HOMEWORK #3

1. Use linear interpolation, cubic spline interpolation, polynomial interpolation, and Chebyshev approximation to approximate the value of log(x) on the interval [a, 1]. Compare the accuracy of the four approaches. How well do these four approaches approximate the value of log(x) outside the specified interval?

2. Use bilinear interpolation, bicubic spline interpolation, and Chebyshev approximation (see, in particular, Algorithm 6.4 on p. 238 of Numerical Methods in Economics) to approximate the value of log(x) log(y) for (x, y) ∈ [a, 1] × [a, 1]. Compare the accuracy of the three approaches (for values of x and y both inside and outside the specified ranges).

3. Use cubic spline integration and Monte Carlo integration to calculate an approximation to \int_{a}^{1} \log(x) \, dx. For Monte Carlo integration, report an estimated error. (Hint: The antiderivative of log(x) is x log(x) − x.)

4. Use cubic spline integration and Monte Carlo integration (both with and without antithetic variates) to calculate an approximation to E[e^x], where x ∼ N(µ, σ^2). For Monte Carlo integration, report an estimated error. (Hint: Since y = e^x has a log-normal distribution, E[e^x] = e^{µ+σ^2/2}.)

5. Use cubic spline integration, Monte Carlo integration, and Gaussian quadrature to calculate an approximation to E[x^n y^m], where n and m are positive integers and (x, y) has a joint normal distribution with mean µ and (non-diagonal) covariance matrix Σ. For Monte Carlo integration, report an estimated error.