Reassessing Aggregate Welfare under Professional Licensing and Certification∗

Mark A. Klee†

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Abstract

I compare the welfare effects of professional licensing and certification in Shapiro’s (1986) moral hazard model of services. Shapiro (1986) shows that licensing can dominate certification, assuming that certification reveals a provider’s human capital investment exactly. This occurs if certification leads to excessive investment in human capital as a signaling device. I consider an alternative form of certification that reveals only whether providers’ investment exceeds some threshold. I show that this form of certification weakly dominates licensing when it sends the same information as licensing about providers’ human capital investments. Although this certification policy limits consumers’ capability to distinguish providers, it also limits the cost of signaling in equilibrium.

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†Department of Economics, Yale University, Box 208264, New Haven, CT 06520-8264. Email: mark.klee@yale.edu
1 Introduction

Asymmetric information is a common concern in professional service markets. Specifically, consumers are often limited in their capability to assess the quality of service providers. If consumers cannot identify high quality providers, they cannot reward these providers for their additional skills or effort. Asymmetric information thus diminishes the incentive to provide high quality services, resulting in an inefficiently low average level of quality. Professional licensing and certification are two regulations that are capable of mitigating the effects of asymmetric information. In this paper, I compare the welfare effects of these two kinds of policies.

Professional licensing policies require service providers to satisfy a list of human capital standards in order to practice. For instance, all barbers in the US are required to obtain a license. Prospective barbers can acquire a license by attending an approved barber training program and passing an exam, among other things. Those who do not meet these standards are not allowed to practice the licensed occupation.

Certification of professionals represents an alternative to licensing. Certification policies accredit service providers that satisfy a list of human capital standards. The fundamental difference between certification and licensing is that certificates are not required by law in order to practice. For instance, all barbers in the UK may practice. Some choose to distinguish themselves by obtaining a certificate issued by the government. Barbers can acquire a certificate by attending an approved barber training program and passing an exam.

Despite the similarity of these forms of regulation, professional licensing is much more common in the US. Kleiner and Krueger (2009) estimate that in 2008, 29% of workers were required by law to obtain a license. By contrast, they estimate that 6% of workers held a certificate issued by some level of government.

1Licensing is also common relative to other major institutions in the US labor market. Kleiner (2000) notes that more workers exercised a licensed profession than earned the minimum wage or were members of a union in 2000.
Economists tend to advocate certification over licensing as a means of addressing asymmetric information problems. If certification is indeed preferable to licensing, this suggests that replacing the current widespread licensing policies with certification may yield substantial welfare gains. However, previous theoretical work comparing licensing and certification does not provide a clear-cut prediction. Shapiro (1986) develops a moral hazard model of professional services. He assumes that certification reveals each provider’s human capital investment exactly. He finds that licensing can dominate certification. This occurs if certification leads to excessive investment in human capital as a signaling device.

In this paper, I consider an alternative form of certification. My starting point is the observation that Shapiro (1986) assumes that certification and licensing differ in two ways. First, certification permits providers with human capital below the licensing standard to practice. Second, certification allows consumers to distinguish levels of human capital above the licensing standard. I assess which of these differences is responsible for the possibility that licensing dominates certification. I do so by assuming that certification reveals only whether a provider’s human capital investment exceeds the licensing standard. This form of certification permits providers with human capital below the licensing standard to practice, but it does not allow consumers to distinguish levels of human capital above the licensing standard. I will refer to this form of regulation as “threshold certification” and the form considered by Shapiro (1986) as “precise certification”. I use Shapiro’s (1986) model to show that threshold certification weakly dominates licensing when it reveals the same information as licensing about providers’ human capital. This occurs because threshold certification allows consumers to identify some providers’ unobservable quality, but it limits the incentive that induces excessive investment in human capital as a signaling device.

Although threshold certification weakly dominates licensing, it is not optimal.

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2See Arias and Scafidi (2009), Summers (2007), and Svorny (2004) for examples.
among the set of all regulations. It is beyond the scope of this paper to charac-
terize the optimal mechanism, but I do take some additional steps in this direction.
First, I show that a regulator can improve welfare by certifying both providers whose
human capital investment meets the licensing standard and those who pay a bribe.
This bribe replicates the effect of increased human capital investment on average
provider quality. Furthermore, it generates revenue for regulators, which improves
welfare. I then explore the welfare effects of increasing the bribe and the certification
threshold. I find that a regulator can weakly improve welfare by increasing these
objects until high quality providers become indifferent between the two certification
options.

The remainder of this paper proceeds as follows. In Section 2 I describe Shapiro’s
(1986) model, first in the absence of regulation and then under licensing and precise
certification. Section 3 introduces threshold certification and discusses how it enters
the environment. I begin Section 4 by presenting equilibrium under perfect informa-
tion. I then present private information equilibrium under licensing, precise certifi-
cation, and threshold certification. Section 5 compares the welfare associated with
the allocations implemented by licensing and certification. In Section 6 I consider an
alternative form of certification and explore its welfare effects. Section 7 concludes.

2 Shapiro’s (1986) Model

Shapiro (1986) presents a moral hazard model of professional services in order to
examine the effect of regulation on welfare through providers’ quality choices. The
model is a static one, in which ex ante homogeneous providers choose both human
capital and quality. Providers with more human capital require less costly additional
effort to provide high quality. Consumers differ in their preferences for quality. Con-

\footnote{I follow the notation introduced in Shapiro (1986) with the exception of a few changes for simplicity of exposition. I also change assumptions about the model’s timing and interpretation. I have shown that the model is isomorphic to all of these changes.}
sumers cannot observe some providers’ quality, nor can they observe any provider’s human capital investment.\footnote{Atkeson, Hellwig and Ordoñez (2010) present a similar dynamic model. An important difference between that paper and Shapiro (1986) relates to consumers’ information about provider quality. In Atkeson et al. (2010), providers have reputations which evolve according to a Brownian Motion, as opposed to the deterministic process assumed by Shapiro (1986). Providers choose whether to exit the market on the basis of this reputation. The distribution of quality thus evolves over time in a more interesting manner than in Shapiro (1986). However, various forms of certification and licensing are distinguished more easily in the simpler model presented in Shapiro (1986).}

2.1 Environment

A. Service providers

There is free entry of \textit{ex ante} homogeneous potential service providers. They choose human capital \(K \in \mathbb{R}^+\) and whether to supply ("provide") high or low quality services. The total cost to a provider with human capital \(K\) of providing services of quality \(q \in \{L, H\}\) is \(C_q(K)\), which is itself composed of two costs:

\[
C_q(K) = K + c_q(K).
\]

Human capital is costly to acquire. In addition, there is some cost of effort \(c_q(K)\) associated with providing quality \(q\). Human capital aids in the production of either level of quality: providers with more human capital require less costly effort to provide either level of quality. Specifically, \(c_q(K)\) is assumed to be decreasing and convex \((c'_q < 0, c''_q > 0)\). Additionally, high quality services require more costly effort than low quality services at any level of human capital \([c_H(K) > c_L(K)]\). Although high quality is more costly for all \(K\), the incremental cost of high quality decreases in the human capital investment \([c_H(K) - c_L(K)]\) decreasing]. Human capital investments are indirectly related to quality through this mechanism.\footnote{Wiswall (2007) posits an alternative model in which human capital affects production more directly by serving as an input in the production function.} Figure \[\] illustrates the characteristics of the cost of effort for each level of quality. The investment that
minimizes the total cost of providing quality \( q \) is denoted \( K_q \), and its associated cost is denoted \( C_q \).

B. Consumers

Perfectly inelastic demand for services comes from consumers of measure 1. Consumers choose which kind of service to purchase. They receive a common value from low quality services, which I normalize to zero. However, they differ in the extent to which they value high quality. Consumers’ marginal valuations of quality \( \theta \) are drawn from some distribution with cdf \( F(\theta) \) defined over the support \([a, b]^{\text{6}}\). All consumers prefer high quality to low quality, so \( 0 < a < b \).

C. Timing and Information Structure

In the first stage of this static model, providers decide on human capital and quality. After these decisions are made, each provider’s quality is revealed to consumers with some probability \( w \). In the second stage of this static model, consumers choose services given the available information about service quality[^7]. Consumers cannot observe any provider’s human capital investment in the absence of regulation, so providers whose quality was not revealed are indistinguishable.

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[^6]: This set of assumptions differs from its analog in Shapiro (1986). He assumes that consumers differ in their valuations of both low and high quality. The valuations drawn by an individual imply a marginal valuation of quality which is heterogeneous across consumers. This set of assumptions differs from mine only because it implies that the payoff levels of a given service vary across consumers with the same \( \theta \). However, if the common valuation of low quality services that I assume is set to be the mean of the distribution of low quality valuations in Shapiro (1986), aggregate welfare will be unaffected by these differing assumptions.

[^7]: The timing and information structure that I describe here differ from their analogs posited by Shapiro (1986). The model presented by Shapiro is a dynamic one. Providers have finite working lives which consist of two periods: youth and maturity. Youth begins at market entry and lasts some fraction \( 1 - w \) of working life. Providers spend the remainder of their working lives in maturity. During youth, providers’ quality is unobservable to consumers. They have not practiced long enough to have acquired a reputation. Providers establish themselves during this period, so that quality is revealed at the start of maturity. My assumption that a provider’s quality is randomly revealed merely translates this mechanism into a static environment. The set of necessary conditions for equilibrium that I describe in Section[^4] is the same set described by Shapiro (1986). However, the dynamic representation yields a better interpretation of the states of the world in which quality is and is not revealed.
D. Market Structure

As a result of the information structure, the market for services is segmented into three submarkets on the basis of expected service quality. The free entry assumption ensures that each submarket is competitive. The information available to consumers allows them choose from three different kinds of service: observably low quality, observably high quality, and unobservable quality. Thus, providers are assigned to submarkets by the quality revelation process. For example, providers of either quality level whose quality was not revealed are assigned to the submarket for services of unobservable quality. These providers receive a price that is independent of their quality.

2.2 Professional Licensing

Shapiro (1986) considers a regulator that uses a licensing policy to address the problems of asymmetric information. This regulator imposes a minimum standard which all providers must meet. By assumption, the regulator cannot observe a provider’s quality, but can observe a provider’s human capital. Consequently, licensing cannot prohibit the entry of low quality providers. Rather, it requires providers to satisfy a minimum standard of human capital, denoted $K_R$. Formally, licensing enters the model through a minimum constraint on the human capital decision. The only direct effect of licensing is to increase the total cost of providing quality to $C_q(K_R)$. This occurs only if the licensing constraint binds for a provider of quality $q$. We can thus interpret a laissez-faire policy as a special case of licensing in which the constraint on providers does not bind.
2.3 Precise Certification

Before introducing the form of certification that I consider, it will be useful to introduce certification as discussed in Shapiro (1986) to serve as a benchmark. Certification enters the model by publicizing each provider’s investment $K$, while leaving the human capital decision unconstrained. Certification status offers consumers additional information about providers whose quality was not revealed. They use this signal to update their beliefs about these providers’ service quality.

This policy fundamentally differs from licensing in two ways. First, precise certification does not restrict entry into the occupation; it leaves the human capital decision unconstrained. Second, precise certification transmits a signal which allows consumers to distinguish providers for whom $K \geq K_R$. Providers choose from an infinitely large set of signals. By contrast, licensing requires all providers to invest at least $K_R$, and reveals no additional information.

3 Threshold Certification

I now consider a second form of certification which leaves the human capital decision unconstrained. In contrast to precise certification, this form of certification reveals only whether a provider’s human capital exceeds some threshold. Under threshold certification, providers choose from only two possible signals: certified and uncertified.

A convenient feature of threshold certification is that a regulator can choose the threshold to be $K_R$. In this case, certification reveals the same information as licensing about a provider’s human capital. In particular, licensing assures consumers that any provider has a human capital investment of at least $K_R$, while threshold certification assures consumers that this is true of any certified provider. As a result, threshold certification differs from licensing only because it does not restrict entry into the occupation; providers may forego increasing $K$. Although these providers do not
become certified, they may still practice. As a result of this close comparison, I first consider a regulator who sets the certification threshold to $K_R$.

4 Equilibrium

In this section, I first define the perfect information equilibrium. I then summarize Shapiro’s (1986) characterization of equilibrium under professional licensing and precise certification. I also characterize a threshold certification equilibrium. In general, an equilibrium requires that providers’ and consumers’ choices be optimal, and that submarkets clear. For the sake of simplicity, I consider only regulations for which $K_L \leq K_R \leq K_H$. The constraint imposed by licensing on the human capital decision binds for low quality providers but not for high quality providers. Laissez-faire is a special case of licensing in which $K_L = K_R$.

