SAVING AND INVESTMENT IN A CLOSED ECONOMY

1. Preliminaries
2. Timing
3. What pins down the real interest rate?
   • Irving Fisher’s Two-period Capital Theory
   • Extend to more than two periods
4. Back to the real world: what is $r$?
5. Tobin’s $q$
6. Equilibrium in the goods market (or saving market)
7. Why is investment so volatile?
8. Fiscal policy and crowding out

TIMING

• Assume that in order to create capital next year (or next quarter, or whatever time frame we are using), resources must be used up this year (or quarter, or whatever). Again, this is an assumption we will maintain throughout the course.
• Intuition: The assumption that it takes time to set up capital is crucial. If benefits come later, but the costs are paid now, buyers of capital (“investors”) will desire to invest less when interest rates are high.

BUT WHAT PINs DOWN THE REAL INTEREST RATE?

• We return to our two-period model from last time, but today we are going to add production to the model.
Recall our agent lives for two periods: 1 and 2. This agent gets utility from consumption in each period:
$$\max_{C_1, C_2} U(C_1, C_2)$$
• But let’s assume for the time being that instead of being handed income each period, this agent has access to a production function:
$$AF(K)$$
Assume the agent comes into this world with a certain a given capital stock ($K_1$). The agent produces $AF(K_1)$ stuff during the initial time period. Of that stuff she can either eat it or save it.

• In particular, assume $AF(K) = AK^\alpha$
There are two ways to interpret this production function.
1. The production function is Cobb-Douglas, and we set $N=1$,
2. We are thinking about a per-capita output
$$\frac{Y}{N} = \frac{AK^\alpha N^{1-\alpha}}{N} = AK^\alpha N^{-\alpha} = A \left( \frac{K}{N} \right)^\alpha = Ak^\alpha$$
where $k$ is the capital to labor ratio. In this second case we want to think of $C$ as representing per capita consumption and $K$ as representing the capital to labor ratio.
• So in period 1, the agent’s budget constraint is:
  \[ C_1 + K_2 = AF(K_1) \]
• In period 2, there is no saving decision. The agent just eats everything he or she produces:
  \[ C_2 = AF(K_2) \]
• So the agent’s problem is
  \[
  \max_{C_1, C_2} U(C_1, C_2)
  \]
  subject to:
  \[ C_1 + K_2 = AF(K_1) \]
  \[ C_2 = AF(K_2) \]
• Relative price of \( K_2 \) to \( C_1 \) is 1. That is, the price of capital \( p_K \) is 1.

How do we solve this problem?
1. Substitute the constraints into the utility function:
  \[
  \max_{K_2} U(AF(K_1) - K_2, AF(K_2))
  \]
  The choice variable is now \( K_2 \) which is the saving from period 1 to period 2.
2. Get the first-order necessary condition for a maximum by taking a derivative with respect to \( K_2 \):
  \[
  U_1(C_1, C_2)(-1) + U_2(C_1, C_2)AF'(K_2) = 0
  \]
3. rearrange terms
  \[
  \frac{U_1(C_1, C_2)}{U_2(C_1, C_2)} = AF'(K_2)
  \]

This pins down the interest rate
• recall from last class that we derived:
  \[
  \frac{U_1(C_1, C_2)}{U_2(C_1, C_2)} = 1 + r
  \]
• We can also solve the one period problem of a firm maximizing the value of production (in terms of goods).
  \[
  \max_K AF(K) - (1 + r)K
  \]
  This implies
  \[
  AF'(K) = 1 + r
  \]

**MULTI-PERIOD THEORY OF CAPITAL**

• Assume that agent’s problem goes on forever. It is no longer just a two period problem. Furthermore assume that capital does not completely depreciate each period, but only the fraction \( \delta \) wears out each period.
• Consider the problem of someone in the business of renting out capital. Let’s weight the costs and benefits.
• Assume the marginal cost of purchasing additional dollar of capital today is 1 (i.e. \( p_K = 1 \))
• Marginal benefit of additional dollar of capital
  – the benefits aren’t going to show up until next period
  – direct benefit next period \( \frac{MPK}{1+\delta} \)
  – plus benefit from selling next period \( \frac{(1-\delta)(1+\delta)}{1+\delta} \).
Setting marginal benefit equal to marginal cost:

\[ 1 = \frac{MPK_{t+1}}{1 + r_t} + \frac{1 - \delta}{1 + r_t} \]

So

\[ 1 + r_t = MPK_{t+1} + (1 - \delta) \]

**User Cost of Capital**

- Rewrite the previous equation as:
  \[ MPK_{t+1} = r_t + \delta. \]
- We call the right hand side the user cost of capital.
  - See equation 4.3 on page 129 of Abel and Bernanke. Recall in this example the price of capital \((pK)\) is equal to 1.
- It is the real cost of using a unit of capital for one period.
- The utility maximizing level of capital is the level such that the marginal product of capital equals the user cost. (i.e. marginal benefit equals marginal cost).

