Economic Growth I

1. Economic Growth Facts
2. Growth Accounting
   - Hong Kong and Singapore revisited
   - The Productivity Slowdown
3. The Solow Growth Model
4. Steady-states
Economic Growth Facts

- **Fact 1:** There are huge differences in the levels of output across countries.
- Of the 6 billion people on this planet, perhaps 1 billion live in absolute poverty – barely able to survive from day to day.
- In contrast the richest 1 percent, living in the affluent North, garner about 20 percent of world income.
<table>
<thead>
<tr>
<th>country</th>
<th>population (millions)</th>
<th>GNP per capita (in U.S.$)</th>
<th>rank</th>
<th>televisions per 1,000</th>
<th>% of children under 5 who face malnutrition</th>
<th>under 5 mortality rate (per 1,000)</th>
<th>life expectancy at birth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Norway</td>
<td>5</td>
<td>$38,730</td>
<td>3</td>
<td>884</td>
<td>0</td>
<td>4</td>
<td>79</td>
</tr>
<tr>
<td>United States</td>
<td>288</td>
<td>$35,400</td>
<td>6</td>
<td>938</td>
<td>0</td>
<td>8</td>
<td>77</td>
</tr>
<tr>
<td>Brazil</td>
<td>174</td>
<td>$2,830</td>
<td>91</td>
<td>349</td>
<td>7</td>
<td>37</td>
<td>69</td>
</tr>
<tr>
<td>India</td>
<td>1049</td>
<td>$470</td>
<td>161</td>
<td>83</td>
<td>66</td>
<td>90</td>
<td>63</td>
</tr>
<tr>
<td>Burkina Fasso</td>
<td>12</td>
<td>$250</td>
<td>187</td>
<td>79</td>
<td>33</td>
<td>207</td>
<td>43</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>country</th>
<th>fertility rate (births per woman)</th>
<th>maternal mortality ratio (per 100,000 births)</th>
<th>% of children 10-14 in work force</th>
<th>annual deforestation (square kilometers)</th>
<th>carbon dioxide emissions (metric tons per capita)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Norway</td>
<td>1.8</td>
<td>6</td>
<td>0</td>
<td>-180</td>
<td>16.6</td>
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<tr>
<td>United States</td>
<td>2.1</td>
<td>8</td>
<td>0</td>
<td>-5,886</td>
<td>20.8</td>
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<tr>
<td>Brazil</td>
<td>2.4</td>
<td>160</td>
<td>15</td>
<td>25,544</td>
<td>1.6</td>
</tr>
<tr>
<td>India</td>
<td>2.9</td>
<td>437</td>
<td>13</td>
<td>-72</td>
<td>1.0</td>
</tr>
<tr>
<td>Burkina Fasso</td>
<td>6.3</td>
<td>930</td>
<td>48</td>
<td>320</td>
<td>0.1</td>
</tr>
</tbody>
</table>
• **Fact 2:** There are wide differences in growth rates across countries.

• **Subfact 2:** Small growth rate difference matter
  
  – The “rule of 69.”
  
  – What this “rule” states is that given a growth rate, you can find number of years it takes for incomes to double by dividing the growth rate by 69.
An Indian will, on average, be twice as well off as his/her grandfather. A South Korean on the other hand will be 32 times as well off as his/her grandfather.

These facts led Robert Lucas, a winner of the Nobel Prize to state:

I do not see how one can look at figures like these without seeing them as possibilities. Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia’s or Egypt’s? If so, what exactly? If not, what is it about the ‘nature of India’ that makes it so? The consequences from human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else.
Growth Accounting

Deriving the growth accounting equation

Begin the production function:

\[ Y_t = A_t F(K_t, N_t) = A_t K_t^\alpha N_t^{1-\alpha} \]

Take natural logs

\[ \ln Y_t = \ln A_t + \alpha \ln K_t + (1 - \alpha) \ln N_t \]  \hspace{1cm} (1)

Go back one period:

\[ \ln Y_{t-1} = \ln A_{t-1} + \alpha \ln K_{t-1} + (1 - \alpha) \ln N_{t-1} \]  \hspace{1cm} (2)

If we subtract equation (2) from equation (1) we get

\[ \ln Y_t - \ln Y_{t-1} = (\ln A_t - \ln A_{t-1}) + \alpha \ln K_t - \ln K_{t-1}) + (1 - \alpha)(\ln N_t - \ln N_{t-1}). \]
\[
\frac{\Delta Y_t}{Y_t} = \frac{\Delta A_t}{A_t} + \alpha \frac{\Delta K_t}{K_t} + (1 - \alpha) \frac{\Delta N_t}{N_t}
\]  