4.1 Perfect Information

Before proceeding to discuss the model with regulation, I describe the perfect information equilibrium. Providers choose whether to enter, and simultaneously choose $q$ and $K$ to maximize payoffs, given prices. The payoffs to a quality $q$ provider with human capital investment $K$ are $P_q - C_q(K)$.

Consumers choose between two submarkets, given prices. The payoff to any consumer from low quality services is $-P_L$. The payoff to a type $\theta$ consumer from high quality services is $\theta - P_H$. Even though all consumers value high quality services more than low quality services ($a > 0$), consumers may receive a larger payoff from low quality services, depending upon relative prices.

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8In contrast to Shapiro (1986), I present an interpretation that describes equilibrium in terms of market clearing conditions rather than conditions requiring beliefs to be consistent with providers’ decisions. The set of equilibrium conditions that I describe here is equivalent to the one presented in Shapiro (1986).

9Consequently, I describe equilibrium under laissez-faire and licensing simultaneously.
Ex ante homogeneous providers must be indifferent between any \((q, K)\) pair chosen in equilibrium. Otherwise, all providers would have a profitable deviation, in which case this is not an equilibrium. Free entry of providers with an outside option of value zero pins down the following necessary conditions for a quality \(q\) provider in a perfect information equilibrium:

\[ P_q \leq \bar{C}_q. \]

With perfect information, only two levels of human capital are chosen in equilibrium: \(K_L\) and \(K_H\). Providers choose \(K\) to minimize the cost of providing quality \(q\).

For consumers, the payoff from high quality is an increasing function of \(\theta\), whereas the payoff from low quality is not a function of \(\theta\). Consider the type \(\theta^*\) consumer who is indifferent between low quality and high quality:

\[ \theta^* = P_H - P_L. \]

The marginal consumer benefits from high quality services sufficiently to exactly offset their larger price. The optimal consumption policy is thus a cutoff rule defining two regions of the preference distribution. Consumers of type \(\theta > \theta^*\) strictly prefer the high quality submarket. Consumers of type \(\theta < \theta^*\) strictly prefer the low quality submarket.

Additionally, submarkets must clear. Demand for high quality services comes from the mass of consumers with \(\theta > \theta^*\). The supply of these services comes from the measure of providers that are of high quality. These must be consistent in equilibrium.

In summary, an equilibrium is given by the set \(\{K_L, K_H, P_L, P_H, \theta^*\}\) such that:

1. Providers earn zero profits \((P_q \leq \bar{C}_q)\);

2. Consumers’ decisions are summarized by a cutoff rule with critical value given
by $\theta^* = P_H - P_L$; and

3. All submarkets clear.

### 4.2 Professional Licensing

Now consider a licensing policy which sets a minimum standard $K_L \leq K_R \leq K_H$. Providers choose whether to enter, and simultaneously choose $q$ and $K$ to maximize expected payoffs, given prices. Providers whose quality is revealed receive a price that depends upon their quality choice, $P_q$. Providers whose quality is not revealed receive a common price, $P_0$. The expected payoffs to a provider of quality $q$ and human capital $K \geq K_R$ are then given by $wP_q + (1-w)P_0 - C_q(K)$. Consequently, the market structure can potentially give providers the incentive to choose high quality, even though these services are more costly to provide.

Consumers choose between three submarkets, given prices and beliefs: observably low quality, unobservable quality, and observably high quality. The expected payoffs to a type $\theta$ consumer from observably low quality services and observably high quality services are the same as under perfect information: $-P_L$ and $\theta - P_H$, respectively. Beliefs about the service quality of a provider whose quality was not revealed are denoted $s$. The expected payoff to a type $\theta$ consumer from services of unobservable quality is then $s\theta - P_0$. This consumer encounters a high quality provider with probability $s$. The consumer’s expected valuation of these services is then $s\theta$.

*Ex ante* homogeneous providers must be indifferent between any option chosen in equilibrium. The following necessary conditions for equilibrium are pinned down by
free entry of low and high quality providers, respectively.\footnote{Recall that \( K_R = K_L \) in the laissez-faire case, which implies \( C_L(K_R) = \bar{C}_L \). If the licensing constraint binds, \( C_L(K_R) > \bar{C}_L \).}

\[
wp_L + (1 - w)p_0 \leq C_L(K_R), \quad (1) \\
wp_H + (1 - w)p_0 \leq \bar{C}_H. \tag{2}
\]

These conditions must hold with equality if both quality levels are chosen in equilibrium. Note that only low quality providers must increase their investment to meet the licensing standard \( K_R \). They invest \( K_R \) exactly, which is the constrained cost-minimizing investment.

Shapiro (1986) shows that the optimal consumption policy is a cutoff rule which defines three regions of the preference distribution. The cutoffs are determined by the following indifference conditions, which are necessary for equilibrium assuming \( 0 < s < 1 \):

\[
s\theta_L = P_0 - P_L, \quad (3)
\]

\[
(1 - s)\theta_H = P_H - P_0. \tag{4}
\]

These conditions identify the consumer types for whom the marginal benefit of consuming higher quality in expectation is exactly counterbalanced by the marginal cost of paying a premium for this higher quality. Type \( \theta_L \) consumers are indifferent between the submarkets of observably low quality and unknown quality. Type \( \theta_H \) consumers are indifferent between the submarkets of unknown quality and observably high quality. Consumers with \( \theta \leq \theta_L \) prefer the observably low quality submarket. Consumers with \( \theta \in (\theta_L, \theta_H) \) prefer the unknown quality submarket. Consumers with \( \theta \geq \theta_H \) prefer the observably high quality submarket.

The optimal behavior of consumers and providers must be consistent in equilib-
rium. First, the submarket for services of observably high quality must clear. This introduces the following necessary condition for equilibrium:

\[ 1 - F(\theta_H) = ws. \]  

(5)

Demand for services of observably high quality is given by the mass of consumers with marginal valuations \( \theta \geq \theta_H \). In equilibrium, this must be consistent with the measure of high quality providers whose quality was revealed, \( ws \). This equilibrium requirement is illustrated in Figure 2, which summarizes the optimal consumption rule and how consumers are matched to providers. The consumer types with marginal valuations on the right half of this figure purchase observably high quality services. This mass of consumers must be the same size as the mass of providers of observably high quality.

Second, the submarket for services of unobservable quality must clear. This introduces the following necessary condition for equilibrium:

\[ F(\theta_H) - F(\theta_L) = 1 - w. \]

(6)

Demand for services of unobservable quality is given by the mass of consumers with marginal valuations \( \theta \in (\theta_L, \theta_H) \). In equilibrium, this must be consistent with the measure of providers whose quality was not revealed, \( 1 - w \). These providers may be of either quality level. This equilibrium requirement is also illustrated in Figure 2. The consumer types with marginal valuations in the middle of this figure purchase unknown quality services. This mass of consumers must be the same size as the mass of providers whose quality was not revealed. If the submarket for observably high quality services and the submarket for unknown services clear, the submarket for

\[ ^{11} \text{This imposes the equivalence between consumers’ beliefs and average provider quality in equilibrium for the sake of exposition. Measure } s \text{ of providers are of high quality. Of these, a fraction } \omega \text{ have their quality randomly revealed.} \]
observably low quality services must also clear.

In summary, Shapiro (1986) defines a licensing equilibrium with $K_L \leq K_R \leq K_H$ as being given by the set $\{K_R, K_H, P_L, P_0, P_H, s, \theta_L, \theta_H \}$ such that:

1. Low quality providers $[wP_L + (1 - w)P_0 \leq C_L(K_R)]$ and high quality providers $[wP_H + (1 - w)P_0 \leq \bar{C}_H]$ earn zero profits in expectation;

2. Consumers’ decisions are summarized by a cutoff rule with critical values implied by $s\theta_L = P_0 - P_L$ and $(1 - s)\theta_H = P_H - P_0$; and

3. All submarkets clear $[1 - F(\theta_H) = ws$ and $F(\theta_H) - F(\theta_L) = 1 - w]$.

4.3 Precise Certification

Now consider a precise certification policy which publicizes $K$ and leaves the human capital decision unconstrained. Providers’ expected payoffs differ from those under licensing in two ways. First, they receive a price $P(K)$ that depends on human capital when their quality is not revealed. Consumers can use certification status to distinguish these providers. Second, providers need not invest $K \geq K_R$. The expected payoffs to a provider of quality $q$ and human capital $K$ are then given by $wP_q + (1 - w)P(K) - C_q(K)$.

Consumers’ expected payoffs differ from those under licensing only for services of unobservable quality. They observe $K$ for a provider whose quality was not revealed. This creates a different submarket for each signal $K$. Conditional beliefs about a provider transmitting signal $K$ are denoted $\phi(K)$. The expected payoff to a type $\theta$ consumer from services of unobservable quality conditional on the signal $K$ is then $\phi(K)\theta - P(K)$. This consumer encounters a high quality provider of investment $K$ with probability $\phi(K)$. The consumer’s expected valuation of these services is then $\phi(K)\theta$. 
Before characterizing equilibrium behavior, it will be useful to discuss two refinements that Shapiro (1986) places on consumers’ beliefs $\phi(K)$ for signals off the equilibrium path. First, he assumes that $\phi(K)$ is increasing. Second, he assumes that consumers believe any provider signaling $K$ to be of high quality with certainty $[\phi(K) = 1]$ if:

$$wP_L + (1 - w)P_H \leq C_L(K).$$

When this condition holds the signal $K$ is not profitable for low quality providers, even if these providers received the most generous possible price when their quality is not revealed ($P_H$).

As Shapiro (1986) shows in Propositions 5 and 6, these equilibrium refinements ensure that the unique equilibrium is a separating equilibrium. Precise certification grants providers sufficient scope to signal that consumers can identify the quality of any provider in equilibrium. Consequently, these refinements simplify the characterization of both providers’ and consumers’ behavior in equilibrium.

For some parameter values, precise certification implements the perfect information allocation. Shapiro’s (1986) Proposition 5 establishes that this occurs if and only if:

$$w\bar{C}_L + (1 - w)\bar{C}_H \leq C_L(K_H).$$  \hspace{1cm} (7)

When this condition holds, it is unprofitable for low quality providers to invest $K_H$ and attempt to pool with high quality providers. In this case, the description of equilibrium behavior corresponds to the description in Section 4.1.

When this condition does not hold, low quality providers can profitably invest $K_H$ in order to pool with high quality providers. Although this investment is costly, low quality providers would receive a premium of $\bar{C}_H - \bar{C}_L$ when their quality is not
revealed. When condition (7) does not hold, the expected premium is enough to compensate low quality providers for the cost of signaling. In this case, Shapiro’s (1986) Proposition 6 shows that the unique equilibrium is one in which high quality providers distinguish themselves by investing $K_S$ such that:

$$w\bar{C}_L + (1-w)C_H(K_S) = C_L(K_S).$$

By design, the signal $K_S$ does not give low quality providers a profitable deviation to pool with high quality providers. In the resulting separating equilibrium, the free entry conditions pin down $P_L \leq \bar{C}_L$ and $P_H \leq C_H(K_S)$.

Since precise certification gives rise to a separating equilibrium, consumers can identify the quality of all providers. Consequently, consumers’ optimal policy resembles the one associated with a perfect information equilibrium. Consumers’ equilibrium behavior is summarized by a cutoff rule which defines only two regions of the preference distribution. The cutoff is given by the consumer of type $\theta^*$ for whom:

$$\theta^* = P_H - P_L.$$

Finally, all submarkets must clear in equilibrium.

In summary, Shapiro (1986) defines a precise certification equilibrium in which condition [7] holds as being given by the set \{\(K_L, K_H, P_L, P(K), P_H, \phi(K), \theta^*\)\} such that:

1. Providers’ earn zero profits \(P_q \leq \bar{C}_q\);

2. Consumers’ decisions are summarized by a cutoff rule with critical value given by $\theta^* = P_H - P_L$;

3. All submarkets clear; and
4. \( \phi(K_L) = 0, \phi(K_H) = 1, \phi(K) \) is increasing, and \( \phi(K) = 1 \) for all \( K \) that do not occur in equilibrium for which \( wP_L + (1-w)P_H \leq C_L(K) \).

The corresponding definition when condition (7) does not hold is one in which \( K_S \) replaces \( K_H \) in the statement of equilibrium objects, in the free entry condition, and in the characterization of beliefs.

### 4.4 Threshold Certification

Now consider a threshold certification policy which publicizes only whether \( K \geq K_R \) and leaves the human capital decision unconstrained. As in Section 4.2 I focus on \( K_L \leq K_R \leq K_H \) for simplicity. Providers’ payoffs differ from those under licensing in two ways. First, providers whose quality is not revealed receive a price that is independent of quality, but which varies by certification status. Uncertified providers receive price \( P_U \), while certified (or equivalently, accredited) providers receive \( P_A \).