**From Desired Capital to Investment**

- But this is in \((K_{t+1}, 1 + r)\) space. The book’s graphs are in \((I_t, 1 + r)\) space.
- Abel and Bernanke use the following equation to get things into \((I_t, 1 + r)\) space.
  \[ K_{t+1} = (1 - \delta)K_t + I_t \]
  This is the law of motion for capital. They then rewrite this equation in the following form:
  \[ I_t = K_{t+1} - (1 - \delta)K_t \]
  So now the equation says investment is how much of next year’s capital stock is bought this year as opposed to being left over from previous years.
- Let’s define two objects:
  - **gross investment** new capital \((I_t)\)
  - **net investment** gross-investment minus depreciation \((I_t - \delta K_t)\)

**Back to the Real World: What is \(r\)?**

What is the interest rate? What is the Marginal Product of Capital?

- Three month Treasury bill rate? Return on the stock market?
- So why does our model have a single interest rate, but in the real world there are lot of interest rates?
- In our economy there is no uncertainty. In the real world, there is a lot of uncertainty, and there are a lot of different types of uncertainty? default risk, inflation risk, labor income risk.
- The theory of finance ultimately comes down to this equation
  \[ \frac{U'(C_t)}{U'(C_{t+1})} = AF'(K_{t+1}) + (1 - \delta) = 1 + r_t \]
- Macro and finance closely intertwined.
**Tobin’s q**

- Tobin’s q is: $q = \frac{V}{p_K K}$ where
  - $V$ = stock market value of the firm
  - $p_K$ = the replacement cost of the firm’s capital
  - $K$ = the firm’s capital stock
- In words
  - $q =$ market value of installed capital
  - replacement cost of installed capital
- When $q > 1$ firms should invest since the value of capital exceed the price of acquiring additional capital
- When $q < 1$ firms should not invest since the value of capital is less than the price of acquiring additional capital

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**Equilibrium in the Goods Market (or Saving Market)**

Recall

\[ S_{pvt} = (Y - T + TR + INT) - C \]

where

- $T =$ Taxes paid to government
- $TR =$ transfers received from government
- $INT =$ Interest payments on the government debt

and government savings as:

\[ S_{gvt} = T - (G + TR + INT) \]

which implies national savings as

\[ S = S_{pvt} + S_{gvt} = Y - C - G \]

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**Why is Investment so Volatile?**

Consider a temporary bad shock to $\Lambda$, the productivity parameter.

- $Y$ decreases so disposable income decreases. Small decrease in total lifetime wealth of households. Mostly savings decreases. Supply curve for savings shifts in almost as much as the loss in disposable income.
- Gives answer to why some industries which sell investment goods are so cyclical. *The shoe industry is much less cyclical than the tractor or construction industry.*
- The PIH says that as individuals we vary savings a lot to keep consumption smooth when income is variable. This holds for the whole economy as well. Since investment goods are how a country saves, the investment good sector gets bumped around while the consumption goods sector is relatively smooth.
The Effects of Fiscal Policy in Our Model

- Do increases in government spending "crowd out," or displace investment by the private sector?
- The answer depends on whether the increase is temporary (short-run) or permanent (long-run).
- Let's write down the government budget constraint or government budget identity for a two period economy:
  \[ G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r} \]
- Since \( Y = C + I + G \), either \( C \) or \( I \) must go down when \( G \) goes up.

A Temporary (one-time) increase in \( G \) and \( T \) of $1000 per person occurs in the first period.
- Government saving unchanged.
- Disposable income of households goes down by $1000 per person in the first period.
- Consumption decrease by far less than the amount of the tax (PIH), say $500 per person.
- Recall from last class, since you consume shares of total wealth each period...
  - Implies total savings curve shifts in (or to the left) by almost $500 ($500).

In a closed economy, a temporary increase in government chiefly "crowds out" or displaces private investment.

But how much?

A permanent (both periods) increase in \( G \) and \( T \) of $1000 per person, per year.
- Government saving unchanged
- Disposable income of households goes down by $1000 per person
- Lifetime wealth of households decrease by $1000 per person each period or roughly $2000 over both periods.
- Consumption decreases by as much as the tax.
- Implies total saving curve shifts in (or to the left) by not very much.

Permanent or long-term increase in \( G \) displaces consumption.