Problem Set 3: Due in class Thursday, April 6

1. Consider the following single good cash-in-advance economy with capital. The utility function of the representative agent is given by

\[ \sum_{t=1}^{\infty} \beta^t U(C_t), \]

where \(U' > 0, U'' < 0\) and \(0 < \beta = \frac{1}{1+\rho} < 1\), where \(\rho\) is the pure rate of time preference. The agent maximizes this utility subject to a budget constraint and a cash-in-advance constraint. The budget constraint is:

\[ C_t + K_{t+1} + M_t/P_t = f(K_t) + (1 - \delta)K_t + (M_{t-1} + G_t)/P_t \]

where \(K_t\) is the capital stock, \(M_t\) are nominal balances, \(C_t\) is consumption, \(P_t\) is the price level, \(G_t\) is the nominal money transfer received at the beginning of the period, and \(\delta\) is the depreciation rate of capital. The production function \(f(\cdot)\) has the standard properties: \(f' > 0\) and \(f'' < 0\).

The cash-in-advance constraint is

\[ C_t \leq (M_{t-1} + G_t)/P_t. \]

Define superneutrality as the property that changes in the rate of the nominal money supply have no affect any real variables including the level of real balances.

(a) Show that in the economy described above, money is superneutral.

(b) Does the Cooley-Hansen CIA model exhibit superneutrality? Explain.