This differs from licensing because certification status partially allows consumers to distinguish providers whose quality was not revealed. The second difference relative to licensing is that uncertified providers can invest as they would have in the absence of regulation. Payoffs to uncertified providers of quality \( q \) and human capital \( K < K_R \) are \( wP_q + (1-w)P_U - C_q(K) \). Payoffs to certified providers of quality \( q \) and human capital \( K \geq K_R \) are \( wP_q + (1-w)P_A - C_q(K) \).

Consumers’ expected payoffs differ from those under licensing only for services of unobservable quality. Under threshold certification, consumers observe two kinds of providers whose quality is unobservable: certified and uncertified. This creates two submarkets for services of unobservable quality. Conditional beliefs about a provider transmitting signal \( e \in \{U,A\} \) are denoted \( s_e \). A type \( \theta \) consumer receives the following payoffs from services of unknown quality, given a provider’s certification
status $e$:

$$s_e \theta - P_e.$$

Threshold certification equilibrium behavior resembles that of a licensing equilibrium. Given $K_L \leq K_R \leq K_H$, high quality providers can acquire a certificate simply by choosing their cost-minimizing investment level $K_H$. All high quality providers choose $K_H$ in equilibrium. Consequently, consumers can identify uncertified providers as being of low quality ($s_U = 0$) regardless of whether their quality is revealed. Nevertheless, when $C_L(K_R) - \bar{C}_L$ is sufficiently high, some low quality providers choose to forego certification. Consumers are indifferent between the services of an uncertified provider and those of an observably low quality provider, since they are of the same expected quality. Consumers offer the price $P_U = P_L$ to uncertified providers in equilibrium. *Ex ante* homogeneous providers must be indifferent between any options in equilibrium. This introduces the following free entry condition for uncertified low quality providers:

$$P_L \leq C_L. \quad (8)$$

This necessary condition for equilibrium is accompanied by similar free entry conditions for certified low quality providers and high quality providers. When the regulator sets the threshold to $K_R$, these two conditions closely resemble their analogs under licensing. These conditions are given by the following for certified low quality providers:

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$^{12}$If $K_R = K_L$, all providers can acquire a certificate simply by choosing their cost-minimizing investment level. In other words, providers all send the same signal of quality. In this case threshold certification, licensing, and *laissez-faire* all implement the same allocation.

$^{13}$I more specifically characterize the case in which some low quality providers forego certification in Appendix A.
providers and high quality providers, respectively:

\[ wP_L + (1 - w)P_A \leq C_L(K_R), \]  
\[ wP_H + (1 - w)P_A \leq \bar{C}_H. \]  

In contrast to equilibria under the other policies, three levels of human capital are chosen in a threshold certification equilibrium: \( K_L, K_R, \) and \( K_H. \) All three free entry conditions must hold with equality if both quality levels are chosen and if some, but not all, low quality providers choose not to become certified in equilibrium. The fraction of all providers who choose to forego certification is denoted \( \phi. \)

Since all uncertified providers are of low quality (\( s_U = 0 \)), there are three submarkets that consumers choose from in equilibrium. Services of known low quality are supplied by providers of observably low quality or uncertified providers. Services of unknown quality are supplied by certified providers of unobservable quality. Services of observably high quality are supplied by certified providers of observably high quality. As a result, the optimal consumption policy is summarized by the same form of cutoff rule that is associated with a licensing equilibrium. For ease of comparison, I use the same notation for these cutoffs under both licensing and threshold certification:

\[ s_A \theta_L = P_A - P_L, \]  
\[ (1 - s_A) \theta_H = P_H - P_A. \]  

The potential presence of uncertified providers requires some modifications of the market clearing conditions derived for licensing. The market clearing condition for
the submarket of observably high quality now reads:

\[ 1 - F(\theta_H) = ws_A(1 - \phi). \] (13)

As in the case of licensing, demand for services of observably high quality is given by the mass of consumers with marginal valuations \( \theta \geq \theta_H \). In equilibrium, this must be consistent with the measure of high quality providers whose quality was revealed, \( ws_A(1 - \phi) \). This expression differs from its analog under licensing because a measure \( \phi \) of providers forego certification. Among certified providers, a fraction \( s_A \) are of high quality. This equilibrium requirement is illustrated in Figure 3, which summarizes the optimal consumption rule and how consumers are matched to providers. The consumer types with marginal valuations on the right half of this figure purchase observably high quality services. This mass of consumers must be the same size as the mass of providers of observably high quality.

The market clearing condition for the submarket of unknown quality now reads:

\[ F(\theta_H) - F(\theta_L) = (1 - w)(1 - \phi). \] (14)

Demand for services of unknown quality is given by the mass of consumers with marginal valuations \( \theta \in (\theta_L, \theta_H) \). In equilibrium, this must be consistent with the measure of providers of unknown quality services, \( (1 - w)(1 - \phi) \). This expression differs from its analog under licensing because a measure \( \phi \) of providers forego certification, and are thus of identifiable quality. A fraction \( 1 - w \) of the remaining providers are of unobservable quality. This equilibrium requirement is also illustrated in Figure 3. The consumer types with marginal valuations in the middle of this figure purchase unknown quality services. This mass of consumers must be the same size as the mass of certified providers whose quality was not revealed. If the submarket for observably high quality services and the submarket for unknown services clear, the submarket
for observably low quality services must also clear.

For some parameter values, regulators following a threshold certification policy can implement the perfect information allocation. By a similar argument to the one employed in Shapiro’s (1986) Proposition 5, this occurs if and only if the following condition holds:

\[ wC_L + (1 - w)C_H \leq C_L(K_R). \]  

(15)

Note that if condition (15) holds, condition (7) does as well, since \( C_L(K) \) is increasing for \( K > K_L \). In this case, precise certification and threshold certification implement the same allocation. When condition (15) does not hold, low quality providers can profitably increase their investment to \( K_R \) in order to become certified. High quality providers cannot distinguish themselves further, since threshold certification publicizes only whether \( K \geq K_R \). In this case, threshold certification yields a partially pooling equilibrium.

In summary, I define a threshold certification equilibrium with \( K_L \leq K_R \leq K_H \) as being given by the set \( \{K_L,K_R,K_H,P_L,P_U,P_A,P_H,s_U,s_A,\theta_L,\theta_H,\phi\} \) such that:

1. Uncertified low quality providers \([P_L \leq C_L]\), certified low quality providers \([wP_L + (1 - w)P_A \leq C_L(K_R)]\), and high quality providers earn zero profits in expectation \([wP_H + (1 - w)P_A \leq C_H]\);

2. Consumers’ decisions are summarized by a cutoff rule with critical values implied by \( s_A\theta_L = P_A - P_L \) and \((1 - s_A)\theta_H = P_H - P_A\); and

3. Submarkets clear for observably high quality \([1 - F(\theta_H) = ws_A(1 - \phi)]\) and unknown quality \([F(\theta_H) - F(\theta_L) = (1 - w)(1 - \phi)]\).

4. \( s_U \) and \( s_A \) are consistent with the fraction of all uncertified and certified providers that are of high quality, respectively.
5 Welfare Comparison

I now present the welfare comparison of licensing and certification. Section 5.1 derives aggregate welfare and sets the stage for the welfare analysis by discussing the inefficiencies associated with asymmetric information in the absence of regulation. Next, Section 5.2 compares the welfare effects of precise certification and threshold certification. Shapiro’s (1986) result that licensing can dominate precise certification presents a strong *prima facie* case that licensing can also dominate threshold certification; relative to threshold certification, precise certification better allows consumers to identify a provider’s quality. This subsection highlights how precise certification can lead to excessive human capital investment as a signaling device and how threshold certification limits this inefficiency. Finally, I build on this insight in Section 5.3 to establish my main result: threshold certification always weakly dominates licensing if the threshold is set to the licensing standard $K_R$.

5.1 Inefficiencies under Asymmetric Information

Free entry among *ex ante* homogeneous providers with an outside option of value zero implies that there are no expected profits in equilibrium. In this case, aggregate welfare consists of consumers’ expected payoffs aggregated according to the optimal consumption rule. I consider only equilibria in which $0 < s < 1$ and $0 < s_A < 1$. This rules out equilibria in which all providers’ quality is known. Regulation is irrelevant in these equilibria.

The allocation resulting from a licensing regime implies the following expression of aggregate welfare:\footnote{14}{Superscript $\ell$ corresponds to licensing, while superscript $t$ corresponds to threshold certification.}

$$W^\ell = \int_a^{\theta_L^\ell} (-P_L^\ell) dF(\theta) + \int_{\theta_L^\ell}^{\theta_H^\ell} (s^\ell \theta - P_0^\ell) dF(\theta) + \int_{\theta_H^\ell}^{b} (\theta - P_H^\ell) dF(\theta).$$
The first integral aggregates the payoff of known low quality services over the region of the preference distribution that consumes these services. The second integral aggregates the expected payoff of services of unknown quality over the region of the preference distribution that consumes these services. The third integral aggregates the payoff of observably high quality services over the region of the preference distribution that consumes these services. The corresponding welfare expression associated with threshold certification replaces the licensing equilibrium outcomes with their analogs under threshold certification, recalling that $s_U = 0$.\footnote{The corresponding welfare expression for a precise certification equilibrium would lack the second term in this expression. The unique equilibrium is a separating equilibrium, so consumers can identify the quality of any provider.}

Before conducting the following welfare comparisons, it will be helpful to introduce the inefficiencies associated with asymmetric information. Shapiro (1986) notes when discussing the equilibrium without regulation that these are twofold. First, there is a mismatch inefficiency for any level of average quality. Consumers of type $\theta \in (\theta_L, \theta_H)$ prefer the submarket in which which services are supplied by a mixture of low quality and high quality providers. These providers cannot be distinguished in the absence of regulation. By assumption, they are allocated to consumers at random. In contrast, under perfect information high quality providers are matched with the consumers in $(\theta_L, \theta_H)$ who have the highest marginal valuations $\theta$. The market clearing condition for these services (6) ensures that the mismatch inefficiency is relevant for an exogenous measure $1 - w$ of consumers in the absence of regulation.

Second, Shapiro (1986) notes that average quality provision is inefficiently low in equilibrium; the social marginal benefit of high quality exceeds its social marginal cost. The marginal benefit of the type $\theta_L$ and $\theta_H$ consumers from services of higher quality in expectation must exactly offset the marginal cost of purchasing these services in
equilibrium. From the indifference conditions (3) and (4):

\[ s\theta_L + (1 - s)\theta_H = P_H - P_L. \]

In the presence of asymmetric information, high quality providers receive \( P_0 \) when their quality is not revealed. This does not compensate them for the additional cost incurred to provide high quality. From the free entry condition (2), providers only choose high quality in equilibrium if they receive \( P_H > \bar{C}_H \) when their quality is revealed. From the free entry conditions (1) and (2) with \( K_R = K_L \):

\[ P_H - P_L = \frac{\bar{C}_H - \bar{C}_L}{w}. \]

The marginal consumers must benefit from high quality sufficiently to offset this premium in equilibrium. However, this premium exceeds the social marginal cost of high quality, which is given by \( \bar{C}_H - \bar{C}_L \). Asymmetric information thus drives a wedge between the social marginal benefit and social marginal cost of high quality in equilibrium. Regulation is capable of mitigating these inefficiencies.

5.2 Comparing Threshold and Precise Certification

Threshold certification and precise certification differ only in that they reveal differing degrees of information. In some cases, the degree of information conveyed by certification status does not affect the equilibrium; both implement the perfect information allocation. This occurs when both condition (7) and condition (15) hold. If condition (7) holds but condition (15) does not, precise certification dominates threshold certification by granting providers access to more informative signals.\(^{16}\) If neither condition holds, threshold certification can dominate precise certification by limiting excessive

\(^{16}\)This assumes that the certification threshold is the licensing standard \( K_R \). However, if it were set to \( K_H \), threshold certification and precise certification would implement the same allocation.
human capital investment. This subsection compares both forms of certification in the latter two cases.

First, consider when condition (7) holds and condition (15) does not. In this case, precise certification implements the perfect information allocation, whereas threshold certification gives rise to a partially pooling equilibrium. Precise certification dominates both threshold certification and licensing through two welfare effects. First, precise certification yields a separating equilibrium, which removes the mismatch inefficiency altogether. Second, this separating equilibrium is not supported by inefficient human capital investment. This removes the wedge between the social marginal benefit of high quality and its social marginal cost, restoring the optimal average level of quality. The limited capability to signal quality offered by threshold certification decreases welfare when the threshold is $K_R$. High quality providers could have distinguished themselves without excessively investing in human capital.

