The Classical Model of Output Determination

A large income is the best recipe for happiness I ever heard of

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Preliminaries

- The classic dichotomy.
- The analysis we do today will lay the groundwork for thinking about issues like:
  - What determines how much a country can produce?
  - Who gets the income from production? How much goes to compensate workers, and how much goes to compensate the owners of capital?
  - Why do some counties grow? What determines the rate of economic growth?
  - How fast can a country grow without sparking inflation?
  - What determines the productivity of inputs?
The Production Function

\[ Y = AF(K, N) \]  

\( Y \) = real output (real GDP)  
\( K \) = the stock of capital  
\( N \) = total worker hours  
\( A \) = total factor productivity

\( Y, K \) and \( N \) are observable. \( A \) can be thought of as a residual number calculated from the observed values of \( Y, K, \) and \( N \) and an assumed form for the function \( F. \)
**What makes a good function** \( F(K,N) \)?

1. It should be increasing in \( K \) and \( N \)

2. It should exhibit decreasing returns to increases either \( K \) or \( N \) alone.

3. Complementarity: increasing \( K \) should increase the marginal product of \( N \), and increasing \( N \) should increase the marginal product of \( K \).

4. Constant returns to increasing \( K \) and \( N \) together.

5. Factor shares are constant.
   - Factor share of labor
     \[
     \frac{\text{wage} \times N}{AF(K,N)} \approx .70
     \]
   - Factor share of capital
     \[
     \frac{\text{rental price of capital} \times K}{AF(K,N)} \approx .30
     \]
The Cobb-Douglas Production Function

\[ F(K, N) = K^\alpha N^{1-\alpha} \text{ where } 0 < \alpha < 1 \]  \hspace{1cm} (2)

1. Since \( \alpha \) and \( 1 - \alpha \) are greater than zero, \( F \) is increasing in \( K \) and \( N \).
2. Since \( \alpha \) and \( 1 - \alpha \) are less than one, decreasing returns to increasing \( K \) and \( N \) alone.
3. Complementary holds.
4. Since \( \alpha + (1 - \alpha) = 1 \), increasing both \( K \) and \( N \) by \( x\% \) increase \( F \) by \( x\% \).
5. Factor shares are constant. Share of labor equals \( 1 - \alpha \). Share of capital equals \( \alpha \).

Let’s note a few other things:

- \( y = Y/N \) is called average labor productivity.
- \( Y/K \) is called average capital productivity.
Interpreting $A$

Since $Y$, $K$, and $N$ are observed, you can impute $A$ from

$$A = \frac{Y}{K^\alpha N^{1-\alpha}}$$

(3)

You can also take logs

$$\log A = \log(Y) - \alpha \log K - (1 - \alpha) \log N$$

(4)

The number $A$ itself has little meaning since it depends on, among other things, the units $K$ and $N$ are measured in.

Percent differences in $A$ have a direct interpretation either

- across countries
- across time

The assumption of the Cobb-Douglas production function above allows us to compare the total factor productivity across countries or time periods with different $K$ and $N$ values.
What could explain persistent differences in $A$ across countries?

- Are capital and labor in the right places?
- Are efficient production technologies being used?
- Are good management decisions being made?
- international trade – are tariffs forcing a country to produce goods it is not very efficient at producing?

Once can consider $A$ to move to temporary *supply shocks* such as:

- an innovation in a production technique
- new inventions
- weather or natural disasters
- changes in prices of imported raw materials
Labor Demand

• Assumptions
  – hold capital stock fixed
  – workers are all alike
  – labor market is competitive
  – firms maximize profits

• Analysis at the margin.
  – A firm compares the costs and benefits of hiring one extra worker.
  – If the real wage \( w \) is greater than the marginal product of labor \( MPN \), the firm is paying the marginal worker more than the worker produces, so a profit-maximizing firm will reduce the number workers to increase profits.
  – If \( w < MPN \), the marginal worker produces more than he or she is being paid, so a profit maximizing firm will increase the number of workers.
Mathematically, the firm’s problem is:
\[
\max_{N} AF(K, N) - wN
\]

Cobb-Douglas production function.
\[
\max_{N} AK^\alpha N^{1-\alpha} - wN
\]

The first order condition for a maximum is
\[
(1 - \alpha)AK^\alpha N^{-\alpha} - w = 0
\]
or
\[
w = (1 - \alpha)AK^\alpha N^{-\alpha}
\]
or
\[
w = (1 - \alpha)A \left(\frac{K}{N}\right)^\alpha
\]
or
\[
w = (1 - \alpha)Ak^\alpha
\]
where \(k \equiv \frac{K}{N}\) that is the capital-to-labor ratio.

