Economic Growth III

1. Preliminaries
2. Endogenous Growth: The AK Model
3. The Economics of Ideas
4. Corruption and Capital Accumulation
5. Taxes and Capital Accumulation
6. An Assessment of Growth Theory

Productivity isn’t everything, but in the long run run it is almost everything
Paul Krugman

**Endogenous Growth: The AK Model**

- The production function is the same as we have worked with throughout the course:
  \[ y_t = A_t k_t^\alpha \]  
  where
  \[ y_t = \text{output per worker} \]
  \[ k_t = \text{capital-to-labor ratio} \]
  \[ A_t = \text{total factor productivity} \]

- But in this model we assume that total factor productivity is endogenous and that it depends on the capital-to-labor ratio. Specifically:
  \[ A_t = Ak^{1-\alpha}_t \]

- Productivity might be affected by the capital-to-labor ratio for several reasons

- If substitute equation (2) into equation (1), we can rewrite the per-worker production function as:
  \[ y_t = Ak^{1-\alpha}_t^\alpha k_t^\alpha = Ak_t \]

  Now we multiple both sides of this equation by \( N_t \):
  \[ y_t N_t = Ak^\alpha_t N_t \]
  \[ Y_t N_t = A K_t N_t \]
  \[ Y_t = AK_t \]

- We have the standard expenditure identity for a closed economy without government:
  \[ Y_t = C_t + I_t \]

- The laws of motion for capital and labor are:
  \[ K_{t+1} = (1 - \delta) K_t + I_t \]
  for a given \( K_0 \).
  \[ N_{t+1} = (1 + n) N_t \]

- Let \( s \) be the fraction of income that is saved (invested) each period, so
  \[ I_t = s Y_t \]
  and
  \[ C_t = (1 - s) Y_t \]

- So the law of motion of capital can be written as:
  \[ K_{t+1} = (1 - \delta) K_t + s Y_t \]
  or
  \[ K_{t+1} = (1 - \delta) K_t + s AK_t \]

- If we divide both sides of the equation by \( N_t \) we get:
  \[ \frac{K_{t+1}}{N_t} = (1 - \delta) \frac{K_t}{N_t} + s A K_t \]
  \[ N_{t+1} \frac{K_{t+1}}{N_t} = (1 - \delta) \frac{K_t}{N_t} + s A K_t \]
  \[ (1 + n) \bar{k}_{t+1} = (1 - \delta) \bar{k}_t + s AK_t \]
  \[ \bar{k}_{t+1} = \frac{(1 - \delta)}{1 + n} \bar{k}_t + \frac{s}{1 + n} AK_t \]
We can re-write this expression as

\[ \frac{k_{t+1}}{k_t} = \frac{(1 - \delta) + sA}{1 + n} \]

This equation is the key. It shows that in the AK model, the capital-to-labor ratio (and hence output per worker) will grow steadily over time. To get things in terms of percentage changes, we subtract the number one from both sides of the equation:

\[ \frac{k_{t+1} - 1}{k_t} = \frac{(1 - \delta) + sA}{1 + n} - 1 \]

The AK model differs from the traditional Solow model in several key ways.

1. Growth in total factor productivity is determined within the model (endogenously).
2. In the Solow growth model, only exogenous productivity change can lead to long-run growth. In the AK model, per-worker output growth can continue forever—assuming the expression above is positive—without any exogenous changes in productivity.
3. In the Solow growth model, one time increases in the savings rate, \(s\), the population growth rate, \(n\), or total factor productivity, \(A\), lead to only temporary changes in the growth rate. In the AK model changes in these variables lead to permanent changes in the growth rate of output.
4. The AK model does not imply convergence.

The difference between objects (goods) and ideas

- First let's define some terms:
  - rival — one person’s use of the good precludes another person from using the good.
  - nonrival — one person’s use of the good does not preclude another person from using the good.
  - excludable — a person can prevent anyone from enjoying the benefit of the good.
  - nonexcludable — a person can not prevent anyone from enjoying the benefit of the good.

- Examples:
  - a public good — national defense — nonrival and nonexcludable
  - a private good — a pair of shoes — rival and excludable
  - a fish in the ocean — rival but largely non-excludable
  - an encoded satellite broadcast — nonrival but excludable

- In general
  - Objects are rival goods while ideas are non rival goods.
  - Ideas can be excludable through patent and copyright laws.

- Human capital, as traditionally defined, is both rival and excludable.

- However ideas such as those that come out of basic research and development are nonrival and nonexcludable.

The Economics of Ideas

But human knowledge or capital are catch-all phrases that are used describe both

- embodied skills, such as the ability to use a word processor or operate a piece of machinery, and

- disembodied knowledge, such as software code or the blue print for a machine.

But recently Paul Romer has emphasized the role of ideas as distinct objects separate from human capital. Romer defines ideas as “the instructions that let us combine limited physical resources in ways that are more valuable.” Romer uses the example of two children’s toys to make his point

1. The Play-Dough Fun Factory
2. The Chemistry set
3. A major improvement in super-conductors
4. Shirt assembly
Corruption and Capital Accumulation

- Question: Does corruption help or hurt economic growth? No much of a debate but...
  - It might help if...
    - "speed money" helps individuals avoid bureaucratic delays.
    - bureaucrats get paid for performance.
- But for corruption to help growth bureaucratic regulation have to cumbersome to begin with.