Next, consider when neither condition (7) nor condition (15) holds. Shapiro (1986) shows in Proposition 6 that precise certification brings about a separating equilibrium. However, this equilibrium is supported by high quality providers’ excessive investment in human capital as a signaling device. This inefficiency mirrors that of the well known job market signaling model presented in Spence (1973). Although precise certification removes the mismatch inefficiency entirely, it is associated with inefficiently low quality in equilibrium. Recall that when condition (7) does not hold, high quality providers invest $K_S > K_H$. In this case, the free entry condition pins down $P_H = C_H(K_S)$. High quality providers require a premium in equilibrium, relative to the perfect information case, to compensate for their increased investment. This drives a wedge between the social marginal benefit and social marginal cost of high quality; markets clear with inefficiently low average quality. Average quality is pinned down by the cutoff rule and the observation that submarkets must clear in
equilibrium. As noted in Section 4.3, this cutoff is given by:

\[ \theta^* = C_H(K_S) - \bar{C}_L > \bar{C}_H - \bar{C}_L. \]

This cutoff implies that a smaller measure of consumers chooses high quality in equilibrium when condition (7) does not hold than in the perfect information equilibrium. Consequently, there is scope for threshold certification to dominate precise certification in this case. Threshold certification offers providers a limited capability to signal quality, thereby limiting inefficient human capital investment in equilibrium. Shapiro (1986) shows in Proposition 8 that when the incentive to increase investment is particularly strong under precise certification, it is dominated by licensing. In Section 5.3 I show that threshold certification weakly dominates licensing when the threshold is set to the licensing standard \( K_R \). This in turn implies that threshold certification can dominate precise certification.

In summary, precise certification differs from licensing along two dimensions. First, it does not restrict entry into the occupation. Second, it reveals a provider’s human capital investment exactly. Shapiro (1986) shows in Proposition 8 that licensing can dominate precise certification. By contrast, threshold certification differs from licensing only in that it does not restrict entry into the occupation. I next show that the capability to forego increasing \( K \) allows threshold certification to weakly dominate licensing. From this, we can infer that revealing additional information about providers’ human capital can be associated with welfare losses.\(^{17}\)

\(^{17}\)Lizzeri (1999) draws a similar conclusion. That paper shows that a certification intermediary can improve its profits by withholding information. The optimal policy of this intermediary is one that closely resembles threshold certification: regulators reveal only whether a provider’s quality exceeds some threshold. However, that conclusion is the result of a crucial assumption about how the certification intermediary collects revenue. In Lizzeri (1999), the intermediary contracts with providers to certify quality. Albano and Lizzeri (2001) note that if instead the intermediary contracted with consumer, the optimal policy would be to reveal quality fully.
5.3 Comparing Threshold Certification and Licensing

I now show that threshold certification weakly dominates licensing when it reveals the same information as licensing about a provider’s human capital. This dominance is strict if \( s'_A > s^f \). On the other hand, consumers are indifferent between certification and licensing when \( s'_A = s^f \) and \( \phi = 0 \). All providers obtain a license, although this is not required.

**Proposition 1.** Certification weakly dominates licensing \((W^t \geq W^f)\).

See Appendix A for the proof of this proposition. The proof proceeds in two steps. In the first step, I show that if \( s'_A \geq s^f \), \( W^t \geq W^f \). The proof of this lemma stems from the recognition that there are three sets of variables which may differentially affect welfare in licensing and certification equilibria: prices, expected quality, and the consumption rule. I first show that differences in equilibrium prices do not directly affect welfare. I then show that assumed differences in expected quality yield welfare improvements for threshold certification. Differing consumption rules result in further welfare improvements. In the second step of the proof of Proposition 1, I show that \( s'_A \geq s^f \) in all equilibria.

One way to interpret this result is to consider the effects of licensing and threshold certification on the two inefficiencies associated with asymmetric information. These two policies reveal the same information about a provider’s human capital, so they both increase inefficiently low average quality through the same mechanism. Assuming \( K_L < K_R \leq K_H \), licensing increases the total cost of providing low quality service. Similarly, threshold certification increases the total cost of providing low quality service for certified providers. Both policies thus reduce the incremental cost of providing high quality. In response to this, high quality providers require a smaller \( P_H \).

---

\(^{18}\)More generally, if \( K_L < K_H < K_R \), both policies increase the cost of providing both levels of service quality. However, the assumption that \( c_H(K) - c_L(K) \) is decreasing ensures that the incremental cost of high quality has decreased \([C_H(K_R) - C_L(K_R)] < C_H - C_L\].
the free entry conditions under licensing (1) and (2) with \( K_R > K_L \):

\[
P_H - P_L = \frac{\bar{C}_H - C_L(K_R)}{w}.
\]

The premium \( P_H - P_L \) is reduced as a result of licensing and threshold certification. Consequently, so is the marginal benefit of the indifferent consumers that exactly offsets this premium in equilibrium. This shrinks the wedge between the social marginal benefit and social marginal cost of high quality in equilibrium. Through this mechanism, both policies indirectly increase average quality in equilibrium.\(^{19}\)

By treating the information content of licensing and certification symmetrically, I can focus on the welfare consequences of the entry limitations imposed by licensing.\(^{20}\) Threshold certification removes the restriction on entry by allowing low quality providers to practice without increasing \( K \). Consumers can identify uncertified providers’ quality in equilibrium, even if their quality is not revealed. This endogenously shrinks the supply of unknown quality services, as reflected in the unknown quality market clearing condition (14).\(^{21}\) Threshold certification thereby reduces the measure of providers who are randomly allocated. By leaving providers’ human capital decision unconstrained, threshold certification mitigates the mismatch inefficiency associated with asymmetric information.

\(^{19}\)By contrast, licensing in Leland (1979) and Wiswall (2007) affects the distribution of quality more directly. In Leland (1979), regulators set a minimum quality standard which truncates the distribution. In Wiswall (2007), regulators require providers to meet a minimum human capital standard. This affects the distribution of quality directly by improving inputs into the production function, rather than by reducing the incremental cost of providing high quality.

\(^{20}\)Note that the free entry assumption assures that there are no monopoly rents in a licensing equilibrium. Threshold certification would dominate licensing to an even greater extent if licensing were associated with monopoly rents.

\(^{21}\)In this way, the capability to forego certification acts like an increase in \( w \). Using this insight, we can interpret Proposition 1 as an application of Proposition 2 in Shapiro (1986) and Theorem 2 in Shapiro (1983).
6 A Welfare Improving Alternative

Threshold certification weakly dominates licensing by changing only who may practice. However, this policy is not necessarily optimal among the set of all certification policies. Whenever threshold certification is associated with partial pooling, some low quality providers invest resources into human capital in excess of $K_L$. This suggests that welfare would be improved further if these resources were extracted by the regulator. Consequently, I consider an alternative to threshold certification in which a regulator certifies both providers who pay a bribe $\tau_0$ and those with $K \geq K_R$. Any revenue that the regulator raises represents a surplus which may be included in the welfare calculation. The payoffs of quality $q$ providers that choose to pay the bribe are given by the following:

$$wP_q + (1 - w)P_A - \tau_0 - C_q(K).$$

Note that consumers still have the capability to distinguish providers of unobservable quality using binary certification status. The only difference is that certification status no longer ensures consumers that all certified providers have $K \geq K_R$.

I consider a special case in which the bribe is chosen to be $\tau_0 = C_L(K_R) - \bar{C}_L$, where $K_R$ is the licensing standard. Furthermore, I assume that all low quality providers that choose to become certified in equilibrium do so by paying the bribe.\textsuperscript{23} Given

\textsuperscript{22}This bribe could equivalently be interpreted as a tax. These interpretations only differ in who receives the extracted resources. If $\tau_0$ is interpreted as a bribe, the most natural assumption is that the regulator receives the resources. If $\tau_0$ is interpreted as a tax, the most natural assumption is that the resources are redistributed to consumers.

\textsuperscript{23}For $\tau_0 = C_L(K_R) - \bar{C}_L$, low quality providers are indifferent between paying the bribe and increasing $K$ to become certified, even out of equilibrium. Low quality providers’ indifference between the certification options yields a continuum of equilibria. These equilibria differ in the fraction of low quality certified providers that pay the bribe.
this assumption, the regulator’s payoff can be stated as:

\[(1 - s_A)(1 - \phi)\tau_0.\]

In this case, the equilibria associated with this certification policy and with threshold certification have the same definition.

*Ex ante* homogeneous providers must be indifferent between the options chosen in equilibrium. This introduces the following necessary condition for equilibrium.

\[wP_L + (1 - w)P_A - \tau_0 \leq \bar{C}_L.\] (16)

This condition must hold with equality if any low quality provider chooses to become certified in equilibrium. High quality providers prefer to acquire a certificate by choosing their cost-minimizing investment \(K_H\). As a result, they are unaffected by the bribe. Low quality providers that choose not to become certified act as they would have in the absence of regulation. Consequently, the free entry conditions (8) and (10) are also relevant under this policy for uncertified low quality and high quality providers, respectively. Additionally, the consumers’ indifference conditions (11) and (12) must hold in equilibrium, and the market clearing conditions (13) and (14) must hold as well.

Introducing the capability to pay a bribe to become certified thus weakly improves welfare\(^{24}\).

**Proposition 2.** \(W^r > W^t\).

The proof of this proposition is given in Appendix \(^B\). Intuitively, the increased

\(^{24}\text{Consequently, the assumption that all low quality providers who become certified do so by paying the bribe is unnecessary in the presence of a welfare-maximizing regulator. Instead, this occurs endogenously in equilibrium. The welfare gain is strictly increasing in the fraction of certified low quality providers paying the bribe. The regulator thus has an incentive to limit the possibility of becoming certified by investing in } K \geq K_R\). For instance, the regulator may limit the fraction of certificates granted on the basis of human capital to } s_A\).
investment that is induced by certification affects only the total cost of providing low quality. A bribe can replicate this effect. When providers become certified by paying a bribe, the increase in the total cost of providing low quality is revenue rather than a sunk cost of investment. This revenue improves the regulator’s surplus, whereas the sunk cost does not. Human capital has no other effect in this model, so it is welfare improving to pay the increased total cost in the form of a bribe.

Given that regulators can weakly improve welfare by offering the option of purchasing a certificate through a bribe, what level of bribe is optimal? I next show that it is weakly welfare improving to increase the bribe and the certification threshold until \( \tau_0 = C_L(K_H) - \bar{C}_L \).

**Proposition 3.** \( \frac{\partial W}{\partial \tau_0} \geq 0 \) for \( \tau_0 \leq C_L(K_H) - \bar{C}_L \).

See Appendix C for the proof of this proposition. Proposition 3 shows that it is weakly welfare improving to increase \( \tau_0 \) and the certification threshold until high quality providers are indifferent between the two methods of acquiring a certificate. On one hand, this increase in \( \tau_0 \) yields a welfare gain for consumers of unknown quality services. They encounter high quality providers more frequently. On the other hand, there may be a countervailing welfare loss for the regulator. Fewer low quality providers become certified in response to the increase in \( \tau_0 \), which may reduce revenue. However, Proposition 3 establishes that the welfare gains outweigh any welfare losses.

### 7 Conclusion

In this paper, I introduce an alternative form of certification into the moral hazard model of professional services presented in Shapiro (1986). This form of certification

\footnote{The proof given in Appendix C applies to the case of a discrete distribution of consumer types and for changes in \( \tau_0 \). The extension of this proof to the continuous type case is work in progress.}
reveals only whether a provider’s human capital investment exceeds the licensing standard. I show that this form of certification weakly dominates licensing. This implies that threshold certification can dominate precise certification as presented in Shapiro (1986); regulators can improve welfare by withholding information. Precise certification grants providers more scope to signal quality. However, this carries with it the incentive to invest in human capital as a signaling device, which can increase prices in equilibrium without increasing average quality. Although threshold certification limits providers’ scope to signal quality, it also limits the cost of signaling. Consequently, when the incentive to invest in the costly signal is sufficiently strong, threshold certification dominates precise certification.

I show that regulators can further improve welfare by certifying both providers that pay a bribe and those whose investments meet some threshold. This bribe replicates the effect of human capital investment on average quality, and any revenue generates surplus for the regulator. I also show that regulators can weakly improve welfare by increasing this bribe and its associated certification threshold until high quality providers are indifferent between the two methods of acquiring a certificate.

