Problem Set 1 Solutions

The problem set is “not graded.” We will look it over to make sure you did the problems adequately, but they will not be corrected. The answers will be given on a posted answer sheet. The problem set is primarily designed to ensure that you keep up with the work and understand the major concepts. Each test will definitely use one of the problems from either graded or non-graded problem sets.

Please read the rules on “Rules on problem set joint work” in the folder “Problems.”

1. There are three major statistical agencies of the US government: the Bureau of Economic Analysis, the Bureau of Labor Statistics, and the Census Bureau. This question is designed to help familiarize you with some of these data sources.

   a. Go to the BEA website (www.bea.gov), and click the following links: “Gross Domestic Product,” “Interactive Tables: GDP and the National Income and Product Account (NIPA) Historical Tables,” and “Begin using the data….” Take a moment to examine the data that are available through this page. Use table 1.1.1 to get data on quarters since 2007 (use “options” to change the date range). In which quarters did the US experience a decline in Real GDP? In which quarter was this decline the greatest?

   Real GDP declined in 2008-Q1, 2008-Q3, 2008-Q4, 2009-Q1, and 2009-Q2. The largest decline came in 2008-Q4. In this quarter, real GDP fell 8.9% at an annual, seasonally adjusted rate.

   b. According to the NBER definition, a recession begins at the peak of the business cycle and ends at the trough. Using table 1.1.3., which quarters (since 2000) had the highest Real GDP levels? Which had the lowest levels? That is, identify the local peaks and troughs since 2000 (Hint: there are two local peaks and two local troughs). Go to the NBER business cycle website (www.nber.org/cycles/cyclesmain.html). Do your peak and trough dates correspond to those of the NBER? (Extra Credit: Any guesses as to why they might differ?)

   Examining real GDP levels since 2000, we see that real GDP grew from 2001-Q4 to its all-time high in 2007-Q4. Real GDP generally declined through 2009-Q2. Since then, real GDP has risen in each quarter. So, it seems that 2007-Q4 and 2009-Q2 are, respectively, the peak and trough of the most recent business cycle.

   Real GDP consistently increased in the late 1990s and early 2000s but fell in 2001-Q1. It rose in 2001-Q2, reaching its local maximum, and then fell in 2001Q3. As the peak was in 2001-Q2 and real GDP grew in 2001-Q4, it seems reasonable to call 2001-Q3 the trough. However, there were never consecutive quarters of negative growth in this episode, so it is not obvious how to date this (quite mild) recession.
Early 2000s Recession
Table 1.1.3 Peak: 2001-Q2
NBER Peak: 2001-Q1
Table 1.1.3 Trough: 2001-Q3
NBER Trough: 2001-Q4

Late 2000s Recession
Table 1.1.3 Peak: 2007-Q4
NBER Peak: 2007-Q4
Table 1.1.3 Trough: 2009-Q2
NBER Trough: 2009-Q2

Our early 2000s dates differ from those of the NBER. In general, there are two reasons dates from table 1.1.3. may differ from those of the NBER. First, the BEA revises old GDP numbers as new data become available and improve the accuracy of the measurement of output. The NBER does not revise business cycle dates, even when the data would warrant doing so. Second, the NBER does not just use GDP numbers in dating business cycles. The NBER also consider, for example, employment figures and industrial production.

c. Go to the BLS website (www.bls.gov), and click “Subject Areas.” Take a moment to look at the data available from this page. Click “Consumer Price Index,” and under “CPI databases,” click the “Top Picks” of “All Urban Consumers.” First, retrieve data from “U.S. All items,” and format the data to be shown as monthly percent changes. In which months since 2007 did the US economy experience deflation, according to the CPI?

CPI Monthly % change
2007 AUG -0.2
2007 DEC -0.1
2008 AUG -0.4
2008 SEPT -0.1
2008 OCT -1.0
2008 NOV -1.9
2008 DEC -1.0
2009 JUL -0.2
2009 DEC -0.2
2010 JUN -0.1
2011 JUN -0.1

d. One criticism of the CPI is that it includes items with very volatile prices, such as food and energy items, and including such items produces a volatile inflation rate. Therefore, economists also look at “All items less food and energy,” which is also known as “Core CPI.” In which months since 2007 did the US economy experience deflation according to this measure?

Core CPI Monthly % change
2008 NOV -0.2
2008 DEC -0.3
2009 NOV -0.2
e. (Extra Credit) In determining monetary policy, the Federal Reserve focuses on “core” measures of inflation. Why do you think this is? [We have not discussed this, so use your judgment.]

The Fed is primarily concerned about medium-run and long-run inflation trends. Short-run movements in food and energy prices are not persistent, and are often reversed. This means that a one-time shock to food and energy prices will generally not get transmitted into medium-run increases in wage or price inflation. On the other hand, core inflation often reflects increases in wage inflation, which do get transmitted into medium-run inflation, and the Fed watches these and reacts to these quickly and decisively. [Note: We have not covered this, and this is not in your readings, but this represents current thinking at the Fed.]

2. Consider the following actual data on computer and non-computer prices and quantities:

<table>
<thead>
<tr>
<th>Sector</th>
<th>Output (billions 2005$)</th>
<th>Price (2005 = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1958</td>
<td>2010</td>
</tr>
<tr>
<td>Computers</td>
<td>0.00000153</td>
<td>179.46</td>
</tr>
<tr>
<td>Non computers</td>
<td>2,575.4</td>
<td>12,908.5</td>
</tr>
</tbody>
</table>

a. Calculate the Laspeyres and Paasche quantity indexes for GDP in each period. Calculate the Fisher index for output growth between the two periods.

