_t) \cdot (r_t - \delta) \) denotes the gross return on capital after taxes. Notice that depreciated capital is not taxed. You can think that if \( r_t \) is the revenue from lending capital to producers, then \( \delta \) is the cost that the owner of capital faces due to depreciation. Taxation is only levied on net income from capital.

2. Firms maximize profits:

\[
\{K_t, n_t\}_{t=0}^{\infty} = \arg \max_{\{K_t', n_t'\}_{t=0}^{\infty}} \{F(K_t', n_t') - w_t \cdot n_t' - r_t \cdot K_t'\}
\]

3. The government’s budget constraint is satisfied:

\[
\sum_{t=0}^{\infty} p_t \cdot g_t = \sum_{t=0}^{\infty} p_t \cdot [\tau_t \cdot w_t \cdot n_t + \theta_t \cdot (r_t - \delta) \cdot K_t]
\]

4. Markets clear:

\[
c_t + g + K_{t+1} = F(K_t, n_t) + (1 - \delta) \cdot K_t
\]

We will first focus on studying this problem in the case when \( \theta_t = \theta \), \( \tau_t = \tau \) for all \( t \). Then the steady state equilibrium can be found by solving the consumer’s problem. The first order conditions are:

\[
\beta^t \cdot u_c(c_t, n_t) = \lambda^t \cdot p_t
\]

\[
p_t = R^K_{t+1} \cdot p_{t+1}
\]

Then

\[
u_c(c_t, n_t) = \beta \cdot u_c(c_{t+1}, n_{t+1}) \cdot R^K_{t+1}
\]

The steady state \( c_t = c, n_t = n, R^K_t = R^K \) satisfies

\[
\beta \cdot R^K = 1
\]

and

\[
R^K = 1 + (1 - \theta) \cdot (r - \delta)
\]

where \( r \) is the steady state factor payment. Assuming that the production technology exhibits constant returns to scale, \( r = F_1(K, n) \) is consistent with
equilibrium and with market clearing. In addition, under this assumption $F_1$ is a function of $\frac{K}{n}$. Therefore,

$$R^K = 1 + [1 - \theta] \cdot \left[ f_1 \left( \frac{K}{n} \right) - \delta \right]$$

and we can solve for $\frac{K}{n}$ (notice that this ratio will depend on the taxing policy $\theta$). To solve for labor, we need the first order conditions with respect to that decision variable, and those will involve the corresponding tax rate $\tau$.

Next we turn to the government. Its decision problem amounts to choosing the sequence

$$\pi = \{\pi_t\}_{t=0}^\infty \equiv \{\theta_t, \tau_t\}_{t=0}^\infty$$

in order to maximize the consumer’s welfare, while meeting its budget. Now, the solution to the individual’s problem showed that the tax choice will induce an optimal behavior on the part of the consumer, as a function of that choice. Therefore, we may define for every tax sequence $\pi$, an allocation rule

$$x(\pi) = \{c_t, K_t, n_t\}_{t=0}^\infty$$

that comes from the consumer’s response to the tax sequence $\pi$.

The taxing policy also determines a sequence of prices:

$$w(\pi) = \{p_t, r_t, w_t\}_{t=0}^\infty$$

these are the prices supporting $x(\pi)$ as a competitive equilibrium.

Then for any $\pi$, there is a competitive equilibrium which from this point onwards we will denote by

$$x(\pi) = \{c_t(\pi), K_t(\pi), n_t(\pi)\}_{t=0}^\infty$$

$$w(\pi) = \{p_t(\pi), r_t(\pi), w_t(\pi)\}_{t=0}^\infty$$

With these elements, we can introduce a useful tools to study this problem:

**Definition 3 (Ramsey equilibrium)** A Ramsey equilibrium is a tax policy $\pi$ (that the government chooses optimally so as to be in budgetary equilibrium), an allocation rule $x(\pi)$, and a price rule $w(\pi)$ such that:

(i) $\pi$ maximizes:

$$\sum_{t=0}^\infty \beta^t \cdot u [c_t(\pi), n_t(\pi)]$$

subject to the government’s budget constraint and with allocations and prices given by $x(\pi)$ and $w(\pi)$. 

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(ii) For every alternative policy $\pi'$, $x(\pi')$ and $w(\pi')$ constitute a competitive equilibrium given policy $\pi'$.

(iii) $\theta_0 = \bar{\theta}_0$

This is an important restriction. The initial level of tax on capital income must be exogenously given. Otherwise, if the government could choose $\theta_0$ arbitrarily high, and $\tau_t = \theta_t = 0 \ \forall t \geq 1$. Taxing initial capital would be like a lump-sum tax, since initial capital is in the nature of a "sunk" investment, which can not be modified.

Then there are two approaches to this problem.

