September 15, 2005

Per Russell and Tony Smith

With Heterogeneous Agents

Quantitative Macroeconomic Models
Are the allegations founded?

"Macroeconomists ignore the possibility that economic policies affect the rich and the poor, or the employed and the unem-
ployed, in different ways."

"Macroeconomists pretend that everyone is identical. Who can believe the resulting analysis?"

Heard in the streets/econ dept corridors:

"Two allegations"
3

the R-A model, or irrelevant in the cases where the new model behaves like

simply not founded •

So the truth of the matter: the allegations are either

Imperfect insurance—imperfect microeconomic underpinnings—relevant heterogeneity, and
representative-agent (R-A) model change when more realistic

2. Precisely, to investigate how the properties of the standard
macroeconomic events and policy:

1. Precisely, to answer questions about how different segments

The whole purpose of the recent research I am reviewing here:
the new positive and normative insights.

• Just use a few examples: no laundry-list discussion of all.

one topic we are working on right now.

— Outline some new directions, with particular emphasis on

learn.

illustrate, with some simple applications, what one can

• how the new methods work and

— Discuss what has been accomplished:

• Focus on the methodological advances:

What I will and will not do in this talk.
sometimes also find that it has new aggregate features.

sues concerning the cross-section of consumers, but we-

3. Not only is the new model very useful for analyzing is-

aggregate „approximate

2. Explanation: It is feasible because of so-called „approximate

more heterogeneous.

a very large class of models with substantial (realistic and

1. It turns out, after all, that it is feasible to solve and analyze

The main messages of this talk are three (3):

els with heterogeneity, say, a realistic wealth/income distribution.

A huge road block: analyzing dynamic general-equilibrium mod-

Issues
The structure of the talk

1. Illustrates: approximate aggregation, new aggregate features.

2. Briefly discuss an open question that jumps out: What explains the huge wealth inequality in the data?

2. Pilot study: new research on asymmetric information and macro.

excellent for illustration and pilot studies.

Use a 2-period baby model: though less quantitative, it is

Covers some key findings from that model:

has quantitatively solid microeconomic underpinnings.

in the applied consumption/labor literature. This means it
classical) GE model. It builds in heterogeneity as modeled

Describe the new core model—an \( \infty \)-horizon dynamic (neo-
—Mkt's clear: bonds (total 0), capital (total $\text{b}$), labor (total $\text{l}$).

—Firm: $\text{z}^* t$ is stochastic.

- At: $\eta + t_0(n_m + e) + z \eta + t_q = t_1 + t_q q + t_q q \eta + q$

- s.t. $\maximize E$ $\text{b}$

—Consumer: picks stochastic processes for $(c', n', q')$ to

(by implication due to incomplete mkt's income and wealth

- Labor productivity/employment status and

Core model: heterogeneity only in

Basic setting: uninsurable idiosyncratic risk
Impossible?

dimensional variable today into one tomorrow. \( \ell \) evolves over time; map an \( \ell \infty \)

\( \ell \) on the entire \( \ell \)

the aggregates---(\( \ell \), \( \ell \), and hence \( \ell \), and \( \ell \) depend

et assets and to work differ in the population. Given (\( \ell \), \( \ell \), \( \ell \)), all

Here, due to incomplete insurance, propensities to save in differ-

in the German class.

only: \( \ell \) and \( \ell \). But it fails here, even using identical preferences

\( \ell \) would mean that aggregates depend on aggregates

\( \ell \) of wealth and individual shocks.

and the joint distribution of the economy's state variable: \( \ell \) and the joint distri-

Recursive equilibrium: Individual decisions (and aggregates) de-
It needed.

• Update perceptions (and add more complexity in perceptions)

• Draw aggregate shocks and simulate an economy with many agents. Use results to assess accuracy of individuals’

• Find optimal individual behavior implied by these perceptions:

  edly rational: a simple law of motion for it.

  Specifically: individual perceptions of how it evolves that are bound-

  solution by simulation. The algorithm:

  this class of models, numerical solution is possible. We use

HOWEVER: despite high dimensionality, it turns out that, in
Surprising validity since: holds for much wider class of models than 1998 setup.

there with bulk of wealth have almost identical propensities. However, the very poorest have quite different propensities. However,

rules are almost linear in wealth. make savings propensities vary much with wealth: decision

For a broad class of preferences, consumption risk does not

Why simple perception works: approximate aggregation.

be a „self-fulfilling prophecy“.

This is simply how this economy behaves: it does not seem to

is almost exactly confirmed!