The labor demand schedule specifies how many units of labor (such as man-hours) firms would hypothetically like to hire at any given prevailing wage.

The labor demand curve is the MPN curve.

Labor demand schedule slopes downward.
• An outward shift in the labor demand curve means that firms wish to hire more workers than previously at every hypothetical wage level.

Labor demand shifts out when

• A increases
• K increases

Labor demand shifts in when

• A decreases
• K decreases
Labor Supply - The Income-Leisure Tradeoff

• Utility depends on consumption, \( c \) and leisure, \( l \):

\[
\text{Utility} = U(c, l)
\]

• Need to compare the costs and benefits of working an additional day.
  – Cost: loss of leisure time
  – Benefit: more consumption, since income is higher

• Resources of an individual and prices the individual faces:

\[
\begin{align*}
a & = \text{real non-labor wealth} \\
L & = \text{total hours in a day} \\
w & = \text{real wage}
\end{align*}
\]

• So the budget constraint of this individual is

\[
c \leq a + w(L - l)
\]
A digression on indifference curves

- We can graph a person’s preferences for consumption and leisure using indifference curves.
- An indifference curve shows combinations of $c$ and $l$ that yield the same utility.
- So this person is equally happy at any point on an indifference curve.
- Indifference curves have three important properties.
  - Slope downward from left to right: less consumption requires more leisure to keep utility unchanged.
  - Indifference curves that are high and to the right represent higher levels of utility, because more is preferred to less.
  - Indifference curves are bowed toward the origin. You prefer a little bit of everything to a lot of just one thing.
• The optimal choice occurs where the marginal rate of substitution - the tradeoff between consumption and leisure – equals the real wage.

• The value of the extra consumption to the person from working a little more just has to be equal to value of the lost leisure that it takes to generate the consumption.

• The real wage, \( w \), is the amount of consumption that this person can purchase if she gives up an hour of leisure.
Comparative Statics

• An increase in non-labor wealth, $a$
  – For most people, they would probably demand more leisure as well as more consumption. In other words, leisure is a normal good for most people.

• An increase in the wage rate, $w$
  – When the wage rate rises, leisure become more expensive, which by itself leads people to want less of it. (the substitution effect).
  – Since leisure is a normal good, we would then predict that an increase in the wage rate would lead to a decrease in the demand for leisure – and thus an increase in the supply of labor.
  – A normal good must have a negatively sloped demand curve. If leisure is a normal good, then the supply curve of labor must be positively sloped.
  – But things aren’t quite that simple...
Income and Substitution Effects

- At an intuitive level, it may not seem reasonable that an increase in the wage rate would *always* lead to an increase in the supply of labor. If my wage becomes very high I might spend some of my extra income consuming leisure.
- Could get a backward bending supply curve for labor.
- There are two effects – an *income effect* and a *substitution effect*.
- The income effect is the tendency of workers to supply less labor in response to greater wealth.
- The substitution effect is the tendency of workers to supply more labor in response to a higher real wage.
An increase in the real wage shifts the budget constraint. The kink in the constraint remains fixed. The real wage has increased so the relative cost of leisure has risen relative to consumption. The substitution effect captures the movement along the indifference curve in response to the increase in the real wage. The relative cost of leisure has increased so the relative price of leisure has risen or fall, because of the opposite substitution and income effects. The income effect is the movement from one indifference curve to another holding the real wage constant.
• Note an increase in non-wage income, \( a \), has a pure income effect, with no substitution effect.

  – Most people demand more leisure. In other words, leisure is a normal good for most people.

• In the case of an increase in the real wage, \( w \), which effect will dominate – the income effect or the substitution effect? This an empirical issue.

• In general, as countries become wealthier and their citizen earn higher wages, their citizen supply less labor.

  – Backward-bending labor supply curve.

• But in the short run the substitution effect dominates. As we said, this implies an upward sloping supply curve for labor.
The aggregate labor supply curve

- The aggregate labor supply curve is the sum of the labor supplied by everyone in the economy.

