CONSUMPTION AND SAVING IN A CLOSED ECONOMY

1. The consumption-saving decision
2. The relative price of consumption today versus consumption tomorrow
3. Inter-temporal choice: Irving Fisher
   - graphically
   - mathematically
4. Permanent income hypothesis: Milton Friedman
5. Ricardian equivalence: Robert Barro

Consumption is the sole end and purpose of all production

Adam Smith
CONSUMPTION AND SAVINGS

- Recall

\[ S_{pvt} = (GDP + NFP - T + TR + INT) - C \]  

- personal disposable income - consumption.
- GDP: Gross Domestic Product
- NFP: Net Factor Payments from Abroad
- T: Taxes Paid to the Government
- TR: Transfers Received from the Government
- INT: Interest Payments on the Government Debt

- So by definition, disposable income must be either spent on consumption or saved.
There are two common meanings to the word saving:

1. refraining from consuming all of current output now in order to produce and consume more in the future.

2. the acquisition of new assets.

These two meanings are consistent.
• What determines the proportion of income that is consumed by a given household?

• Recall Keynesian consumption function from Econ 116

\[ C = c_0 + c_Y Y \]

• Is current income the whole story?

• In general, most economic analysis of spending decision focus on two factors: prices and wealth. At what rate can households trade-off between goods, and how much in total can they buy.

• For the consumption-saving decision, these factors are:
  – the price of consuming today as opposed to the future
  – overall lifetime wealth
Interest Rates

- Nominal interest rate denoted $i$
- The inflation rate is denoted $\pi$
- Approximate formula for realized real interest rate $= i - \pi$.
- Exact formula for realized real interest rate $= \frac{1+i}{1+\pi} - 1$.
- Why is $i - \pi$ a good approximation? Well, because $\log(1 + x) \approx x$.

Difference between realized and expected real interest rate

- Expected Real Interest Rate is denoted $r$.
- Expected Inflation Rate is denoted $\pi^e$.
- Approximate formula for expected real interest rate, $r = i - \pi^e$.
- Exact formula for expected real interest rate $r = \frac{1+i}{1+\pi^e} - 1$. 
The price of consumption today versus the future

- Ignoring taxes, the variable which determines *how much* future consumption is given up by consumption today is the expected real interest rate.

- Allowing for taxes, the actual relevant variable which determines how much future consumption is given up by consumption today is the expected *after-tax* real interest rate.

- From before, the expected real interest rate is:

\[ r = \frac{1 + i}{1 + \pi^e} - 1 \]  

or

\[ r = i - \pi^e \]
A Digression on Present Values

- Present value is the value of payments to be made in the future in terms of today’s dollars or goods.
- Example: At an interest rate of 10%, $12,000 today invested for one year is worth $12,000 \times 1.10 = $13,200. So the present value of $13,200 in one year is $12,000 (if the interest rate is 10%).
- In general,

\[
P V = \frac{\text{future value}}{1 + r}
\]
INTER-TEMPORAL CHOICE: IRVING FISHER

- Assumptions
  1. no taxes,
  2. no money, so no inflation
  3. no uncertainty

- Consider an environment in which agents live for two periods:
  - in period 1 they are young
  - in period 2 they are old
• We assume an individual gets utility from consumption \( (C_t) \) in each period. He or she is an utility maximizer so he or she wishes to:

\[
\max_{C_1,C_2} U(C_1, C_2)
\]

• For now we assume he or she can borrow and lend freely across the two periods at an interest rate \( r \). The agent faces the following budget constraint:

\[
C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}
\]

• Note everything in this equation is units of “goods in period 1”. The term \( \frac{1}{1+r} \) is relative price of goods between the two periods and converts the units from the second period into the first.

• This budget line equates the present value of lifetime consumption (PVLC) to the present value of lifetime resources (PVLR).
The agent’s constrained maximization problem

So this individual agent maximizes his/her utility over $C_1$ and $C_2$ given the budget constraint. Let’s think about the problem both graphically and mathematically.