(I) The government directly solves

$$\max_{\{\theta_t, \tau_t\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t \cdot u(c_t, n_t) \right\}$$

subject to

1. $\beta^t \cdot u_c(c_t, n_t) = \lambda \cdot p_t$
2. $\beta^t \cdot u_n(c_t, n_t) = -\lambda \cdot p_t \cdot (1 - \tau_t) \cdot w_t$
3. $p_t = R_{K+1} \cdot p_{t+1} = [1 + (1 - \theta_t) \cdot (r_t - \delta)] \cdot p_{t+1}$
4. $r_t = F_K(K_t, n_t)$
5. $w_t = F_n(K_t, n_t)$

where $\theta_0 = \bar{\theta}_0$

$$\sum_{t=0}^{\infty} p_t \cdot g = \sum_{t=0}^{\infty} p_t \cdot [\tau_t \cdot w_t \cdot n_t + \theta_t \cdot (r_t - \delta) \cdot K_t]$$

where (*) are the first order conditions which, together with the market clearing conditions (**), define a competitive equilibrium; and (***) is the government’s own budget constraint.

(II) Instead of the previous huge system, we could solve the problem in a smarter way by having government solve:

$$\max_{\{c_t, K_{t+1}, n_t\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t \cdot u(c_t, n_t) \right\}$$

subject to

1. $c_t + g + K_{t+1} = F(K_t, n_t) + (1 - \delta) \cdot K_t$
2. $\sum_{t=0}^{\infty} \beta^t \cdot [u_c(c_t, n_t) \cdot c_t + u_n(c_t, n_t) \cdot n_t] = \frac{u_c(c_0, n_0)}{R_K^K \cdot K_0}$

where $R_K^K = 1 + [1 - \bar{\theta}_0] \cdot [F_K(K_0, n_0) - \delta]$

The claim is that solving the problem (II) is equivalent to solving (I). Then the two sole constraints in (II) must contain the same information as the huge system of constraints in (I).
In addition, notice that in the system (II), the government’s decision variables are not the tax sequence $\pi$ any more, but directly the consumption-capital accumulation-labor supply path $\{c_t, K_{t+1}, n_t\}_{t=0}^{\infty}$. Thus, for the two problems to be equivalent, it must be the case that by choosing these three paths subject to the two constraints in (II), we must be indirectly choosing all other variables, in particular taxes.

In particular, this means that any sequence $\{c_t, K_{t+1}, n_t\}_{t=0}^{\infty}$ satisfying the two constraints in (II), has to be part of a competitive equilibrium vector. We will now show that this is true. Define prices using the usual guesses:

- $r_t = F_K(K_t, n_t)$
- $w_t = F_n(K_t, n_t)$
- $p_0 = 1$
- $p_t = \beta \cdot \frac{u_c(c_t, n_t)}{u_c(c_0, n_0)}$

Let taxes on labor income be determined by the equation

$$(1 - \tau_t) \cdot F_n(K_t, n_t) = -\frac{u_n(c_t, n_t)}{u_n(c_0, n_0)}$$

and taxes on capital income by

$$u_c(c_t, n_t) = \beta \cdot u_c(c_{t+1}, n_{t+1}) \cdot [1 + (1 - \theta_{t+1}) \cdot (F_K(K_{t+1}, n_{t+1}) - \delta)]$$

So, are the conditions for a competitive equilibrium met?

* Market clearing: Yes, since $\{c_t, K_{t+1}, n_t\}_{t=0}^{\infty}$ was assumed to satisfy the two restrictions in (II), and one of those was precisely market clearing.

* Consumers’ and firms’ first order conditions: Yes, they are satisfied. This follows from the way prices were determined.

* Individuals’ budget constraints: If we use the second restriction in (II) and substitute prices back in, then this restriction will become exactly an individual’s budget constraint.

* Government’s budget constraints: If individual’s budget constraints are met, and markets clear, then we must have that the government’s constraint is also met. This argument is similar to a Walras’ law type of reasoning.

It looks like we have immensely simplified system (I) into system (II). However, this is not for free. Two drawbacks from the alternative approach to the problem must be highlighted:
1. The constraint set looks "weirder" than in our usual maximization problem. In particular, the equation in the second constraint might have a very arbitrary shape. The requirements for sufficiency of the first order conditions, therefore, will not necessarily be met. Points solving problem (II) will have to be cross-checked to make sure that they maximize the objective function.

2. Do you think that it is possible to apply dynamic programming techniques to solve (II)? Is it possible to write this as a recursive problem? Unfortunately, the answer is no. It is not possible to formulate (II) recursively.

Notice that second drawback that we have mentioned goes beyond the mathematical aspects involved. What does the impossibility of formulating (II) recursively tell us about the economic nature of the problem we are dealing with? The answer is that the problem is not stationary, because any solution to it can not be time-consistent. If we rename any \( t > 0 \) as \( t = 0 \), and act as if the economy was starting at that point, then the government would be willing to revise the decisions taken originally for that \( t \). In particular, the decision regarding taxing capital income.

This implies that any solution to (II) is a non-credible promise on the part of the government, since it will be willing to modify its original plan at every point in time. The way we overcome this drawback is that we assume that there is some sort of commitment device (enforceable laws, for example), which is assumed. Commitment to a plan is not endogenous in this setup. However insightful it may be, this approach has this as its main weakness.