This paper opens a number of avenues for future research. My findings suggest that regulators may weakly improve welfare by replacing current licensing regulations with a certification policy that simply publicizes which providers would have met the licensing standard. I plan to estimate the welfare effects of such a policy in future work. I progress in this direction by estimating the relationship between professional licensing policies and training in Klee (2010). A necessary condition for certification to yield welfare gains relative to licensing is that licensing regulations must bind. I thus examine whether there is scope for this certification policy to improve welfare. My findings suggest that welfare gains are more likely to come from alternatives to this certification policy. This underscores the need to characterize the optimal regulatory regime, which I plan to do in future work.
Finally, I show in this paper that Shapiro’s (1986) result that licensing can dominate certification depends crucially on his assumption about the information conveyed by licensing. While I employ a model that allows for a clear comparison of licensing and certification, the model also contains numerous stylized components. In future work, I plan to conduct a similar welfare comparison in the presence of more realistic mechanisms that cast doubt on the superiority of certification. For example, I plan to compare licensing and certification in the presence of negative externalities and rational inattention. These are often cited as justification for licensing.
Figure 1: The effort cost of providing quality $q \in \{L, H\}$
Figure 2: Summary of licensing equilibrium

Figure 3: Summary of certification equilibrium
References


Svorny, Shirley, “Licensing Doctors: Do Economists Agree?,” Econ Journal

A Proof of Proposition 1

The proof of this claim proceeds in two steps. In Lemma 1, I show that if \( s_A^t \geq s^\ell \), then certification strictly dominates licensing. In Lemma 2, I show that \( s_A^t \geq s^\ell \) in all equilibria. These lemmas combine to imply that certification weakly dominates licensing in all equilibria.

**Lemma 1.** If \( s_A^t \geq s^\ell \), then \( W^t \geq W^\ell \).

*Proof.* The strategy for this proof stems from the recognition that there are three sets of variables which may differentially affect welfare in licensing and certification equilibria: prices, expected quality, and the consumption rule. I first show that differences in equilibrium prices do not directly affect welfare. I then show that assumed differences in expected quality yield welfare improvements for certification. Differing consumption rules result in further welfare improvements.

The free entry conditions (1) and (2) define a set of price triples \((P_L, P_0, P_H)\) that may occur in a licensing equilibrium. The indifference conditions (3) and (4) simply pin down which element of this set occurs in equilibrium. All members of this set generate the same aggregate revenue for providers, given the consumption policy associated with a licensing equilibrium. To see this, note that the aggregate revenue raised by a licensing equilibrium can be expressed as:

\[
\rho = F(\theta_L^L)P_L^L + [F(\theta_H^L) - F(\theta_L^L)] P_0^L + [1 - F(\theta_H^L)] P_H^L.
\]

This expression aggregates the price of each kind of service using the measure of consumers in that submarket in equilibrium. The free entry conditions (1) and (2)
imply that this can be expressed equivalently as follows:

\[
\rho = F(\theta^L) \left( \frac{C_L(K_R)}{w} - \frac{(1 - w)P_0^L}{w} \right) + \left[ F(\theta^H) - F(\theta^L) \right] P_0^L \\
+ \left[ 1 - F(\theta^H) \right] \left( \frac{C_H}{w} - \frac{(1 - w)P_0^H}{w} \right).
\]

Next, I apply the market clearing condition for unknown quality \([6]\) twice to the \(P_0^L\) components of the first and last terms of this expression. All \(P_0^L\) terms then cancel, yielding the following expression for the aggregate revenue raised in a licensing equilibrium:

\[
\rho = F(\theta^L) \frac{C_L(K_R)}{w} + \left[ 1 - F(\theta^H) \right] \frac{C_H}{w}.
\]

This expression does not depend on the particular price triple pinned down by the consumer indifference conditions in equilibrium. Rather, any price triple satisfying the free entry conditions \([1]\) and \([2]\) would raise the same revenue given the consumption policy associated with a licensing equilibrium. Consequently, \(W^\ell\) could be expressed equivalently in terms of any price triple satisfying the free entry conditions \([1]\) and \([2]\). The free entry conditions \([9]\) and \([10]\) ensure that the price triple \((P_L^t, P_A^t, P_H^t)\) which occurs in a threshold certification equilibrium is necessarily a member of this set. Using this insight, we can express \(W^\ell\) in the following way:

\[
W^\ell = \int_a^{\theta_L^t} (-P_L^t) \, dF(\theta) + \int_{\theta_L^t}^{\theta_H^t} (s^t \theta - P_A^t) \, dF(\theta) + \int_{\theta_H^t}^b (\theta - P_H^t) \, dF(\theta).
\]

Next, if \(s^t \leq s_A^t\), then consumers in the unknown quality submarket in a licensing regime would weakly benefit from encountering high quality providers as frequently
as their counterparts under threshold certification:

\[
W^\ell \leq \int_a^{\theta_L^t} (-P_L^t) \, dF(\theta) + \int_{\theta_L^t}^{\theta_H^t} (s_A^t \theta - P_A^t) \, dF(\theta) + \int_{\theta_H^t}^b (\theta - P_H^t) \, dF(\theta).
\]

This inequality implies that licensing is weakly dominated by a policy that offers consumers the equilibrium payoffs associated with threshold certification but constrains them to follow the optimal consumption policy associated with licensing. However, this consumption rule may be suboptimal under threshold certification, given the differences in prices and expected quality. A necessary condition for a threshold certification equilibrium to exist is that consumers choose services optimally. This implies that:

\[
W^t = \int_a^b \max \{ -P_L^t, s_A^t \theta - P_A^t, \theta - P_H^t \} \, dF(\theta),
\]

\[
\geq \int_a^{\theta_L^t} (-P_L^t) \, dF(\theta) + \int_{\theta_L^t}^{\theta_H^t} (s_A^t \theta - P_A^t) \, dF(\theta) + \int_{\theta_H^t}^b (\theta - P_H^t) \, dF(\theta).
\]

This implies that if \( s_A^t \geq s^t \), \( W^t \geq W^\ell \)

I now show that \( s_A^t \geq s^\ell \) in all equilibria. The proof of this claim proceeds in three steps. First, I show that this is true when low quality providers can profitably forego certification. Second, I show that this is true when low quality providers cannot profitably forego certification. Third, I show that a certification equilibrium is unique. Therefore, no equilibria exist in which \( s_A^t < s^\ell \).

**Lemma 2.** \( s_A^t \geq s^\ell \).

**Proof.** Case 1: If \( P_L^t > \bar{C}_L \), \( s_A^t > s^\ell \).

In this case, licensing equilibrium behavior does not constitute an equilibrium under certification. The assumption on \( P_L^t \) violates the free entry condition for uncertified low quality providers. This is a necessary condition for a threshold certification equilibrium to exist. Consequently, an equilibrium must feature \( \phi > 0 \).
Next, denote the equilibrium marginal cost \( P_H - P_0 \) to type \( \theta_H \) consumers in a licensing equilibrium \( \Delta_H(P_L) \). This equilibrium object varies with \( P_L \) across equilibria. To characterize this relationship, first apply the free entry condition \( [2] \) for high quality providers and the free entry condition \( [1] \) for low quality providers. We could apply the free entry conditions \( [9] \) and \( [10] \) for a threshold certification equilibrium to find the same relationship in terms of \( P_L \) rather than \( P_0 \):

\[
P_H - P_0 = \frac{(1 - w)\bar{C}_H - C_L(K_R) + wP_L}{w(1 - w)}.
\]

From this expression, we can identify \( \Delta_H(P_L) \) as being larger in equilibria with larger \( P_L \). Intuitively, if low quality providers receive a higher price when their quality is revealed, they require a lower price when their quality is not revealed in order to compensate them for investing \( K_R > K_L \). Similarly, high quality providers observe this decreased price when their quality is not revealed, and require a larger \( P_H \) to compensate them for the additional total cost of providing high quality. Given the assumption that \( P_L^t > \bar{C}_L \) and its implication that \( \phi > 0 \), we can infer that \( P_L^t > P_L^t \) from the free entry condition \( [8] \). This in turn implies \( \Delta_H(P_L^t) > \Delta_H(P_L^t) \). For a given (conditional) expected quality \( s \), this implies:

\[
1 - F\left(\frac{\Delta_H(P_L^t)}{1 - s}\right) < 1 - F\left(\frac{\Delta_H(P_L^t)}{1 - s}\right).
\]

The inequality above imposes the indifference conditions \( [4] \) and \( [12] \) for a licensing and threshold certification equilibrium, respectively, to state \( \theta_H \) in terms of prices and expected quality. Intuitively, price differences in this case increase the demand for observably high quality services under threshold certification relative to licensing, for a given \( s \). Additionally, \( \phi > 0 \) implies that for a given (conditional) expected quality \( s \) the supply of observably high quality services is larger under licensing than certification. Although the fraction of high quality providers among those who choose
\( K \geq K_R \) is the same for a given \( s \), a smaller measure of providers choose \( K \geq K_R \) under threshold certification than under licensing \([ws > ws(1 - \phi)]\). Since these inequalities hold for any given \( s \), they also hold for \( s^\ell \):

\[
1 - F\left(\frac{\Delta_H(P_L^\ell)}{1 - s^\ell}\right) > ws^\ell(1 - \phi).
\]

Thus, the submarket of observably high quality services requires a higher expected quality to clear under certification than under licensing \((s_A^\ell > s^\ell)\). Intuitively, increasing \( s \) reduces the improved expected quality of observably high quality services relative to services of unknown quality. This offsets the increased demand caused by differences in relative prices between licensing and threshold certification. This also offsets the reduced supply resulting from the smaller measure of providers under threshold certification that choose \( K \geq K_R \).

**Case 2:** If \( P_L^\ell \leq \bar{C}_L \), \( s_A^\ell = s^\ell \)

When this condition holds, it is an equilibrium under a threshold certification regime for providers and consumers to act as if they were in a licensing regime. All providers become certified \((\phi = 0)\), though this is not required. Evaluating the certification equilibrium conditions at the licensing equilibrium allocation:

\[
P_L^\ell \leq \bar{C}_L,
\]

\[
wP_L^\ell + (1 - w)P_0^\ell = C_L(K_R),
\]

\[
wP_H^\ell + (1 - w)P_0^\ell = \bar{C}_H,
\]

\[
s^\ell \theta_L^\ell = P_0^\ell - P_L^\ell,
\]

\[
(1 - s^\ell)\theta_H^\ell = P_H^\ell - P_0^\ell,
\]

\[
1 - F(\theta_H^\ell) = ws^\ell,
\]

\[
F(\theta_H^\ell) - F(\theta_L^\ell) = 1 - w.
\]
The free entry condition for uncertified low quality providers holds with weak inequality by assumption. If this equilibrium condition does not hold with equality, \( P_L^\ell \) may be an equilibrium price under certification so long as \( \phi = 0 \). All other conditions for a certification equilibrium with \( \phi = 0 \) must hold in a licensing equilibrium. This shows that if \( P_L^\ell \leq \bar{C}_L \), then the licensing and threshold certification equilibria coincide, in which case \( s_A^t = s^\ell \).

In summary, Case 1 shows that if \( P_L^\ell > \bar{C}_L \), then \( s_A^t > s^\ell \). Case 2 shows that if \( P_L^\ell \leq \bar{C}_L \), then \( s_A^t = s^\ell \). If these equilibria are unique, this completes the proof that \( s_A^t \geq s^\ell \).

The strategy of the uniqueness proof is to first show that there are at most two certification equilibria: one with \( \phi > 0 \) and one with \( \phi = 0 \). I then show that both of these cannot exist for a given set of parameter values. I consider only threshold certification equilibria in which \( 0 < s_A < 1 \), since regulation is irrelevant otherwise.

Regardless of whether \( \phi > 0 \), the indifference conditions (11) and (12) for a threshold certification equilibrium and the free entry condition (9) imply:

\[
s_A \theta_L = \Delta_L(P_L) = \left( \frac{C_L(K_R) - wP_L}{1 - w} \right) - P_L,
\]

\[
(1 - s_A) \theta_H = \Delta_H(P_L).
\]

The first of these equations expresses the marginal cost to a consumer of moving from the submarket of observably low quality to the submarket of unknown quality services. From this expression, it is clear that \( \Delta_L(P_L) \) is decreasing. Recall that previously in this proof I showed that \( \Delta_H(P_L) \) is increasing. Demand for services of unknown quality comes from the mass of consumers with marginal valuations \( \theta_L < \theta < \theta_H \). Consequently, these two equations can be used to express demand for services of unknown quality in terms of \( s_A \) and \( P_L \).

Similarly, the supply of services of unknown quality is given by \((1 - w)(1 - \phi)\)
The market clearing condition for observably high quality services implies a relationship between \( \theta_H \) and \( s_A \). Stating \( \theta_H \) in terms of \( P_L \) and \( s_A \) yields the following expression for supply of services of unknown quality in terms of \( s_A \) and \( P_L \):

\[
1 - \phi = \frac{1}{ws_A} \left[ 1 - F\left( \frac{\Delta_H(P_L)}{1 - s_A} \right) \right].
\]

We can then combine these expressions using the market clearing condition for unknown quality services. This implies a relationship between \( P_L \) and \( s_A \) which must hold in equilibrium:

\[
F\left( \frac{\Delta_H(P_L)}{1 - s_A} \right) - F\left( \frac{\Delta_L(P_L)}{s_A} \right) = (1 - w) \left( \frac{1}{ws_A} \right) \left[ 1 - F\left( \frac{\Delta_H(P_L)}{1 - s_A} \right) \right].
\]

There may be many pairs of \( P_L \) and \( s_A \) for which this equality is satisfied. However, for a given \( P_L \), demand for services of unknown quality is strictly increasing in \( s_A \), whereas supply of these services is strictly decreasing in \( s_A \). Consequently, for a given \( P_L \), a unique \( s_A \) clears the submarket of unknown quality services. In a certification equilibrium with \( \phi > 0 \), uncertified low quality providers pin down \( P_L = \bar{C}_L \). This unique \( P_L \) pins down a unique \( s_A \). As a result, we can conclude that there is at most one certification equilibrium with \( \phi > 0 \).