- Where is the optimal level of consumption?
- Measured private saving is

$$S_{pvt} = Y_1 - C_1$$

- If the agent choose a point where

$$C_1 < Y_1,$$

we say that person is a lender.

- If the agent choose a point where

$$C_1 > Y_1,$$

we say that person is a borrower.
THE EFFECT OF CHANGES IN INCOME AND INTEREST RATES ON CONSUMPTION AND SAVING

The effect on consumption of a change in income (current or future) or interest rates depends only on how the change affects the present value of lifetime resources (PVLR).

- An increase in current income ($Y_1$)
- An increase in future income ($Y_2$)
- A rise in the interest rate ($r$)
SOLVING THE AGENT’S PROBLEM MATHEMATICALLY

The individual’s problem once again is:

\[
\max_{C_1, C_2} U(C_1, C_2)
\]

subject to:

\[
C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}
\]

To solve the problem we substitute the constraint into the objective function so agent’s problem becomes:

\[
\max_{C_2} U \left( Y_1 + \frac{Y_2}{1+r} - \frac{C_2}{1+r}, C_2 \right)
\]

We take the first derivative with respect to \( C_2 \):

\[
U_1(C_1, C_2) \left( \frac{-1}{1+r} \right) + U_2(C_1, C_2) = 0
\]
where $U_1$ is derivative of $U$ with respect to the first argument, and where $U_2$ is derivative of $U$ with respect to the second argument.

Rearrange terms:

$$\frac{U_1(C_1,C_2)}{U_2(C_1,C_2)} = 1 + r$$

So the agent chooses $C_1$ and $C_2$ such that the marginal rate of substitution between consumption today for consumption tomorrow is equal to the interest rate $1 + r$. 
AN EXAMPLE WITH A SPECIFIC UTILITY FUNCTION

- Set $U(C_1, C_2) = \ln(C_1) + \beta \ln(C_2)$ where $\beta < 1$
- Why is $\beta < 1$?
  - We assume agents discount future consumption at the rate $\beta$. That is, they prefer current consumption to future consumption all other things held equal.
- We write the agent’s problem as
  \[
  \max_{C_1, C_2} \ln(C_1) + \beta \ln(C_2)
  \]
  subject to:
  \[
  C_1 + \frac{C_2}{1 + r} = Y_1 + \frac{Y_2}{1 + r}
  \]
HOW DO WE SOLVE THIS PROBLEM?

1. Substitute the constraint into the objective function so the problem becomes:

\[
\max_{C_2} \ln \left( Y_1 + \frac{Y_2}{1+r} - \frac{C_2}{1+r} \right) + \beta \ln(C_2)
\]

2. Take the derivative with respect to \( C_2 \):

\[
\frac{1}{Y_1 + \frac{Y_2}{1+r} - \frac{C_2}{1+r}} \left( \frac{-1}{1+r} \right) + \frac{\beta}{C_2} = 0
\]

3. Rearrange terms

\[
\frac{1}{Y_1 + \frac{Y_2}{1+r} - \frac{C_2}{1+r}} \left( \frac{1}{1+r} \right) = \frac{\beta}{C_2}
\]

4. Cross-multiply

\[
\frac{C_2}{1+r} = \beta \left( Y_1 + \frac{Y_2}{1+r} - \frac{C_2}{1+r} \right)
\]
5. Collect the $C_2$ terms

\[ \frac{1 + \beta}{1 + r} C_2 = \beta \left( Y_1 + \frac{Y_2}{1 + r} \right) \]

6. Solve for $C_2$

\[ C_2 = (1 + r) \left( \frac{\beta}{1 + \beta} \right) \left( Y_1 + \frac{Y_2}{1 + r} \right) \]

7. Using the budget constraint to solve for $C_1$ yields

\[ C_1 = \left( \frac{1}{1 + \beta} \right) \left( Y_1 + \frac{Y_2}{1 + r} \right) \]

8. Saving in this model economy is

\[
S = Y_1 - C_1 \\
= Y_1 - \left( \frac{1}{1 + \beta} \right) \left( Y_1 + \frac{Y_2}{1 + r} \right)
\]
So what’s the point of all this math?