Taxes and Capital Accumulation

- Let
  \[ 1 + r = \text{required gross rate of return for putting off consumption.} \]
  \[ \tau = \text{tax rate on rental of capital.} \]
  \[ \text{MPK} = \text{marginal product of capital = rental price of capital} \]
- Suppose you are in the steady state and are considering investing one more dollar.
- Sacrifice today = $1.
- Future gains discounted back to today's dollars are:
  \[
  \text{Future gains} = \frac{\text{MPK}(1 - \tau)}{1 + r} + \frac{(1 - \delta)\text{MPK}(1 - \tau)}{(1 + r)^2} + \frac{(1 - \delta)^2\text{MPK}(1 - \tau)}{(1 + r)^3} + \ldots
  \]
  \[
  = \text{MPK}(1 - \tau) \left( \frac{1 + \beta}{1 + r} + \frac{(1 - \delta)^2}{(1 + r)^2} \right)
  \]
  Remember from high school math ...
  \[
  \frac{1}{1 - \beta} = 1 + \beta + \beta^2 + \beta^3 \ldots
  \]


- Data from Business International (now the Economist Intelligence Unit).
- He looks at three factors to develop a bureaucratic efficiency index
  1. Legal system – Judiciary – Efficiency and integrity of the legal environment as it affects business, particularly foreign firms.
  2. Bureaucracy and Red Tape – The regulatory environment foreign firms must face when seeking approvals and permits. The degree to which it represents an obstacle to business.
  3. Corruption – The degree to which business transactions involve corruption or questionable payments.

\[
1 + \frac{1 - \delta}{(1 + r)} + \frac{(1 - \delta)^2}{(1 + r)^2} + \ldots = \frac{1}{1 - \frac{(1 + r)}{(1 + r)}}
\]

\[
= \frac{(1 + r) - (1 - \delta)}{(1 + r)}
\]

\[
= \frac{r}{\delta}
\]

So

\[
\text{Future gains} = \frac{\text{MPK}(1 - \tau)}{1 + r} \left( \frac{(1 + r)}{r + \delta} \right)
\]

As usual, we assume a Cobb-Douglas production function:

\[
\text{MPK} = \frac{\partial Y}{\partial K} = \alpha AK^{\alpha-1}N^\beta
\]

So

\[
\text{Future gains} = \frac{\alpha AK^{\alpha-1}N^{1-\alpha}(1 - \tau)}{r + \delta}
\]
Since marginal cost of increased investment must equal the marginal benefit:
\[ 1 = \alpha AK^{\alpha-1}N^{1-\alpha}(1 - \tau) \]
\[ \frac{r - \delta}{\alpha A(1 - \tau)} = k^{\alpha-1} \]
\[ \frac{r + \delta}{\alpha A(1 - \tau)} \]
\[ \left( \frac{r + \delta}{\alpha A(1 - \tau)} \right)^{\frac{1}{\alpha}} = k \]
\[ y = Ak^\alpha \]
\[ = A \left( \frac{\alpha A(1 - \tau)}{r + \delta} \right)^{\alpha/(1 - \alpha)} \]
\[ = (1 - \tau)^{\alpha/(1 - \alpha)} A \left( \frac{\alpha A}{r + \delta} \right)^{\alpha/(1 - \alpha)} \]

So what is the big deal?
- Let say \( \alpha = 1/3 \), and \( \tau = 1/3 \) then
\[ (1 - \tau)^{\alpha/(1 - \alpha)} = (1 - 1/3)^{1/3} \]
\[ = \left( \frac{2}{3} \right)^{1/3} \]
\[ = \frac{\sqrt[3]{2}}{\sqrt[3]{3}} \]
\[ \approx 0.82 \]

- If \( \alpha = 2/3 \) then
\[ (1 - \tau)^{\alpha/(1 - \alpha)} = (1 - 1/3)^{2/3} \]
\[ = \left( \frac{2}{3} \right)^{2/3} \]
\[ = \frac{\sqrt[3]{4}}{\sqrt[3]{9}} \]
\[ \approx 0.44 \]

But before you start sending letters to Congress ...
- The government has to raise revenue somehow – by taxing labor, capital, or consumption. Each of the taxes cause distortions.
- Lump-sum taxes do not cause distortions, but they can cause riots (ask Margaret Thatcher).
- Have to run the same basic experiment with labor and consumption taxes.
- You may also care about the distribution of income.

An assessment of growth theory
Recall the stylized facts of economic growth. What did we hit?
1. There are huge differences in the levels of output across countries.
2. There are wide differences in growth rates across countries.
3. Output per-capita shows continuing growth
4. The growth rate of output is not fully explained by growth factor of inputs. Growth accounting leaves a residual.
5. Across countries, how much a country grows on average over time shows no correlation with how rich or poor the country is. That is, it is not the case that poor countries grow faster than rich countries or vice-versa.