In addition, there is at most one certification equilibrium with \( \phi = 0 \). This cannot be shown by extending the proof of the previous claim, since there is not necessarily a unique \( P_L \) for \( \phi = 0 \). However, the necessary conditions summarizing a threshold certification equilibrium coincide with those summarizing a licensing equilibrium when \( \phi = 0 \). Shapiro (1986) shows in Proposition 1 that a laissez-faire equilibrium is unique. This proof is easily extended to show the uniqueness of a licensing equilibrium, and thereby a threshold certification equilibrium with \( \phi = 0 \). Indeed, the laissez-faire equilibrium is simply a special case of a licensing equilibrium in which \( K_R = K_L \). When \( K_R > K_L \), only the total cost of providing low quality changes.
This changes the equilibrium, but does not affect the uniqueness of the equilibrium.

What remains to be shown in order to prove the uniqueness of a certification equilibrium is that no set of parameter values can support both of these equilibria. The proof of this point proceeds by contradiction. Suppose that some set of parameter values can support both a certification equilibrium with $\phi > 0$ and a licensing equilibrium with $P_L \leq C_L$. This implies that $P^t_L \leq P^t_L$. This inequality implies that, for a given (conditional) expected quality $s$:

$$F \left( \frac{\Delta_H(P^t_L)}{1 - s} \right) - F \left( \frac{\Delta_L(P^t_L)}{s} \right) \leq F \left( \frac{\Delta_H(P^t_L)}{1 - s} \right) - F \left( \frac{\Delta_L(P^t_L)}{s} \right).$$

For a given $s$, price differences increase the demand for unknown quality services at both margins under threshold certification relative to licensing. Additionally, $\phi > 0$ implies that the supply of services of unknown quality is larger under licensing than certification $[1 - w > (1 - w)(1 - \phi)]$. Since these inequalities hold for any given $s$, they also hold for $s^t$:

$$F \left( \frac{\Delta_H(P^t_L)}{1 - s^t} \right) - F \left( \frac{\Delta_L(P^t_L)}{s^t} \right) > (1 - w)(1 - \phi).$$

Thus, the submarket of unknown quality requires less expected quality to clear under certification than licensing ($s^t_A < s^t$). Intuitively, decreasing $s$ reduces the improved expected quality of unknown quality services relative to observably low quality services. Similarly, decreasing $s$ increases the improved expected quality of observably high quality services relative to services of unknown quality. This offsets the increased demand for services of unknown quality at both margins.

The implications that $P^t_L \leq P^t_L$ and $s^t > s^t_A$ imply that $\theta^t_H < \theta^t_H$. From the indifference conditions (4) and (12) for type $\theta_H$ consumers in a licensing and threshold

---

I assume this for ease of notation. The equilibrium behavior of licensing with $P^t_L \leq C_L$ and certification with $\phi = 0$ coincide. Consequently, this is equivalent to examining whether some set of parameter values can support a certification equilibrium with $\phi > 0$ and one with $\phi = 0$. 

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certification equilibrium, respectively:

$$\frac{\Delta_H(P_L^c)}{1 - s^c} < \frac{\Delta_H(P_L^c)}{1 - s_A^c}.$$  

Given this implication, we can also infer that $\theta_L^t < \theta_L^t$. This is implied by the unknown quality market clearing conditions which must hold in a licensing equilibrium with $P_L^t \leq P_L^t$ and a certification equilibrium with $\phi > 0$:

$$F(\theta_H^t) - F(\theta_L^t) = 1 - w > (1 - w)(1 - \phi) = F(\theta_H^t) - F(\theta_L^t).$$

However, $\theta_L^t < \theta_L^t$, $\theta_H^t < \theta_H^t$, and $s_A^t < s^t$ imply a contradiction. The indifference conditions (3) and (4) must hold in a licensing equilibrium, as must their analogs (11) and (12) in any certification equilibrium. The free entry conditions (1) and (2) for low quality certified providers and high quality providers must also hold in a licensing equilibrium, as must their analogs (9) and (10) in a threshold certification equilibrium. This implies:

$$s^t \theta_L^t + (1 - s^t)\theta_H^t = \frac{C_H - C_L(K_R)}{w} = s_A^t \theta_L^t + (1 - s_A^t)\theta_H^t.$$  

This is a contradiction because only one of these equalities can hold. The certification expression places more weight $(1 - s_A^t > 1 - s^t)$ on the largest component $(\theta_H^t > \theta_H^t)$. Consequently, no set of parameter values can support both a certification equilibrium with $\phi > 0$ and a licensing equilibrium with $P_L^c \leq C_L$. From the equivalence of a licensing equilibrium with $P_L^c \leq C_L$ and a certification equilibrium with $\phi = 0$, this implies that a certification equilibrium is unique. \[\square\]

To sum up the proof of Proposition 1, Lemma 1 shows that if $s_A^t \geq s^t$, then certification weakly dominates licensing. Lemma 2 then shows that this is the case in all equilibria. It then follows that certification weakly dominates licensing.
B Proof of Proposition 2

The necessary conditions for a certification equilibrium which offers the option of purchasing the certificate through a bribe are given by the following system. Again assuming $0 < s_A < 1$:

\[
P_L \leq \bar{C}_L,
\]

\[
wP_L + (1 - w)P_A - \tau_0 \leq \bar{C}_L,
\]

\[
wP_H + (1 - w)P_A = \bar{C}_H,
\]

\[
s_A \theta_L = P_A - P_L,
\]

\[
(1 - s_A) \theta_H = P_H - P_A,
\]

\[
F(\theta_H) - F(\theta_L) = (1 - w)(1 - \phi),
\]

\[
1 - F(\theta_H) = w s_A(1 - \phi).
\]

For $\tau_0 = C_L(K_R) - \bar{C}_L$, the capability to become certified by paying a bribe leaves the equilibrium conditions unaffected. The uniqueness of the certification equilibrium thus implies that the introduction of the bribe does not affect the equilibrium value of the set \{P_L, P_A, P_H, \theta_L, \theta_H, s_A, \phi\}. This implies that aggregate welfare under this certification regime, which I denote $W^\tau$, can be expressed as:

\[
W^\tau = \int_{a}^{\theta^*_L} (-P^*_L) dF(\theta) + \int_{\theta^*_L}^{\theta^*_H} (s_A^i \theta - P^*_A) dF(\theta) + \int_{\theta^*_H}^{b} (\theta - P^*_H) dF(\theta)
\]

\[
+ (1 - s_A)(1 - \phi) \tau_0.
\]

The assumption that $\tau_0 = C_L(K_R) - \bar{C}_L > 0$, combined with the uniqueness of a certification equilibrium, implies that $W^\tau = W^t + (1 - s_A)(1 - \phi) \tau_0 \geq W^t$. Introducing the capability to become certified by paying a bribe thus weakly improves welfare. If any low quality provider becomes certified in equilibrium, this strictly improves welfare.
welfare.

C Proof of Proposition 3

To show this, I consider a discrete distribution of $N$ consumer types. Consumer type $i$ has marginal valuation of quality $\theta_i$, where $\theta_1 < \ldots < \theta_N$. Fraction $f_i$ of all consumers are of type $i$. Fraction $\mu_i$ of type $i$ consumers choose the submarket of unknown quality.

The proof of this claim proceeds in three steps. First, I show in Lemma 3 that we need only consider equilibria in which one consumer type is indifferent between two submarkets. Additionally, I consider only small increases in $\tau_0$ that do not affect which consumer type is indifferent. Second, I show in Lemma 4 that welfare is weakly increasing in $\tau_0$ when consumer type $m$ is indifferent between the submarkets of known low quality and unknown quality. Third, I show in Lemma 5 that welfare is strictly increasing in $\tau_0$ when consumer type $n$ is indifferent between the submarkets of unknown quality and observably high quality.

Lemma 3. An equilibrium in which two consumers are indifferent between different submarkets occurs only in a knife-edge case.

Proof. The free entry conditions (10), (8), and (16) for high quality, uncertified low quality, and certified low quality providers, respectively, are still relevant for the case of a discrete preference distribution. The consumer indifference conditions (11) and (12) are also relevant for the case of a discrete preference distribution. To make notation in these equilibrium conditions consistent with a discrete distribution, I denote by $\theta_m$ the consumer type that is indifferent between known low quality services and unknown quality services. Similarly, I denote by $\theta_n$ the consumer type that is indifferent between services of unknown quality and observably high quality services. In addition to these, the following market clearing conditions must hold for observably
high quality services and unknown quality services, respectively:

\[(1 - \mu_n)f_n + \sum_{i=n+1}^{N} f_i = s_Aw(1 - \phi), \quad (17)\]

\[\mu_m f_m + \sum_{i=m+1}^{n-1} f_i + \mu_n f_n = (1 - w)(1 - \phi). \quad (18)\]

The indifference conditions \((11)\) and \((12)\) summarize the consumers’ optimal decision. Consumers of type \(\theta_i, \forall i < m\) strictly prefer the known low quality submarket. Consumers of type \(\theta_i, \forall m < i < n\) strictly prefer the unknown quality submarket. Consumers of type \(\theta_i, \forall i > n\) strictly prefer the submarket of observably high quality. Given this consumption rule, demand for observably high quality services comes from the mass of consumers with \(\theta > \theta_n\), and from the mass of type \(n\) consumers choosing observably high quality services, \((1 - \mu_n)f_n\). Similarly, demand for services of unknown quality comes from the mass of consumers with \(\theta_m < \theta < \theta_n\) and from the mass of type \(m\) and \(n\) consumers choosing services of unknown quality \(\mu_m f_m\) and \(\mu_n f_n\), respectively.

In order to determine the region of the parameter space in which such an equilibrium can exist, I begin by finding the implication of the indifference conditions \((11)\) and \((12)\), which must hold in any equilibrium. By subtracting \(P_L\) from both sides of the free entry condition \((16)\), we obtain:

\[(1 - w)(P_A - P_L) = \bar{C}_L + \tau_0 - P_L,\]

\[P_A - P_L = \frac{\tau_0}{1 - w}.\]

The second equation imposes the free entry condition \((8)\) for uncertified low quality providers. This implies an expression for \(s_A\) in terms of exogenous parameters, given
\( \tau_0 \), which must also hold in equilibrium. From the indifference condition (11):

\[
s_A \theta_m = \frac{\tau_0}{1 - w}.
\]

Similarly, by subtracting \( P_A \) from both sides of the free entry condition (10) for high quality providers and imposing the free entry conditions (8) and (16) for uncertified and certified low quality providers, respectively:

\[
P_H - P_A = \left(1 - w\right) \left(C_H - C_L\right) - \tau_0.
\]

This implies an expression for \( s_A \) in terms of exogenous parameters, given \( \tau_0 \), which must also hold in equilibrium. From the indifference condition (12):

\[
(1 - s_A)\theta_n = \frac{(1 - w) \left( C_H - C_L\right) - \tau_0}{w(1 - w)}.
\]

Thus, we can combine the two expressions of \( s_A \) in terms of exogenous parameters to find an equation which must hold in any equilibrium for which two consumer types are indifferent:

\[
1 - \left(\frac{(1 - w) \left( C_H - C_L\right) - \tau_0}{\theta_n w(1 - w)}\right) = s_A = \frac{\tau_0}{\theta_m(1 - w)}.
\]

However, there are no endogenous variables that may adjust to ensure that this does hold for a given \( \tau_0 \). Rather, it can only hold in the limited region of the parameter space in which the exogenous parameters are defined to have precisely the proper values. Thus, an equilibrium with two types of indifferent consumers can only occur in a knife-edge case.

I now consider the welfare effect of an increase in \( \tau_0 \) only in equilibria in which one type of consumer is indifferent between two kinds of service.
Lemma 4. If the type m consumer is indifferent between the submarkets of known low quality and unobservable quality, welfare is non-decreasing in $\tau_0$.