- We just derived consumption and saving functions.
- Note consumption in both periods depend on PVLR.
- Really the agent’s problem boils down to how to split total lifetime income between the two periods.
- If $S > 0$, this person is a lender, $S < 0$, this person is a borrower.
- Note that for ln utility

$$\frac{C_2}{C_1} = \frac{(1 + r) \left( \frac{\beta}{1+\beta} \right) \left( Y_1 + \frac{Y_2}{1+r} \right)}{\left( \frac{1}{1+\beta} \right) \left( Y_1 + \frac{Y_2}{1+r} \right)} = \beta(1 + r)$$
• So if

\[ \beta > \frac{1}{1 + r} \]

the agent consumes more in the second period than in the first period.

• If

\[ \beta < \frac{1}{1 + r} \]

the agent consumes more in the first period than in the second period.

• If

\[ \beta = \frac{1}{1 + r} \]

the agent consumes equal amounts in both periods.
• Increases in either $Y_1$ or $Y_2$ increase consumption both periods

• How does a change in interest rate effect the agent’s consumption?
  – Second period consumption becomes cheaper relative to first period consumption. Consumption in the first period goes down. Consumption in the second period goes up.
PERMANENT INCOME HYPOTHESIS: MILTON FRIEDMAN

- For a given interest rate, households do not directly care what their current income is when deciding on consumption but spend out of their total lifetime wealth. In other language, you consume out of your permanent income not your transitory income.

- The PIH states that matter is the present value of lifetime resources that matter, not the timing. Furthermore changes in permanent income get smoothed out over the entire lifetime.

- The mathematical example we solved illustrates the PIH; consumption is function of total permanent income - not just current income.
**Example: Stock Market Crash of 1987.**

- October 19, 1987, S&P 500 fell 20% in one day. Already down 16% since August.
- $1 trillion drop in financial wealth
- PIH suggests consumers who want to spread the effects of this loss out over their lifetimes.
- Back of the envelop calculation: Assume $r = 0$. Assume consumers want to reduce their consumption in each of the next 25 years.
- $\frac{1}{25} \times $1 trillion = $40 billion
- About what we saw – no crunch from the crash.
When does the Permanent Income Hypothesis possibly not hold?

- Borrowing constraints.
- Short-sightness
- Are you this rational?
Suppose we add a government to our two-period example. This government spend G amount in goods each period. So the present value of all government spending is

\[ G + \frac{G}{1+r}. \]

So the government has to raise the revenue to cover these expenditures. Suppose the government has three options:

1. Raise just enough taxes each period to cover current period expenditures (balance-budget)
2. Raise all the taxes during period 1 to cover the expenditures in both periods. (surplus)
3. Raise all the taxes during the period 2 to cover the expenditures in both periods. Borrow in period 1. (deficit spending)
Will these three financing arrangements have different effects on the agent’s consumption patterns? Rather than do the math, let’s think through the intuition.

- Taxes just reduce the agent’s income $Y_1$ and/or $Y_2$.
- The agent’s spending decisions, $C_1$ and $C_2$, are functions solely of her PVLR.
- All three of these arrangement are going to reduce the PVLR by the same amount.
- So he or she doesn’t care which one of these arrangement’s the government uses.

So in a world where Ricardian equivalence holds, a balance budget amendment will have no effect on the consumption decisions of the citizens.
When will Ricardian Equivalence not hold?

- Depends on out far out is period 2? Are you going to be around? Do you have a different time horizon than the government?
  - If people don’t care about period 2.
    - old, childless
    - happy to push the tax burden on the next generation
  - Can’t leave bequests
  - Borrowing/lending constraints
  - Taxes aren’t lump-sum
  - people don’t have perfect foresight

- Note that if PIH doesn’t hold, then Ricardian Equivalence doesn’t either.