Problem Set 5: Three Questions on Lucas’ “Mechanics” paper  
Due in Class, Wednesday, February 19, 2003

1. Per capita income in the United States is about 15 times per capita income in India. This ratio has remained fairly stable over the last 40 years. If the neoclassical model reviewed in Section 2 of Lucas’ paper is an accurate model for both India and the U.S. and if both countries have the same technology level, about what must be the ratio of the real wage in the U.S. to the real wage in India?

2. Consider a Robinson Crusoe economy where a single agent has preferences
\[ \sum_{t=0}^{\infty} \beta^t \ln c_t \] with \( 0 < \beta < 1 \)

The agent is endowed with one unit of time per period. He can allocate \( u_t \) fraction to the production of the consumption good, and \( 1 - u_t \) to accumulation of human capital. If he begins period \( t \) with \( x_t \) units of human capital, his production technology is:
\[ C_t = x_t u_t \quad x_{t+1} = \delta x_t^\alpha (1 - u_t)^\gamma, \quad \alpha, \gamma, \delta > 0, \quad \alpha \leq 1. \]

(a) State this agent’s Bellman equation
(b) Show that it has a solution of the form: \( v(x) = A + B \ln(x) \).
(c) Find \( B \). Find the optimal policy (time allocation).
(d) For what values of the parameters will this agent’s consumption exhibit sustained growth? For these parameter values, what is the growth rate?
(e) For what parameter values will consumption converge to a constant level? For these values, what is the long run level of consumption?

3. Suppose the U.S. and Mexico were to agree to a system under which citizens of either country were free to work and live in either place. Use one of the two models presented in Lucas’ paper (but don’t flip between the two) to answer the questions below:

(a) Describe the consequences of this policy for the time paths of income of people currently living in Mexico and the U.S. Discuss labor and capital income separately, and be sure to distinguish between growth effects and level effects. Your answer should be consistent with the evident fact that many people now working in Mexico would prefer to be working in the U.S.
(b) To what extent can these same effects be expected under NAFTA; that is a policy of (let’s say) no labor mobility, but complete trade in final goods and assets.