Finding (Krusell and Smith, 1998):

if $k_{t+1}$ is believed to depend only on $k_t$ and on $z_t$, this belief
Basic positive results

- Wealth Gini counter-cyclical: as in data, less disparities in booms.

- Inequality. This has led to new research (see below).

- Though it has substantial remaining consumption risk, the baseline model has a hard time generating enough wealth:

  • Nontrivial implications for inequality: Long run and dynamics.
  
  • Tallist/walker (saver/hand-to-mouth-consumer) model.
  
  • Of these, have „shortcut“ representations, such as a cap.
  
  • Many extensions do not, though (see below). And, some
  
  • (almost) like it's R-A counterpart: the full-insurance model.
  
  • Baseline setup with idenitical, German preferences behaves
  
  • The model generates time series for aggregates (GDP, prices)
Counter-cyclical idiosyncratic risk can help explain asset prices.

stand why the $P^C$ of the R-A model does not match data.

Indivisible labor: gives aggregation biases that help under-

mouth", and less permanent-income behavior.

aggregate consumption-income correlations (more "hand-to-

Model with heterogeneity in patience: gives very different

Examples:

more frictions, are introduced, very different properties may arise.

No, no, not if other features, such as other heterogeneity or

Does approximate aggregation mean that we might as well stick

Implications of approximate aggregation
Here, micro consumption/labor literature may help us discri-

Yes! Not only normative, but also positive, properties differ.

Does it matter what explains it?

• Poor are poor because they are unlucky.

Huge exogenous wage inequality (left-wing perspective): the poor

Preference heterogeneity (right-wing perspective): the poor

Wealth inequality: what can explain it?
agents make heterogeneous interferences from prices. work toward dynamic model where differentially informed current

non-robustness of R-A model when adding other frictions

approximate aggregation

2. Entrepreneurial production and financing frictions. Illustrates social insurance role of macro policy

robustness of R-A model

approximate aggregation

I. 1998 setup. Illustrates 3 distinct models with different features

using 2-period models
and \( m \) are marginal products of aggregate technology.

Budgets: \( c_1 + \lambda y_2 = \frac{r(y_2)}{r(y_2) + \lambda} y_2 \) and \( c_2 = \frac{r(y_2) - y_2}{r(y_2) + \lambda} y_2 \), where \( \lambda \) is the interest rate in period 2.

Shocks: no aggregate shocks, but \textit{idiosyncratic labor productivity shock.}

\textbf{Exogenous initial wealth distribution \( T \): } \( \xi \) Lognormal.

Technology: \textit{Investment} \( x_2 \) in period 1 \( \Rightarrow \) output \( y_2 \) in period 2.

Preferences: \( n(c_1) + g'(c_2) n(c_2) \) is CRRA.

I: 1998 setup, baby version
Stakes: role for policy?

Highly significant losses from incomplete markets. Potentially high

3. But: significant consumption inequality in period 2. Sig-

2. Aggregates just like the R-A model (cf. top-left corners).

1. Approximate aggregation (look down columns).

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Results (CRRA=3)
\[ \theta \text{ only on mean wealth and parameters } \theta \text{ for } \kappa \text{ that depends on } \kappa \text{ only on mean wealth and parameters.} \]

- Consumers are boundedly rational: they use a forecasting rule.
- Convolution of random variables drawn from known distributions.
- Let the mean and variance of the (lognormal) wealth distribution.
- Now illustrate how solution by simulation works in the 2-period model.
- The previous results are exact.

Assessing solution by simulation.
1. Draw simulated values for the variance of the \( \theta \) im-

2. Using the simulated data, estimate a new value for \( \theta \).

3. Iterate to convergence and assess the fit of the forecasting

\textbf{Solution by simulation:}

\begin{itemize}
  \item To pin down the parameters of the forecasting rule, use
\end{itemize}
simulation does not reflect a self-fulfilling prophecy.

In the two-period model, the equilibrium found by solution by

lation: the deviations nearly vanish (on the order of 0.001)%

also compare the exact solution to the solution by simu-

R² ≈ 0.99999 and residual std. dev. ≈ 0.04%.

In all cases, the fit of the forecasting rule is nearly perfect:

(average wealth Gini = 0.51).

Let the mean and std. dev. of wealth distribution vary by 10%
Savings Decision Rules
(bottom line = no risk; top line = high risk)

Tomorrow's individual capital

Indvidual wealth

0

1

2

3

4

5

æ

-.72

1.78

Savings Decision Rules (bottom line = no risk; top line = high risk)
Slopes of Savings Decision Rules

(top line = no risk; bottom line = high risk)
Tomorrow's Aggregate Capital vs. Today's Mean Wealth

Tomorrow's Aggregate Capital

Today's Mean Wealth

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2.1 | 4.5 |
Very small.