- The aggregate labor supply schedule specifies how many units of labor households would hypothetically wish to provide at any given prevailing current wage, holding all other variables constant.

- Aggregate labor supply slopes upward. In general, workers are willing to supply more man-hours at a higher real wage.

- A main determinant of labor supply is demographics (working age population, immigration, fertility)

- An outward shift in the labor supply curve means that households wish to provide more labor than previously at every hypothetical wage level.
The labor supply curve shifts out when:

- lifetime wealth decreases (a lot like a change in $a$)
- increase in the working age population

The labor supply curve shifts in when:

- lifetime wealth increases
- decrease in the working age population

Note that one possible source of an increase in lifetime wealth is an increase in a worker’s expected future wages.
Labor market equilibrium ignoring unemployment

The hours demanded by firms and the hours workers supply reach equality at the intersection of the labor demand and labor supply curve denoted \((\bar{w}, \bar{N})\). Why?

- If \(w < \bar{w}\) firms wish to hire more workers than are willing to work. Firms bid up the real wage \(w\) to attract workers.

- If \(w > \bar{w}\) firms find more workers willing to work than firms care to hire. Jobseekers drive down the real wage.

Upshot: the labor market moves toward equilibrium \((\bar{w}, \bar{N})\).
**Full Employment Output**

\[ \bar{Y} = AF(K, \bar{N}) \]  

where \( \bar{N} \) is the market clearing quantity in labor market.

- *Key Assumption* \( N \) is determined today. \( K \) is determined in the previous time period. That is, it takes time to build machinery and structures.

  **This assumption is maintained throughout the course.**

- We are going to use this concept of full employment to derive a “full-employment” curve or FE curve during the second half of the course.

- Anything that changes \( \bar{N} \) will change \( \bar{Y} \)

- What’s the effect of a supply shock (a change to \( A \)) on \( \bar{N} \) and \( \bar{Y} \)?

  1. A supply shock raises or lowers output directly by raising or lowering the productivity of capital and labor.

  2. A supply shock also raises or lowers the demand labor. It thus changes \( \bar{N} \). This also either raises or lowers \( \bar{Y} \).
Frictional Unemployment

- Problem with supply and demand model above. No worker who wishes to work another hour at the prevailing wage $\bar{w}$ is unable to find an employer. Unemployment (defined as people looking for work but not working) would be zero!

  - How can this be?

  - Supply and demand is a useful way of thinking about the determination of $\bar{w}$ and $\bar{N}$ but doesn’t tell the whole story.
Example Landlords and tenants

- Search is an important economic activity.
- With constant turnover, there will always be a percentage of apartments that are vacant even though the landlords are willing to rent them at the going rental rate.
- This shows up in almost every market we use supply and demand to analyze.
- In labor markets, such unemployment is called *frictional unemployment*. 
Structural Unemployment

• The *structural unemployed* are the chronically unemployed. These are unemployed people without the skills to get and hold a job.

• Often these people are not counted as unemployed since they become discouraged and drop out of the labor force all together.

• Can all unemployment be explained as frictional or structural? Hard to argue that the Great Depression was just frictional and structural unemployment.
The Natural Rate and Cyclical Unemployment

- Define the *natural rate of unemployment* as the level of unemployment when output and employment are at their full-employment levels.
  - It reflects unemployment that is due to just frictional unemployment and structural unemployment.
  - If we denote the unemployment rate by $u$, then the natural rate is $\bar{u}$ and is the level of unemployment associated with $\bar{N}$ and $\bar{Y}$.

- This is a controversial subject.

- Define *cyclical unemployment* is just the difference between the actual unemployment rate and the natural rate.
Okun’s Law

- Not a law - it is shorthand formula for approximating the cyclical relationship of cyclical unemployment and real cyclical GDP.

\[
\frac{\bar{Y} - Y}{\bar{Y}} = 2.5 \times (u - \bar{u})
\]

Why is the coefficient 2.5 instead of 1?
- Other things happen when cyclical unemployment rises.
  - labor force declines (discouraged workers)
  - hours of work per worker declines
  - average productivity of workers declines

In computer models of the economy used for forecasting or investing, many programs have a version of Okun’s law embedded into the program. Often see it mentioned in monetary policy discussions.