Proof. The free entry conditions (10), (8), and (16) are still relevant for this kind of equilibrium. This kind of equilibrium differs from the kind considered in Lemma 3 because the consumer indifference condition (12) is no longer relevant. Without loss of generality, type $n$ consumers strictly prefer the submarket of observably high quality services ($\mu_n = 0$). Given this difference, the market clearing conditions for services of observably high quality and unknown quality are, respectively:

$$\sum_{i=n}^{N} f_i = s_A w(1 - \phi),$$
$$\mu_m f_m + \sum_{i=m+1}^{n-1} f_i = (1 - w)(1 - \phi).$$

Demand for services of observably high quality now comes from the mass of consumers with $\theta \geq \theta_n$. Similarly, the demand for services of unknown quality now comes from the mass of consumers with $\theta_m < \theta < \theta_n$ and the mass of type $m$ consumers choosing unknown quality $\mu_m f_m$. Aggregate welfare in this equilibrium is given by the following expression for consumers’ payoffs:

$$W^\tau = \sum_{i=1}^{m-1} f_i (-P_L) + (1 - \mu_m) f_m (-P_L) + \mu_m f_m (s_A \theta_m - P_A)$$
$$+ \sum_{i=m+1}^{n-1} f_i (s_A \theta_i - P_A) + \sum_{i=n}^{N} f_i (\theta_i - P_H) + (1 - s_A)(1 - \phi)\tau_0.$$

The first term represents the contribution to aggregate welfare of consumers who

---

$^{27}$Note that if $n - m = 1$, demand for services of unknown quality comes only from type $m$ consumers of mass $\mu_m f_m$.

$^{28}$Note that if $m = 1$, no consumer type strictly prefers observably low quality. The corresponding statement of aggregate welfare would lack the first term in this expression, but the two statements would be the same in all other respects. Similarly, if $n - m = 1$ then no consumer type strictly prefers the submarket of unknown quality. The corresponding statement of aggregate welfare would lack the fourth term in this expression, but the two statements would be the same in all other respects.
strictly prefer known low quality. The fourth and fifth terms are the corresponding contributions of consumers who strictly prefer unknown quality and observably high quality, respectively. The second and third terms represent the payoffs of type $m$ consumers, some fraction $\mu_m$ of whom consume services of unknown quality. The final term represents the surplus of the regulator generated by the bribe.

The welfare effects of an increase in $\tau_0$ are given by the following expression:

$$\frac{\partial W^r}{\partial \tau_0} = \frac{\partial \mu_m}{\partial \tau_0} f_m (-P_L) + \frac{\partial \mu_m}{\partial \tau_0} f_m (s_A \theta_m - P_A) + \mu_m f_m \left( \frac{\partial s_A \theta_m}{\partial \tau_0} - \frac{\partial P_A}{\partial \tau_0} \right)$$

$$+ \sum_{i=m+1}^{n-1} f_i \left( \frac{\partial s_A \theta_i}{\partial \tau_0} - \frac{\partial P_A}{\partial \tau_0} \right) + \sum_{i=n}^{N} f_i \left( -\frac{\partial P_H}{\partial \tau_0} \right)$$

$$+ (1 - s_A)(1 - \phi) - \frac{\partial s_A}{\partial \tau_0} (1 - \phi) \tau_0 - \frac{\partial \phi}{\partial \tau_0} (1 - s_A) \tau_0.$$ 

This expression already imposes the free entry condition (8) for uncertified low quality providers, which tells us that $\frac{\partial P_L}{\partial \tau_0} = 0$. The first and second terms in this expression cancel according to the indifference condition (11). Next, we can use the free entry conditions (8) and (16) to identify how $P_A$ responds to the change in $\tau_0$:

$$P_A = \frac{(1 - w) \bar{C}_L + \tau_0}{1 - w}.$$ 

Similarly, rearranging the free entry condition (10) for high quality providers to identify how $P_H$ responds to the change in $\tau_0$:

$$P_H = \frac{\bar{C}_H - (1 - w) P_A}{w},$$

$$\frac{\partial P_H}{\partial \tau_0} = \frac{(1 - w)}{w} \frac{\partial P_A}{\partial \tau_0} = -\frac{1}{w}.$$
Substituting these into the expression for $\frac{\partial W^\tau}{\partial \tau_0}$ and grouping the $\frac{\partial P_A}{\partial \tau_0}$ terms:

$$
\frac{\partial W^\tau}{\partial \tau_0} = (1 - s_A)(1 - \phi) - \frac{\partial s_A}{\partial \tau_0}(1 - \phi)\tau_0 - \frac{\partial \phi}{\partial \tau_0}(1 - s_A)\tau_0 + \mu_m f_m \frac{\partial s_A}{\partial \tau_0} \theta_m \\
+ \sum_{i=m+1}^{n-1} f_i \frac{\partial s_A}{\partial \tau_0} \theta_i - \left( \mu_m f_m + \sum_{i=m+1}^{n-1} f_i \right) \frac{1}{1 - w} - \sum_{i=n}^{N} f_i \left( -\frac{1}{w} \right).
$$

Next, applying the market clearing conditions for observably high quality services and for services of unknown quality:

$$
\frac{\partial W^\tau}{\partial \tau_0} = (1 - s_A)(1 - \phi) - \frac{\partial s_A}{\partial \tau_0}(1 - \phi)\tau_0 - \frac{\partial \phi}{\partial \tau_0}(1 - s_A)\tau_0 \\
+ \mu_m f_m \frac{\partial s_A}{\partial \tau_0} \theta_m + \sum_{i=m+1}^{n-1} f_i \frac{\partial s_A}{\partial \tau_0} \theta_i - (1 - \phi) + s_A(1 - \phi).
$$

Next, I use condition (17) describing market clearing among observably high quality services to establish a relationship between $\frac{\partial s_A}{\partial \tau_0}$ and $\frac{\partial \phi}{\partial \tau_0}$:

$$
\frac{\partial s_A}{\partial \tau_0}(1 - \phi) - \frac{\partial \phi}{\partial \tau_0} s_A = 0.
$$

This implies that $\frac{\partial W^\tau}{\partial \tau_0}$ can now be expressed as:

$$
\frac{\partial W^\tau}{\partial \tau_0} = - \frac{\partial \phi}{\partial \tau_0} \tau_0 + \mu_m f_m \frac{\partial s_A}{\partial \tau_0} \theta_m + \sum_{i=m+1}^{n-1} f_i \frac{\partial s_A}{\partial \tau_0} \theta_i.
$$

Applying the market clearing condition for observably high quality again, we obtain:

$$
\frac{\partial W^\tau}{\partial \tau_0} = - \frac{\partial s_A}{\partial \tau_0}(1 - \phi) \left( \frac{1}{s_A} \right) \tau_0 + \mu_m f_m \frac{\partial s_A}{\partial \tau_0} \theta_m + \sum_{i=m+1}^{n-1} f_i \frac{\partial s_A}{\partial \tau_0} \theta_i.
$$
Next, I add and subtract $\sum_{i=m+1}^{n-1} f_i \frac{\partial s_A}{\partial \tau_0} \theta_m$ from the welfare expression:

$$\frac{\partial W^\tau}{\partial \tau_0} = -\frac{\partial s_A}{\partial \tau_0} (1 - \phi)(1 - w) \left( \frac{1}{s_A} \right) \left( \frac{\tau_0}{1 - w} \right) + \left( \mu_m f_m + \sum_{i=m+1}^{n-1} f_i \right) \frac{\partial s_A}{\partial \tau_0} \theta_m$$

$$+ \sum_{i=m+1}^{n-1} f_i \frac{\partial s_A}{\partial \tau_0} (\theta_i - \theta_m).$$

Finally, I apply the market clearing condition for services of unknown quality (18) and the indifference condition for type $m$ (11) to the first term in the expression for $\frac{\partial W^\tau}{\partial \tau_0}$. The first two terms of this expression then cancel. We are left with:

$$\frac{\partial W^\tau}{\partial \tau_0} = \sum_{i=m+1}^{n-1} f_i \frac{\partial s_A}{\partial \tau_0} (\theta_i - \theta_m).$$

From this expression, we can see that $\frac{\partial W^\tau}{\partial \tau_0} > 0$. By assumption, $\theta_i > \theta_m$, $\forall m < i < n$. Additionally, we can use the indifference condition for type $m$ consumers to show that $\frac{\partial s_A}{\partial \tau_0} > 0$:

$$\frac{\partial s_A}{\partial \tau_0} \theta_m = \frac{\partial P_A}{\partial \tau_0} > 0.$$

This shows that welfare is non-decreasing in $\tau_0$ if a type $m$ consumer is indifferent between services of observably low quality and those of unknown quality. I next show that welfare is increasing in $\tau_0$ if a type $n$ consumer is indifferent between services of unknown quality and those of observably high quality.

**Lemma 5.** If a type $\theta_n$ consumer is indifferent between the submarket of observably high quality and that of unknown quality, welfare is increasing in $\tau_0$.

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29Note that this term does not appear if $n - m = 1$, since in this case no consumer type strictly prefers the submarket of unknown quality. All other aspects of this proof are unchanged if $n - m = 1$. Consequently, $\frac{\partial W^\tau}{\partial \tau_0} = 0$ if this is the case.
Proof. The free entry conditions (10), (8), and (16) are still relevant for this kind of equilibrium. This kind of equilibrium differs from the kind considered in Lemma 3 because the consumer indifference condition (11) is no longer relevant. Without loss of generality, type $m$ consumers strictly prefer services of unknown quality ($\mu_m = 1$). Given this difference, the market clearing conditions for services of observably high quality and unknown quality are, respectively:

\[
(1 - \mu_n)f_n + \sum_{i=m+1}^N f_i = s_A w(1 - \phi),
\]

\[
\sum_{i=m}^{n-1} f_i + \mu_n f_n = (1 - w)(1 - \phi).
\]

Demand for services of observably high quality now comes from the mass of consumers with $\theta > \theta_n$ and the mass of type $n$ consumers choosing observably high quality $(1 - \mu_n)f_n$. Similarly, the demand for services of unknown quality now comes from the mass of consumers with $\theta_m \leq \theta < \theta_n$ and the mass of type $n$ consumers choosing unknown quality $\mu_n f_n$. Aggregate welfare in this equilibrium is given by the following expression for consumers’ payoffs:

\[
W = \sum_{i=1}^{m-1} f_i (-P_L) + \sum_{i=m}^{n-1} f_i (s_A \theta_i - P_A) + \mu_n f_n (s_A \theta_n - P_A)
\]

\[
+ (1 - \mu_n) f_n (\theta_n - P_H) + \sum_{i=n+1}^N f_i (\theta_i - P_H) + (1 - s_A)(1 - \phi)\tau_0.
\]

The first term represents the contribution to aggregate welfare of consumers who strictly prefer known low quality. The second and fifth terms are the corresponding

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30 Note that if $n = m$, demand for services of unknown quality comes only from type $m = n$ consumers of mass $\mu_n f_n$.

31 Note that if $n = N$, no consumer type strictly prefers observably high quality. The corresponding statement of aggregate welfare would lack the last term in this expression, but the two statements would be the same in all other respects. Similarly, if $n = m$ then no consumer type strictly prefers services of unknown quality. The corresponding statement of aggregate welfare would lack the second term in this expression, but the two statements would be the same in all other respects.
contributions of consumers who strictly prefer unknown quality and observably high quality, respectively. The third and fourth terms represent the payoffs of type $n$ consumers, some fraction $\mu_n$ of whom consume services of unobservable quality. Again, the final term represents the surplus of the regulator that is generated by the bribe.