Let $P_{g,t}$ and $Q_{t}$ be the price and quantity of good $g$ in time $t$. So, $PC_{2010}$ is the price of computers in 2010 and $QNC_{1958}$ is the quantity of non-computers in 1958. With this notation, the Laspeyres (L) and Paasche (A) quantity indexes are:

$$Q_{1958,L} = PC_{1958}Q_{C,1958} + PNC_{1958}QNC_{1958} = 467.29$$
$$Q_{2010,L} = PC_{1958}Q_{C,2010} + PNC_{1958}QNC_{2010} = 133,786,060$$
$$Q_{1958,A} = PC_{2010}Q_{C,1958} + PNC_{2010}QNC_{1958} = 2879.30$$
$$Q_{2010,A} = PC_{2010}Q_{C,2010} + PNC_{2010}QNC_{2010} = 25,589$$

The Fisher index for real output growth is:

$$realG_{Fisher} = ((Q_{2010,L}/Q_{1958,L})(Q_{2010,A}/Q_{1958,A}))^{(1/2)} - 1 = 159.4123, \text{ or } 159.412\%$$

b. In part (a) you used indexes to calculate the overall level of output. How would you define Laspeyres and Paasche price indexes for the overall price level?

Quantity indexes use prices as weights to sum and compare quantities. Price indexes use quantities as weights to sum and compare prices. We can calculate Laspeyres and Paasche price indexes as follows:

$$P_{1958,L} = PC_{1958}QC_{1958} + PNC_{1958}QNC_{1958} = 467.29$$
P2010_L = PC_2010*QC_1958 + PNC_2010*QNC_1958 = 2879.30
P1958_A = PC_1958*QC_2010 + PNC_1958*QNC_2010 = 133,786,050
P2010_A = PC_2010*QC_2010 + PNC_2010*QNC_2010 = 25,589

The Fisher index for inflation is:
Inflation_Fisher = ((P2010_L/ P1958_L)(P2010_A/ P1958_A))^(1/2) – 1 = -.97, or –97%

c. Calculate the Laspeyres and Paasche price indexes for each period from the same data above. Calculate the Fisher index for inflation between the two periods.

See part b.

3. Suppose output in the Japanese economy is given by a Cobb-Douglas production function with three inputs: K is capital (number of machines), L is labor (number of workers), and T is land or “terra.” The production function is:

\[ Y = T^{.1} K^{.2} L^{.7} \]

Assume that initially the levels of T, K, and L are fixed and that wages, capital rentals, and land rents are determined competitively.

a. Derive expressions for competitive wages, capital rentals, and land rents.

With competitive factor markets, wages and rental rates equal marginal products:

- Wages = \( W = .7(T^{.1}K^{.2})/(L^{.3}) \)
- Capital rental rate = \( R = .2(T^{.1}L^{.7})/(K^{.8}) \)
- Land rents = LR = .1(K^{.2}L^{.7})/(T^{.9})

b. What share of total income is paid to workers? What is the share paid to capital owners? What is the share paid to land owners? Derive one of these using the appropriate math.

Since \( Y = T^{.1} K^{.2} L^{.7} \), we see that

- \( W*L/Y = .7 \)
- \( R*K/Y = .2 \)
- \( LR*T/Y = .1 \)

With Cobb-Douglas production, competitive income shares are equal to the respective exponents from the production function.

c. Suppose a terrible tsunami destroys \( \frac{1}{2} \) of Japan’s land. How does this affect wages, capital rentals, and land rents? How does this affect income shares of each factor?

After the Tsunami, the supply of land is \( (.5)T \). We find that:
- Wages and capital rental rates each fall by a factor of \( (.5)^{.1} \)
- Land rents increase by a factor of \( 2^{.9} \)
Since production is Cobb-Douglas and markets are competitive, income shares do not change

d. Suppose the Japanese government decided to invest in capital infrastructure to bring output back to the pre-tsunami level. How much capital would be needed? Calculate the new shares of the three factors. Calculate the relative changes in the factor prices.

After the Tsunami, \( Y = (.5)^1 T^{.1} K^{-.2} L^{-.7} \), so production falls by a factor of \((.5)^1\). For an increase in capital to offset this loss, it must be that \( 1 = (.5)^1 (x)^2 \), where \( xK \) is the new level of capital. So, \( x = (2)^5 \). That is, output will return to its pre-Tsunami level if capital increases by roughly 40%.

Again, due to Cobb-Douglas production, income shares are .1, .2, and .7.

Now, let’s compare the final prices to the pre-Tsunami prices:
- wages = \(.7(2^{.5})^2(T^{.1}K^{-.2})/(L^{-.3}) = .7 (T^{.1}K^{-.2})/(L^{-.3}) \), so wages are as before the Tsunami
- capital rental rate = \(.2(2^{.5})^{-.8} (T^{.1}L^{-.7})/(K^{-.8}) = (.71).2(T^{.1}L^{-.7})/(K^{-.8}) \), so the capital rental rate falls by about 30%.
- land rents = \(.1(2^{.5})^{-9} (K^{-.2}L^{-.7})/(T^{-.9}) = (2).1(K^{-.2}L^{-.7})/(T^{-.9}) \), so land rents double.

After the Tsunami and subsequent capital-driven recovery, output and wages are as before the disaster, but capital rental rates are relatively lower and land rents are relatively higher.