Roll the DICE Again: 
Economic Models of Global Warming

Appendix E

William D. Nordhaus and Joseph Boyer

Yale University
October 25, 1999

Note: This is the “manuscript edition” of the book by the same title to be published by MIT Press. The manuscript edition is for the personal use of readers and may not be sold or used without the written permission of MIT Press.

Version is DICE v.101599
Appendix E. GAMS Code for DICE-99

SETS
T         Time periods /1*35/
   TFIRST(T) First period
   TLAST(T) Last period
tearly(T) First 20 periods
   TLATE(T) Second 20 periods;

TFIRST(T) = YES$(ORD(T) EQ 1);
TLAST(T) = YES$(ORD(T) EQ CARD(T));
tearly(T) = YES$(ORD(T) LE 20);
TLATE(T) = YES$(ORD(T) GE 21);

SCALARS
A1       Damage coeff linear term /-.0045/
A2       Damage coeff quadratic term /.0035/
COST10   Intercept control cost function /.03/
COST2    Exponent of control cost function /2.15/
dmiufunc Decline in cost of abatement function (pct per decade) /-8/
decmiu   Change in decline of cost function (pct per year) /.5/
DK       Depreciation rate on capital (pct per year) /10/
GAMA     Capital elasticity in production function /.30/
K0       1990 value capital trill 1990 US dollars /47/
LU0      Initial land use emissions (GtC per year) /1.128/
SIG0     CO2-equivalent emissions-GNP ratio /.274/
SIGMA    Growth of sigma (pct per decade) /-15.8854/
desig    Decline rate of decarbonization (pct per decade) /2.358711/
desig2   Quadratic term in decarbonization /-.00085/
WIEL     World industrial emissions limit (GtC per year) /5.67/
LL0      1990 world population (millions) /5632.7/
GL0      Initial rowth rate of population (pct per decade) /15.7/
DLAB     Decline rate of pop growth (pct per decade) /22.2/
A0       Initial level of total factor productivity /.01685/
GA0      Initial growth rate for technology (pct per decade) /3.8/
DELAY    Decline rate of technol. change per decade /.000001/
MAT1990  Concentration in atmosphere 1990 (b.t.c.) /735/
MU1990   Concentration in upper strata 1990 (b.t.c) /781/
ML1990   Concentration in lower strata 1990 (b.t.c) /19230/
b11      Carbon cycle transition matrix (pct per decade) /66.616/
b12      Carbon cycle transition matrix /33.384/
b21      Carbon cycle transition matrix /27.607/
b22  Carbon cycle transition matrix /60.897/
b23  Carbon cycle transition matrix /11.496/
b32  Carbon cycle transition matrix /0.422/
b33  Carbon cycle transition matrix /99.578/
TL0  1985 lower strat. temp change (C) from 1900 /.06/
T0   1985 atmospheric temp change (C) from 1900 /.43/
C1   Climate-equation coefficient for upper level /.226/
CS   Eq temp increase for CO2 doubling (C) /2.9078/
C3   Transfer coeffic. upper to lower stratum /.440/
C4   Transfer coeffic for lower level /.02/
SRTP Initial rate of social time preference (pct per year) /3/
DR   Decline rate of social time preference (pct per year) /.25719/
coefopt1 Scaling coefficient in the objective function /333.51/
coefopt2 Scaling coefficient in the objective function /622.78/  

PARAMETERS
  cost1(t)      cost function for abatement
gcost1(t)
ETREE(T)      Emissions from deforestation
GSIG(T)       Cumulative improvement of energy efficiency
SIGMA(T)      CO2-equivalent-emissions output ratio
WEL(T)        World total emissions limit (GtC)
L(T)          Level of population and labor
GL(T)         Growth rate of labor 0 to T
AL(T)         Level of total factor productivity
GA(T)         Growth rate of productivity from 0 to T
FORCOTH(T)    Exogenous forcing for other greenhouse gases
R(T)          Instantaneous rate of social time preference
RR(T)         Average utility social discount rate;

gcost1(T)=(dmiufunc/100)*EXP(-(decmiu/100)*10*(ORD(T)-1));
cost1("1")=cost10;
LOOP(T,
cost1(T+1)=cost1(T)/((1+gcost1(T+1)));)

ETREE(T) = LU0*(1-0.1)**(ord(T)-1);

gsig(T)=(gsigma/100)*EXP ( -(desig/100)*10*(ORD(T)-1) - desig2*10* ((ord(t)-1)**2) ) ;
sigma("1")=sig0;
LOOP(T,
\sigma(T+1) = \frac{\sigma(T)}{(1-g\sigma(T+1))};

\text{WEL(T)} = \text{WIEL} + \text{ETREE(T)};

\text{GL(T)} = \left(\frac{\text{GL0}}{\text{DLAB}}\right) \times (1-\exp(-\left(\frac{\text{DLAB}}{100}\right) \times (\text{ord(t)}-1)));
\text{L(T)} = \text{LL0} \times \exp(\text{GL(t)});