(3)

where

- \( \frac{\Delta Y_t}{Y_t} \) = rate of output growth
- \( \frac{\Delta A_t}{A_t} \) = rate of total factor productivity growth
- \( \frac{\Delta K_t}{K_t} \) = rate of capital growth
- \( \frac{\Delta N_t}{N_t} \) = rate of labor growth

\[ \alpha = \text{elasticity of output with respect to capital} \]
\[ 1 - \alpha = \text{elasticity of output with respect to labor} \]

- A rise of 10\% in total factor productivity \( z \) raises output by 10\%.
- A rise of 10\% in capital \( K \) raises output by \( \alpha \times 10\% \).
- A rise of 10\% in labor \( N \) raises output by \( (1 - \alpha) \times 10\% \).
Four steps to growth accounting

1. Get data of \( \frac{\Delta Y}{Y} \), \( \frac{\Delta K}{K} \), \( \frac{\Delta N}{N} \).

2. Use an estimate of \( \alpha \).

3. Calculate \( \alpha \frac{\Delta K}{K} \) and \( (1 - \alpha) \frac{\Delta K}{K} \).

4. Calculate the contribution of total factor productivity growth as the residual:

\[
\frac{\Delta A_t}{A_t} = \frac{\Delta Y_t}{Y_t} - \alpha \frac{\Delta K_t}{K_t} - (1 - \alpha) \frac{\Delta N_t}{N_t}
\]
### The productivity slowdown

<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Labor growth $\frac{\Delta N}{N}$</td>
<td>0.99</td>
<td>1.79</td>
<td>2.38</td>
<td>1.75</td>
<td>1.40</td>
</tr>
<tr>
<td>Capital growth $\frac{\Delta K}{K}$</td>
<td>3.29</td>
<td>3.66</td>
<td>3.53</td>
<td>2.67</td>
<td>2.51</td>
</tr>
<tr>
<td>TFP growth $\frac{\Delta A}{A}$</td>
<td>1.14</td>
<td>1.74</td>
<td>0.50</td>
<td>0.95</td>
<td>1.67</td>
</tr>
<tr>
<td>Output growth $\frac{\Delta Y}{Y}$</td>
<td>2.98</td>
<td>4.25</td>
<td>3.33</td>
<td>3.05</td>
<td>3.50</td>
</tr>
</tbody>
</table>

Table 2: Accounting for Economic Growth in the U.S.

Four explanations for the productivity slowdown.

1. Measurement
2. The Legal and Human Environment
3. Adoption of New Technology
4. Oil Prices
The Solow Growth Model

- Closed economy, single good produced each period, $Y_t$.
- There are two factors of production, labor, $N_t$ and $K_t$.
- Total factor productivity, $A$, is a constant.
- There is a constant returns-to-scale production function. In particular, we are going to assume that the production function is Cobb-Douglas:

\[
Y_t = AF(K_t, N_t) = AK_t^{1-\alpha} N_t^\alpha
\]

- We have the standard expenditure identity for a closed economy without government:

\[
Y_t = C_t + I_t.
\]
• The law of motion for capital is:

\[ K_{t+1} = (1 - \delta)K_t + I_t \]

for a given \( K_0 \).

• Let \( s \) be the fraction of income that is saved (invested) each period. so

\[ I_t = sY_t \]

and

\[ C_t = (1 - s)Y_t. \]

• So the law of motion of capital can be written as:

\[ K_{t+1} = (1 - \delta)K_t + sY_t. \]
• Assume $N$ grows over time:

$$N_{t+1} = (1 + n)N_t$$

• Let $y_t$ = per capital output. So $y_t \equiv \frac{Y_t}{N_t}$.

$$y_t = \frac{AF(K_t, N_t)}{N_t}$$

$$= AF\left(\frac{1}{N_t}K_t, \frac{1}{N_t}N_t\right)$$

$$= AF\left(\frac{K_t}{N_t}, 1\right)$$

$$= AF(k_t, 1)$$

$$= Af(k_t)$$

where $k_t$ is the capital-to-labor ratio.