The welfare effects of an increase in $\tau_0$ are given by:

$$\frac{\partial W^\tau}{\partial \tau_0} = \sum_{i=m}^{n-1} f_i \left( \frac{\partial s_A}{\partial \tau_0} \theta_i - \frac{\partial P_A}{\partial \tau_0} \right) + \frac{\partial \mu_n}{\partial \tau_0} f_n \left[ s_A \theta_n - P_A \right] + \mu_n f_n \left( \frac{\partial s_A}{\partial \tau_0} \theta_n - \frac{\partial P_A}{\partial \tau_0} \right)$$

$$- \frac{\partial \mu_n}{\partial \tau_0} f_n \left[ \theta_n - P_H \right] + (1 - \mu_n) f_n \left( - \frac{\partial P_H}{\partial \tau_0} \right) + \sum_{i=n+1}^N f_i \left( - \frac{\partial P_H}{\partial \tau_0} \right)$$

$$+ (1 - s_A)(1 - \phi) - \frac{\partial s_A}{\partial \tau_0} (1 - \phi) \tau_0 - \frac{\partial \phi}{\partial \tau_0} (1 - s_A) \tau_0.$$
clearing condition (17). This differs from the corresponding relationship in Lemma 4 because some type \( n \) consumers also choose observably high quality, and this fraction depends on \( \tau_0 \). From the high quality market clearing condition:

\[
\frac{\partial s_A}{\partial \tau_0} (1 - \phi) - \phi \frac{\partial s_A}{\partial \tau_0} = -\frac{\partial \mu_n}{\partial \tau_0} \left( \frac{f_n}{w} \right).
\]

Imposing this relationship on the expression for \( \frac{\partial W^\tau}{\partial \tau_0} \), we obtain:

\[
\frac{\partial W^\tau}{\partial \tau_0} = -\frac{\partial \phi}{\partial \tau_0} + \frac{\partial \mu_n}{\partial \tau_0} \left( \frac{f_n}{w} \right) \tau_0 + \left( \sum_{i=m}^{n-1} f_i \theta_i + \mu_n f_n \theta_n \right) \frac{\partial s_A}{\partial \tau_0}.
\]

Next, establishing a relationship between \( \frac{\partial \phi}{\partial \tau_0} \) and \( \frac{\partial \mu_n}{\partial \tau_0} \) using the unknown quality market clearing condition:

\[
\frac{\partial \mu_n}{\partial \tau_0} f_n = -(1 - w) \frac{\partial \phi}{\partial \tau_0}.
\]

Using this to substitute \( \frac{\partial \phi}{\partial \tau_0} \) out of the expression for \( \frac{\partial W^\tau}{\partial \tau_0} \), and adding and subtracting \( \sum_{i=m}^{n-1} f_i \theta_n \frac{\partial s_A}{\partial \tau_0} \), we obtain:

\[
\frac{\partial W^\tau}{\partial \tau_0} = \frac{\partial \mu_n}{\partial \tau_0} f_n \tau_0 \left( \frac{1}{w} + \frac{1}{1 - w} \right) + \left( \sum_{i=m}^{n-1} f_i + \mu_n f_n \right) \theta_n \frac{\partial s_A}{\partial \tau_0} + \sum_{i=m}^{n-1} f_i (\theta_i - \theta_n) \frac{\partial s_A}{\partial \tau_0}.
\]

Next, using the consumer’s indifference condition (12):

\[
-\frac{\partial s_A}{\partial \tau_0} \theta_n = \frac{\partial P_H}{\partial \tau_0} - \frac{\partial P_A}{\partial \tau_0} = -\frac{1}{w} - \frac{1}{1 - w}.
\]

Using this to substitute into the first term in the expression for \( \frac{\partial s_A}{\partial \tau_0} \), we obtain:

\[
\frac{\partial W^\tau}{\partial \tau_0} = \left( \frac{1}{w} + \frac{1}{1 - w} \right) \left( \frac{\partial \mu_n}{\partial \tau_0} f_n \tau_0 + \sum_{i=m}^{n-1} f_i + \mu_n f_n \right) + \sum_{i=m}^{n-1} f_i (\theta_i - \theta_n) \frac{\partial s_A}{\partial \tau_0}.
\]
Next, I solve for the equilibrium value of \( \mu_n \) in order to find \( \frac{\partial \mu_n}{\partial \tau_0} \). Combining the market clearing conditions (17) and (18) with the consumer’s indifference condition (12), we obtain the marginal benefit to the type \( n \) consumer in terms of \( \mu_n \) and exogenous parameters. Combining the free entry conditions (10), (8), and (16), we obtain the marginal cost to the type \( n \) consumer in terms of exogenous parameters, for a given \( \tau_0 \):

\[
\left[ 1 - \left( \frac{1-w}{w} \right) \left( \frac{(1-\mu_n)f_n + \sum_{i=n+1}^N f_i}{\sum_{i=m}^{n-1} f_i + \mu_n f_n} \right) \right] \theta_n = P_H - P_A = \frac{(1-w)(\bar{C}_H - \bar{C}_L) - \tau_0}{w(1-w)}.
\]

Multiplying both sides of this equation by \( w \left( \sum_{i=m}^{n-1} f_i + \mu_n f_n \right) \), we obtain an expression that must hold in this kind of equilibrium:

\[
[w \mu_n f_n - (1-w)(1-\mu_n)f_n] \theta_n = \left( \sum_{i=m}^{n-1} f_i + \mu_n f_n \right) \left( \bar{C}_H - \bar{C}_L - \frac{\tau_0}{1-w} \right) - \theta_n \left[ w \sum_{i=m}^{n-1} f_i - (1-w) \sum_{i=n+1}^N f_i \right].
\]

We can then take the derivative of this equation with respect to \( \tau_0 \) in order to find \( \frac{\partial \mu_n}{\partial \tau_0} \), after simplifying:

\[
\frac{\partial \mu_n}{\partial \tau_0} f_n \theta_n = \frac{\partial \mu_n}{\partial \tau_0} f_n \left( \frac{\bar{C}_H - \bar{C}_L - \frac{\tau_0}{1-w}}{1-w} \right) - \left( \sum_{i=m}^{n-1} f_i + \mu_n f_n \right) \left( \frac{1}{1-w} \right),
\]

\[
\frac{\partial \mu_n}{\partial \tau_0} = - \left( \sum_{i=m}^{n-1} f_i + \mu_n f_n \right) \left( \frac{1}{1-w} \right) \left( \frac{1}{\theta_n - (\bar{C}_H - \bar{C}_L - \frac{\tau_0}{1-w})} \right).
\]

Substituting this into the first term of the expression for \( \frac{\partial W^*}{\partial \tau_0} \), we obtain:

\[
\frac{\partial W^*}{\partial \tau_0} = \left( \sum_{i=m}^{n-1} f_i + \mu_n f_n \right) \left( 1 - \frac{\tau_0}{\theta_n - (\bar{C}_H - \bar{C}_L - \frac{\tau_0}{1-w})} \right) + \sum_{i=m}^{n-1} f_i (\theta_i - \theta_n) \frac{\partial s_A}{\partial \tau_0}.
\]

If \( \theta_n > \bar{C}_H - \bar{C}_L \), then the first term in this expression is positive. This must be true
in any equilibrium in which some low quality providers choose to become certified. First, note the following necessary condition for any low quality provider to choose to become certified:

\[
\frac{\bar{C}_H - \bar{C}_L - \tau_0}{w} > \bar{C}_H - \bar{C}_L,
\]

\[
w\bar{C}_L + (1 - w)\bar{C}_H > \bar{C}_L + \tau_0.
\]

The second condition above provides insight into why this is a necessary condition for any low quality provider to choose to become certified. Low quality providers only find it profitable to become certified if the premium to pooling with high quality providers is large enough to justify paying the bribe. This is the case when the second inequality above holds. Next, note that from the free entry conditions (10) and (16) for certified high and low quality providers, respectively:

\[
P_H - P_L = \frac{\bar{C}_H - \bar{C}_L - \tau_0}{w}.
\]

To finish showing that the first term in the expression for \( \frac{\partial W_T}{\partial \tau_0} \) is positive, note that the type \( n \) consumer’s indifference condition (12) and the fact that no type \( n \) consumers choose the submarket of observably low quality in equilibrium imply the following:

\[
\theta_n = s_A \theta_n + (1 - s_A)\theta_n > \frac{\bar{C}_H - \bar{C}_L - \tau_0}{w} > \bar{C}_H - \bar{C}_L.
\]

I next show that the second term in the expression for \( \frac{\partial W_T}{\partial \tau_0} \) is also positive. Applying the indifference condition (12) to the second term and redistributing from the first
term:

\[
\frac{\partial W}{\partial \tau_0} = \left(\frac{1}{w} + \frac{1}{1-w}\right) \mu_n f_n \left(1 - \frac{\tau_0}{1-w} \theta_n - \left(C_H - C_L - \frac{\tau_0}{1-w}\right)\right) + \sum_{i=m}^{n-1} f_i \theta_i \frac{\partial s_A}{\partial \tau_0} \\
+ \left(\frac{\sum_{i=m}^{n-1} f_i}{w(1-w)}\right) \left(1 - \frac{\tau_0}{1-w} \theta_n - \left(C_H - C_L - \frac{\tau_0}{1-w}\right)\right) - \left(\frac{\sum_{i=m}^{n-1} f_i}{w(1-w)}\right). 
\]

After canceling we can regroup terms to obtain:

\[
\frac{\partial W}{\partial \tau_0} = \left(\frac{1}{w} + \frac{1}{1-w}\right) \mu_n f_n \left(1 - \frac{\tau_0}{1-w} \theta_n - \left(C_H - C_L - \frac{\tau_0}{1-w}\right)\right) \\
+ \sum_{i=m}^{n-1} f_i \left[\theta_i \frac{\partial s_A}{\partial \tau_0} - \left(\frac{1}{w} + \frac{1}{1-w}\right) \theta_n - \left(C_H - C_L - \frac{\tau_0}{1-w}\right)\right]. 
\]

The first term in this expression is positive, as shown previously. Now I attempt to sign the second term of this expression. The market clearing conditions (17) and (18) for observably high quality and unknown quality imply the following equilibrium relationship between \(s_A\) and \(\mu_n\):

\[
s_A w \left(\sum_{i=m}^{n-1} f_i + \mu_n f_n\right) = (1-w) \left[(1 - \mu_n) f_n + \sum_{i=n+1}^{N} f_i\right]. 
\]

Taking the derivative of this equilibrium relationship, we can discover how \(s_A\) and \(\mu_n\) respond to a change in \(\tau_0\), after simplifying:

\[
\frac{\partial s_A}{\partial \tau_0} w \left(\sum_{i=m}^{n-1} f_i + \mu_n f_n\right) + s_A w f_n \frac{\partial \mu_n}{\partial \tau_0} = -(1-w) f_n \frac{\partial \mu_n}{\partial \tau_0}, \\
\frac{\partial s_A}{\partial \tau_0} = -(\frac{1-w+s_A w}{w}) f_n \frac{\partial \mu_n}{\partial \tau_0}. \\
\frac{\partial s_A}{\partial \tau_0} = -\frac{(1-w+s_A w)}{w} \frac{\partial \mu_n}{\partial \tau_0} \sum_{i=m}^{n-1} f_i + \mu_n f_n. 
\]

After substituting in the expression for \(\frac{\partial \mu_n}{\partial \tau_0}\) derived above, we obtain the following
expression for $\frac{\partial s_A}{\partial \tau_0}$:

$$\frac{\partial s_A}{\partial \tau_0} = \left(1 - w + w s_A\right) \left(\frac{1}{\theta - (C_H - C_L - \tau_0)}\right).$$

Introducing this into the second term of the expression for $\frac{\partial W^r}{\partial \tau_0}$, we obtain:

$$\frac{\partial W^r}{\partial \tau_0} = \left(1 + \frac{1}{1 - w}\right) \mu_n f_n \left(1 - \frac{\tau_0}{\theta - (C_H - C_L - \tau_0)}\right)$$

$$+ \sum_{i=m}^{n-1} \frac{f_i}{w(1 - w)} \left(\theta - (C_H - C_L - \tau_0)\right) \left[\theta_i (1 - w + w s_A) - \frac{\tau_0}{1 - w}\right].$$

This welfare expression may be rewritten as:

$$\frac{\partial W^r}{\partial \tau_0} = \left(1 + \frac{1}{1 - w}\right) \mu_n f_n \left(1 - \frac{\tau_0}{\theta - (C_H - C_L - \tau_0)}\right)$$

$$+ \sum_{i=m}^{n-1} \frac{f_i}{w(1 - w)} \left(\theta - w (P_H - P_A)\right) \left[\theta_i (1 - w + w s_A) - \frac{\tau_0}{1 - w}\right].$$

We know from the consumer’s indifference condition (12) that:

$$\theta_n > (1 - s_A)\theta_n = P_H - P_A > w(P_H - P_A).$$

Additionally, we know that, $\forall m \leq i < n$:

$$\theta_i [1 - w(1 - s_A)] > \theta_i [1 - (1 - s_A)] = s_A \theta_i > \frac{\tau_0}{1 - w}$$

Thus, both the first term and the second term of the expression for $\frac{\partial W^r}{\partial \tau_0}$ are strictly positive, so welfare is increasing in $\tau_0$.

In summary, I have shown that $\frac{\partial W^r}{\partial \tau_0} \geq 0$ when only one consumer type is at a

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32 Note that if $m = n$, no consumer strictly prefers the submarket of unknown quality. As a result, the second term in this expression would not appear if this were the case. All other aspects of this proof are unchanged if $n = m$. Consequently, it is still true that $\frac{\partial W^r}{\partial \tau_0} > 0$. 

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margin. Additionally, I have shown that equilibria in which consumers are at both margins occur only in a knife-edge case. From this, we can conclude that welfare is non-decreasing in response to small changes in \( \tau_0 \) that do not change which type of consumer is indifferent. Two things remain to be shown. The first is whether small changes in \( \tau_0 \) that affect which consumer type is indifferent in equilibrium are weakly welfare improving. The second is whether this result generalizes to the case of continuously distributed consumer types.