Notwithstanding this, we will solve system (II). To that effect, we re-state the problem as

\[
\max_{\{c_t, n_t, K_{t+1}\}_{t=0}^{\infty}, \lambda} \left\{ \sum_{t=0}^{\infty} \beta^t \cdot W(c_t, n_t, \lambda) - \lambda \cdot u_c(c_0, n_0) \cdot R_0^K \cdot K_0 \right\}
\]

s.t.

\[
c_t + g_t + K_{t+1} = F(K_t, n_t) + (1 - \delta) \cdot K_t
\]

\[
W(c_t, n_t, \lambda) = u(c_t, n_t) + \lambda \cdot [u_c(c_t, n_t) \cdot c_t + u_n(c_t, n_t) \cdot n_t]
\]

The term \( u_c(c_0, n_0) \) in the objective function is endogenous, where as \( R_0^K \cdot K_0 \) is exogenous. \( \lambda \) is the Lagrange multiplier of the "implementability" constraint

\[
\sum_{t=0}^{\infty} \beta^t \cdot [u_c(c_t, n_t) \cdot c_t + u_n(c_t, n_t) \cdot n_t] = \frac{u_c(c_0, n_0)}{R_0^K \cdot K_0}
\]

in (II).
We take first order conditions. We should keep in mind that we do not know whether they are sufficient or not, but unfortunately we have no choice but to disregard this problem for the moment. We have

\[\begin{align*}
c_t : \beta_t \cdot W_c(c_t, n_t, \lambda) &= \mu_t \\
n_t : \beta_t \cdot W_n(c_t, n_t, \lambda) &= -\mu_t \cdot F_n(K_t, n_t)
\end{align*}\]

\[W_c(c_t, n_t, \lambda) - W_n(c_t, n_t, \lambda) = F_n(K_t, n_t) \quad t = 1, 2, \ldots\]

\[K_{t+1} : \mu_t = [F_k(K_{t+1}, n_{t+1}) + 1 - \delta] \cdot \mu_{t+1}\]

\[\Rightarrow (\ast) W_c(c_t, n_t, \lambda) = W_c(c_{t+1}, n_{t+1}, \lambda) + \beta \cdot [F_k(K_{t+1}, n_{t+1}) + 1 - \delta] \quad t = 1, 2, \ldots\]

For \( t = 0 \), the first order conditions are different (which reflects the time inconsistency of the choice).

\[\frac{W_c(c_t, n_t, \lambda)}{u_c(c_t, n_t)} = 1 + \lambda \cdot \left[ \frac{u_{cc}(c_t, n_t) \cdot c_t + u_{cn}(c_t, n_t) \cdot n_t}{u_c(c_t, n_t)} + 1 \right]\]

Notice that this expression involves the coefficient of relative risk aversion: \( \frac{u_{cc}(c_t, n_t)}{u_c(c_t, n_t)} \).

If

\[\frac{W_c(c_t, n_t, \lambda)}{u_c(c_t, n_t)} = \frac{W_c(c_{t+1}, n_{t+1}, \lambda)}{u_c(c_{t+1}, n_{t+1})}\]

then the condition (\( \ast \)) above can be written as:

\[u_c(c_t, n_t) = u_c(c_{t+1}, n_{t+1}) \cdot \beta \cdot [F_k(K_{t+1}, n_{t+1}) + 1 - \delta] \quad (EE)\]

This is the "usual" Euler equation - the one a social planner would choose in an economy without taxes. Therefore in a competitive equilibrium,

\[u_c(c_t, n_t) = u_c(c_{t+1}, n_{t+1}) \cdot \beta \cdot R^K_{t+1} \quad (CEE)\]

and

\[R^K_{t+1} = 1 + [1 - \theta_{t+1}] \cdot [F_k(K_{t+1}, n_{t+1}) - \delta]\]

Which implies that \( \theta = 0 \) is required to make the competitive Euler equation (CEE) equal to the planner’s Euler equation (EE).

Notice that the previous argument relies on the hypothesis that

\[\frac{W_c(c_t, n_t, \lambda)}{u_c(c_t, n_t)} = \frac{W_c(c_{t+1}, n_{t+1}, \lambda)}{u_c(c_{t+1}, n_{t+1})}\]

holds. When will this be valid? When will the ratio \( \frac{W_c(c_t, n_t, \lambda)}{u_c(c_t, n_t)} \) remain constant over time?

There are two answers:
1. Clearly this ration will not change in steady state.

2. Some functional forms for the utility representation will also yield such a stationary result. These functions are:

\[
u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} + v(n)\]

or

\[
u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} \cdot (1 - n)^{(1-\sigma)}\]

(Where the total labor endowment is normalized to 1.)

In these cases, the ratio \(W/c_t(n_t, \lambda)/u(c_t, n_t)\) will not depend on time, from \(t = 1\) onwards. Therefore, \(\theta_t = 0\) for \(t \geq 1\) will yield the required result.

3 References

Atkenson, Chari and Kehoe, Taxing capital income: a bad idea,

Barro, Robert, Are government bonds net wealth?, *Journal of political economy*