**Derivation of the Efficiency Wage Condition**

In class, we learned that the condition that determines the efficiency wage is that the marginal efficiency at that wage be equal to the average efficiency per wage. The derivation of this condition is quite straightforward and may build your intuition for the result.

Consider a monopolistic firm that maximizes profit by adjusting both quantity of labor and the wage rate. The only inputs into production are efficiency and number of workers. Formally, $Y = F(e(w)N)$, where $e(w)$ is efficiency, which is an increasing function of the wage rate. If the output price is normalized to one, the firm’s profit is given by:

$$\text{Profits} = F(e(w)N) - wN,$$

which is just revenue minus costs. Two conditions must be met for profit maximization: the first derivative of profits with respect to the wage must equal zero, and the first derivative with respect to labor must equal zero. The two first order conditions are then:

1. F.O.C w/r/t $w$: $F'(e(w)N)e'(w)N - N = 0$
2. F.O.C w/r/t $N$: $F'(e(w)N)e(w) - w = 0$

Condition (1) reduces to $F'(e(w)N) = 1/ e'(w)$, which we can plug into condition (2) to get, $w = e(w)/ e'(w)$, or $e'(w) = e(w)/w$, which is the efficiency wage condition we sought to derive. Again, the efficiency wage is chosen such that the marginal efficiency at that wage is equal to the average efficiency.