\text{ga(T)} = \frac{\text{ga0}}{100} \times \exp\left(-\left(\frac{\text{dela}}{100}\right) \times 10 \times (\text{ORD(T)}-1)\right);
\text{al("1")} = \text{a0};
\text{LOOP(T},
\text{al(T+1)} = \text{al(T)} / \left(1 - \text{ga(T)}\right);
\text{);}

\text{FORCOTH(T)} = (-0.1965 + (\text{ORD(T)}-1) \times 0.13465) \times \left(\text{ORD(T)} \text{ LT 12} \right) + 1.15 \times (\text{ORD(T)} \text{ GE 12});

\text{R(T)} = \left(\frac{\text{srtp}}{100}\right) \times \exp\left(-\left(\frac{\text{DR}}{100}\right) \times 10 \times (\text{ORD(T)}-1)\right);
\text{RR("1")} = 1;
\text{LOOP(T},
\text{RR(T+1)} = \frac{\text{RR(T)}}{\left(1 + \text{R(T)}\right)^{10}};
\text{);}

\text{VARIABLES}
\begin{align*}
\text{Y(T)} & \quad \text{Output} \\
\text{I(T)} & \quad \text{Investment trill US dollars} \\
\text{K(T)} & \quad \text{Capital stock trill US dollars} \\
\text{E(T)} & \quad \text{CO2-equivalent emissions bill t} \\
\text{MIU(T)} & \quad \text{Emission control rate GHGs} \\
\text{MAT(T)} & \quad \text{Carbon concentration in atmosphere (b.t.c.)} \\
\text{MU(T)} & \quad \text{Carbon concentration in shallow oceans (b.t.c.)} \\
\text{ML(T)} & \quad \text{Carbon concentration in lower oceans (b.t.c.)} \\
\text{TE(T)} & \quad \text{Temperature of atmosphere (C)} \\
\text{FORC(T)} & \quad \text{Radiative forcing (W per m2)} \\
\text{TL(T)} & \quad \text{Temperature of lower ocean (C)} \\
\text{C(T)} & \quad \text{Consumption trill US dollars} \\
\end{align*}
\text{UTILITY};

\text{POSITIVE VARIABLES MIU, TE, E, Mat, mu, ml, Y, C, K, I ;}

\text{EQUATIONS}
\begin{align*}
\text{YY(T)} & \quad \text{Output equation}
\end{align*}
CC(T)  Consumption equation
KK(T)  Capital balance equation
KK0(T) Initial condition for K
KC(T)  Terminal condition for K
EE(T)  Emissions process
MMAT0(T) Starting atmospheric concentration
MMAT(T) Atmospheric concentration equation
MMU0(T) Initial shallow ocean concentration
MMU(T) Shallow ocean concentration
MML0(T) Initial lower ocean concentration
MML(T)  Lower ocean concentration
TTE(T) Temperature-climate equation for atmosphere
TTE0(T) Initial condition for atmospheric temperature
FORCE(T) Radiative forcing equation
TLE(T) Temperature-climate equation for lower oceans
TLE0(T) Initial condition for lower ocean
UTIL Objective function;

** Equations of the model

KK(T). K(T+1) =L= (1-(DK/100))**10 *K(T)+10*I(T);
KK0(TFIRST).  K(TFIRST) =E= K0;
KC(TLAST).  .02*K(TLAST) =L= I(TLAST);

EE(T).
E(T)=G=10*SIGMA(T)*(1-(MIU(T)/100))*AL(T)*L(T)**(1-GAMA)*K(T)**GAMA + ETREE(T);
FORCE(T). FORC(T) =E= 4.1*((log(Mat(T)/596.4)/log(2)))+FORCOTH(T);

MMAT0(TFIRST). MAT(TFIRST) =E= MAT1990;
MMU0(TFIRST). MU(TFIRST) =E= MU1990;
MML0(TFIRST). ML(TFIRST) =E= ML1990;
MMAT(T+1). MAT(T+1) =E= MAT(T)*(b11/100)+E(T)+MU(T)*(b21/100);
MML(T+1). ML(T+1) =E= ML(T)*(b33/100)+(b23/100)*MU(T);
MMU(T+1). MU(T+1) =E= MAT(T)*(b12/100)+MU(T)*(b22/100)+ML(T)*(b32/100);

TTE0(TFIRST). TE(TFIRST) =E= T0;
TTE(T+1). TE(T+1) =E= TE(t)+C1*(FORC(t)-(4.1/CS)*TE(t)-C3*(TE(t)-TL(t)));
TLE0(TFIRST). TL(TFIRST) =E= TL0;
TLE(T+1). TL(T+1) =E= TL(T)+C4*(TE(T)-TL(T));
YY(T).  \[ Y(T) = E = \frac{A(T) L(T)^{(1-GAMA)} K(T)^{GAMA}(1-cost1(t)*((MU(T)/100)^{cost2}))}{(1+a1*TE(T)+ a2*TE(T)^2)}; \]