• In the case of a Cobb-Douglas production function:

$$y_t = \frac{Y_t}{N_t}$$

$$= \left(\frac{1}{N_t}\right) AK_t^\alpha N_t^{1-\alpha}$$

$$= A \left(\frac{K_t^\alpha}{N_t^\alpha}\right) \left(\frac{N_t^{1-\alpha}}{N_t^{1-\alpha}}\right)$$

$$= Ak_t^\alpha$$
• The law of motion for the capital-to-labor ratio, $k_t$:

$$k_{t+1} = \frac{K_{t+1}}{N_{t+1}}$$

$$= \frac{(1 - \delta)K_t + I_t}{(1 + n)N_t}$$

$$= \frac{(1 - \delta)K_t + sY_t}{(1 + n)N_t}$$

$$= \frac{(1 - \delta)K_t + sAK_t^\alpha N^{1-\alpha}}{(1 + n)N_t}$$

$$= \frac{(1 - \delta)K_t}{(1 + n)N_t} + \frac{sA}{(1 + n)N_t^\alpha} K_t^\alpha$$

$$= \frac{(1 - \delta)k_t}{(1 + n)} + \frac{s}{(1 + n)}Ak_t^\alpha$$
Let's do some simulations

- Assign values to $\delta$, $s$, $n$, $A$, and $\alpha$. Given a starting value for $k$ we can simulate this economy.

- Let's use 1980 as a starting value. Use data from Table 3.1 on page 65. So

\[
k_{1980} = \frac{K_{1980}}{N_{1980}} = \frac{5569}{99.3} = 56.08
\]

- Set $A = TFP_{1980} = 14.75$. Set $\delta = 0.08$, $s = .12$, $n = 0.015$, and $\alpha = 0.3$

- Just to make sure you understand what we are doing we are setting $k_0 = 56.08$ and using the law of motion for $k_t$ we just derived to compute values for $k_1, k_2, k_3, ...$
**Simulations**

- After about 50 years the capital-to-labor ratio stops growing.
- Forget above doubling every 30. This model does not predict continued growth in per capita output.
- We do get sustained growth in total output.
Figure 1: Simulated time path of capital-to-labor ratio: alpha=0.3, n=0.015
Figure 2: Simulated time path for GDP per worker: alpha = 0.3, n=0.015
Figure 3: Simulated time path of GDP: alpha=0.3, n=0.015
Figure 4: A longer simulated time path of capital-to-labor ratio: alpha=0.3, n=0.015
Figure 5: A longer Simulated time path for GDP per worker: alpha = 0.3, n=0.015
Figure 6: A longer simulated time path of GDP: alpha = 0.3, n = 0.015
What’s going on here?

- Go back to our law of motion for $k_t$.

$$k_{t+1} = \frac{(1 - \delta)}{(1 + n)} k_t + \frac{sA}{(1 + n)} k_t^\alpha$$

- Note that when $k_t = 0$, $k_{t+1} = 0$.

- Take a derivative:

$$\frac{dk_{t+1}}{dk_t} = \frac{(1 - \delta)}{(1 + n)} + \frac{sA \alpha}{1 + n} k_t^{\alpha - 1}$$

- Since $1 - \alpha < 0$, we know when $k_t = 0$, $dk_{t+1}/dk_t = \infty$

- Also since $1 - \alpha < 0$, when $k_t = \infty$, $k_t^{\alpha - 1} = 0$. Thus $dk_{t+1}/dk_t = (1 - \delta)/(1 + n) < 1$. 

Steady-state

- To compute the steady state capital-to-labor ratio we need to do a little algebra:

\[ k^* = (1 - \delta)k^* + \frac{sA}{1 + n}k^{*\alpha} \]

\[ (1 + n)k^* = (1 - \delta)k^* + sAk^{*\alpha} \]

\[ (n + \delta)k^* = sAk^{*\alpha} \]

- This last equation is equation 6.10 in the book.

\[ (n + \delta)k^* = sAk^{*\alpha} \]

\[ \frac{n + \delta}{sA} = k^{*\alpha - 1} \]

\[ k^* = \left( \frac{n + \delta}{sA} \right)^{\frac{1}{\alpha - 1}} \]

Let get the exponent positive

\[ k^* = \left( \frac{sA}{n + \delta} \right)^{\frac{1}{1 - \alpha}} \]

- Growth does occur while \( k_t < k^* \).
- But eventually growth slows to zero.
Comparative statics/dynamics

- A change in \( s \).
- A change in \( \delta \).
- A change in \( n \).
- A one-time change in \( A \).
- Natural and unnatural disasters.
So neither growth in capital nor growth in labor can sustain output growth over a long period time. What’s left? $A$.

At this point you should be thinking there are decreasing marginal benefits to increasing capital and labor – but not $A$.

So it is going to turn out that growth in $A$ can get us sustained growth in output per worker.

But what the heck is $A$? We just computed it as a residual.