– total wealth moving between constrained and unconstrained

– total wealth among constrained close to zero; and

– people choose not to be constrained (because it hurts).

Here, in a long-horizon model, note, however: \( \omega \) and variances in it are free parameters.

If in addition \( \omega \) varies enough, further from aggregation.

With \( \omega \) high enough, many constrained.

Might borrowing constraints weaken those results?
Instead of savings policy in the 2-period setup, an example would be stabilization policy, but we will illustrate general ideas: prices may be altered to reduce de-facto risk, gains from intervention. When markets are incomplete, price effects can allow welfare it's effects on prices. Nevertheless may have an impact on social insurance through the social insurance role of macro policy.

Using the simple model to illustrate
Long-horizon model may thus call for investment subsidy. Initial wealth inequality case implies redistribution.

Thus, lower-than-equilibrium aggregate savings would raise ex ante welfare. Constrained inefficiency of equilibrium.

\[ \text{ex ante, consumer risk } \epsilon \text{ is scaled down.} \]

\[ \text{For up, } \omega \text{ down } \text{ex post, unlucky gain, lucky lose!} \]

\[ \text{Then investment tax. Suppose government induces decrease in savings (say, with an } \text{For simplicity: no initial wealth inequality } \implies \text{all save the same.} \]

Tracking cross-sectional effects of savings policy
Dogmatic market incompleteness.

On the other hand, this setup is decentralized and seems more descriptively accurate than available models with

...Kocherlakota proof.

A methodological complement on macro policy analyses.
$c_1 + k_2 = \mu \text{ and } c_2 = \bar{c}_2 + q + \eta.$

- Budgets:

- Ex post, and entrepreneurial risk cannot be marketed.

- Idiosyncratic technology: \( \xi \), \( \eta \), Lognormal capital, immobile.

Same 2-period model EXCEPT:

2. Entrepreneurial economy
rate much lower with idiosyncratic production risk.

2. However, aggregate losses are not like in the R-A model: risk-free

1. Approximate aggregation again.

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Entreprenurial economy

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Previous (1998) setup, for comparison
ment decisions will differ for this reason as well. Trade occurs.

The beliefs about the ε will differ across consumers, so invest-

They signal-extract and are rational Bayesian updaters.

- They see the price q, which partially reveals ε (and w).
- They do not know the income distribution I; they do not know it.
- They do not know its mean μ. Their own wealth, w, is a signal on it.
- People do not know ε, the average expected ϵ, but they get signal ϵ on it.

Assumptions:
New entrepreneurs inform. Build on entrepreneurial economy. New
entrepreneurs/consumers have net-
So: develop setting where entrepreneurs/consumers have net-

Suspicion: real-world trade and investments are explained more

Heterogeneous information in entrepreneurial economy

3. New Research:
5. Expand parametric class for beliefs if necessary.

4. Assess whether the assumed beliefs are close to accurate.

3. Draw aggregate shocks (ε, m) and simulate equilibria
find function of b.

2. Find optimal investment behavior given beliefs: decisions as
a specific parametric distribution, a parametric vector.

1. Assume beliefs ε̃ are (boundedly rationally) given by
Algorithm: solution by simulation again, as in earlier work. All-
Method: Models with partially revealed information and rational updating
are very hard to solve even numerically (even with 2 periods).

Ongoing work
Line of research can be made quantitative. From present and past prices, that is, with some luck also this formation in fully dynamic settings where agents draw inferences... It may make feasible the study of economies with asymmetric information.

...Thesis is reminiscent of, and appears related to, approximate aggregation.

These two features exactly?

? Being normal, than the resulting equilibrium (almost) confirm to be (i) linear in the Information (or linear) with (ii) the error

It seems that, for the economy studied, if beliefs are assumed...
Many exciting issues left to explore: how macro interacts with labor markets with frictions, credit markets with fric-
tions, etc.; this work is feasible.

● Our original 1998 model can now be solved in a matter of
minutes. Require this kind of training today.

● Much of the work uses numerical methods, but these are

● Evaluate policies for the cross-section of consumers.

● Use the models to understand the equilibrium determina-
tion of inequality and aggregate quantities and prices; and

● Structured aggregate models with realistic understandings.

● Use available insights from the applied literatures to con-

● Vibrant quantitative research area in the intersection of macro

Conclusions and comments