CC(T).  \[ C(T) = E = Y(T)-I(T); \]

UTIL.  UTILITY = E = \sum(T, 10 *RR(T)*L(T)*LOG(C(T)/L(T))/coefopt1) + coefopt2;

** Upper and Lower Bounds: General conditions imposed for stability

MIU.up(T) = 1.0;
MIU.lo(T) = 0.000001;
K.lo(T) = 1;
TE.up(t) = 12;
MAT.lo(T) = 10;
MU.lo(t) = 100;
ML.lo(t) = 1000;
C.lo(T) = 2;

** Emissions control policy. Current setting is for optimal policy.
** Reinstall equation "Miu.fx(t) = .0" for no-control run.

MIU.fx(t) = 0;

** Solution options

option iterlim = 99900;
option reslim = 99999;
option solprint = on;
option limrow = 0;
option limcol = 0;

model CO2 /all/;

solve CO2 maximizing UTILITY using nlp;

** Display of results

display y.l, e.l, mat.l, te.l;

Parameters
Year(t)  Date
Indem(t)  Industrial emissions (b.t.c. per year)
Wem(t)  Total emissions (b.t.c. per year)
S(t)  Savings rate (pct);

Year(t) = 1995 +10*(ord(t)-1);
Indem(t) = e.l(t)-etree(t);
Wem(t) = e.l(t);
S(t) = 100*i.l(t)/y.l(t);

display s;

Parameters
Tax(t)  Carbon tax ($ per ton)
damtax(t)  Concentration tax ($ per ton)
dam(t)  Damages
cost(t)  Abatement costs;

tax(t) = -1*ee.m(t)*1000/(kk.m(t));
damtax(t) = -1*mmat.m(t)*1000/kk.m(t);
dam(t) = y.l(t)*(1-1/(1+a1*te.l(t)+ a2*te.l(t)**2));
cost(t) = y.l(t)*(cost1(t)*(miu.l(t)**cost2));

File d98oute;
D98oute.pc=5;
D98oute.pw=250;
Put d98oute;
Put / "base (no control) run";
Put / "year";
Loop (tearly, put year(tearly)::0);
Put / "output";
Loop (tearly, put y.l(tearly)::3);
Put / "indem";
Loop (tearly, put indem(tearly)::4);
Put / "sigma";
Loop (tearly, put sigma(tearly)::4);
Put / "temp";
Loop (tearly, put te.l(tearly)::3);
Put / "conc";
Loop (tearly, put mat.l(tearly)::3);
Put / "ctax";
Loop (tearly, put tax(tearly)::2);
Put / "discrate"
Loop (tearly, put rr(tearly)::5);
Put / "prod"
Loop (tearly, put al(tearly)::3);
Put / "exogforc"
Loop (tearly, put forcoth(tearly)::3);
Put / "pop"
Loop (tearly, put l(tearly)::3);
Put / "etree"
Loop (tearly, put etree(tearly)::4);
Put / "margy"
Loop (tearly, put yy.m(tearly)::3);
Put / "marge"
Loop (tearly, put cc.m(tearly)::3);
Put / "miu"
Loop (tearly, put miu.l(tearly)::3);
Put / "total emissions";
Loop (tearly, put wem(tearly)::3);
Put / "damages";
Loop (tearly, put dam(tearly)::5);
Put / "abatement cost";
Loop (tearly, put cost(tearly)::5);
Put / "objective function";
Put utility.l::3;

File d98outL;
D98outL.pc=5;
D98outL.pw=250;
Put d98outL;
Put / "base (no control) run";
Put / "year";
Loop (tlate, put year(tlate)::0);
Put / "output";
Loop (tlate, put y.l(tlate)::3);
Put / "indem";
Loop (tlate, put indem(tlate)::4);
Put / "sigma";
Loop (tlate, put sigma(tlate)::4);
Put / "temp";
Loop (tlate, put te.l(tlate)::3);
Put / "conc";
Loop (tlate, put mat.l(tlate)::3);
Put / "ctax";
Loop (tlate, put tax(tlate)::2);
Put / "discrate";
Loop (tlate, put rr(tlate)::5);
Put / "prod";
Loop (tlate, put al(tlate)::3);
Put / "exogforc";
Loop (tlate, put forcoth(tlate)::3);
Put / "pop";
Loop (tlate, put l(tlate)::3);
Put / "etree";
Loop (tlate, put etree(tlate)::4);
Put / "margy";
Loop (tlate, put yy.m(tlate)::3);
Put / "marge";
Loop (tlate, put cc.m(tlate)::3);
Put / "miu";
Loop (tlate, put miu.l(tlate)::3);
Put / "total emissions";
Loop (tlate, put wem(tlate)::3);
Put / "damages";
Loop (tlate, put dam(tlate)::5);
Put / "abatement cost";
Loop (tlate, put cost(tlate)::5);