Alternative Methods for Measuring Productivity Growth Including Approaches When Output is Measured With Chain Indexes

William D. Nordhaus\textsuperscript{1}

June 24, 2002

Abstract

The present study is a contribution to the theory of the measurement of productivity growth. It first examines the welfare-theoretic basis for measuring productivity growth and shows that the ideal welfare-theoretic measure is a chain index of the productivity growth rates of different sectors which uses current nominal output weights. Second, it lays out a technique for decomposing productivity growth which separates aggregate productivity growth into four factors – the pure productivity effect, the effect of changing shares or Baumol effect, the effect of different productivity levels or Denison effect, and a fixed-weight drift term. Finally, it shows how to apply the theoretically correct measure of productivity growth and indicates which of the four different components should be included in a welfare-oriented measure of productivity growth. The study concludes that none of the measures generally used to measure productivity growth is consistent with the theoretically correct measure.

\textsuperscript{1} I am grateful for helpful comments by Erwin Diewert, Paul Samuelson, and Thijs ten Raa. This version is an augmented version of a similar paper dated November 20, 2000. The major change are a correction of an erroneous approximation formula, an addition of a section dealing with chain-weighted output indexes, and updating of the empirical estimates. Version is welf_062402.doc
Measuring productivity growth has been a growth industry within economics for half a century. Over this period, there have been substantial changes and improvements in the construction of the underlying data and methods. Particularly notable are improvements in measuring output and prices and in implementing improved indexes, notably the use of “superlative” price and output measures by government statistical agencies.²

Productivity growth is usually taken to be an obvious index of welfare. Paul Krugman put it succinctly, “Productivity isn't everything, but in the long run it is almost everything.”³ The link between productivity growth and economic welfare is actually not obvious. There has, however, been surprisingly little attention to the construction of productivity measures.

The present paper is part of a larger study which is devoted to analytical and empirical questions in productivity measurement.⁴ ⁵ It makes three contributions to understanding the measurement of productivity. First, it examines the welfare-theoretic basis for measuring productivity growth. Second, it lays out a technique for decomposing productivity growth which divides aggregate productivity trends into four factors that contribute to the growth in economy-wide productivity. Finally, it discusses the appropriate way to apply the ideal welfare-theoretic measure in practice.

The major practical result of this study is that current measures of productivity growth are generally inappropriate from the point of view of reflecting economic welfare. We propose an alternative measure of productivity

---


growth rate, the chain index productivity growth rates, which better approximates the ideal index.

I. Welfare Aspects of Productivity Measures

We begin with the question of the ideal approach to measuring productivity growth. We approach this issue using the tools of index number theory.\(^6\) For simplicity, we assume that all output is devoted to consumption goods and that consumption goods are immediately used up (i.e., there are no durable goods).\(^7\) We further assume that the appropriate measure of real income is a smooth utility function of the form

\[
U_t = U(C_{1t}, C_{2t}, ..., C_{nt})
\]

where \(C_{it}\) is the flow of services from consumption goods at time \(t\), and there are \(n\) goods, \(i = 1, ..., n\). We do not assume any particular form for \(U\), but we do assume that the utility function is smooth and homothetic. Under this assumption, we can construct indexes of real income changes by taking the weighted average growth of individual components.

It will be convenient to simplify by assuming that each good is produced by primary factors alone, so \(C_{it} = F_i(S_{it})\), where \(F_i\) is a constant returns to scale production function for industry \(i\) and \(S_{it}\) is a scalar index of inputs into the industry \(i\) (for example, \(S\) might be a Cobb-Douglas function of the relevant inputs). If \(A_{it}\) is total factor productivity in sector \(i\), we can then write the production function as \(C_{it} = A_{it} S_{it}\).

For the discussion in this section, we assume that the economy is characterized by perfect competition, that all factors are priced at their marginal products, and that all goods are priced at their marginal costs. This assumption removes influences of imperfect competition and the distortions that may arise from indirect taxation. Finally, we assume that households have identical utility functions and endowments.

In addition, we make three simplifying normalizations. First, each household supplies one unit of the composite input, \(S\). Second, we normalize the price of the


composite input $S$ to be unity. These normalizations imply that each household has 1 unit of income. Third, we assume that initial price and level of productivity are equal to 1. Under these assumptions, the price of each good is given by:

\[ q_{it} = 1/A_{it}. \]

We now consider the expenditure function, $V$, that comes from maximizing the utility function in (1) subject to the budget constraint:

\[ E_t = V(q_{1t}, q_{2t}, \ldots, q_{nt}, U_t) \]

where $E_t$ is expenditure. Note that the income term has been suppressed because we normalize income to be unity. Differentiating (3) with respect to time yields:

\[
\dot{E}_t = (\partial V/\partial q_{1t}) \dot{q}_{1t} + (\partial V/\partial q_{2t}) \dot{q}_{2t} + \cdots + (\partial V/\partial q_{nt}) \dot{q}_{nt}
\]

where a raised dot over a variable indicates a time derivative. Using the properties of the expenditure function, we have

\[
\dot{E}_t = C_{1t} \dot{q}_{1t} + C_{2t} \dot{q}_{2t} + \cdots + C_{nt} \dot{q}_{nt}
\]

Dividing by $E_t$ and multiplying and dividing each term on the right hand side by the relevant $q_{it}$ yields

\[ \dot{E}_t/E_t = C_{1t} q_{1t} \left[ \dot{q}_{1t} / q_{1t} \right] / E_t + C_{2t} q_{2t} \left[ \dot{q}_{2t} / q_{2t} \right] / E_t + \cdots + C_{nt} q_{nt} \left[ \dot{q}_{nt} / q_{nt} \right] / E_t \]

Taking the logarithmic derivative of (2) and substituting into (4) yields:

\[ g(E_t) = - \left[ \sigma_{1t} g(A_{1t}) + \sigma_{2t} g(A_{2t}) + \cdots + \sigma_{nt} g(A_{nt}) \right] \]

where $\sigma_{it} = C_{it} q_{it} / E_t$ = the share of good $i$ in total nominal spending at time $t$. In what follows, we use the notation that $g(x_t) = \dot{x}_t / x_t = \text{the rate of growth of } x_t$ for either continuous or discrete variables as appropriate.

We now proceed to determine the growth in real income due to changes in the total factor productivities in different industries. Defining real income as $R_t$, the growth in real income can be calculated as the growth in $R_t$ over time. Since (5) represents the
decline in total expenditure or income necessary to attain a constant utility, by homotheticity the growth in real income that can be attained with the actual consumption shares, productivity levels, and prices is therefore:

\[ g(R_t) = \sigma_{1t} g(A_{1t}) + \sigma_{2t} g(A_{2t}) + \ldots + \sigma_{nt} g(A_{nt}) \]

In words, the growth rate of real income or real output is the chain-weighted index of sector-level productivity growths. The weights in the index are the current nominal shares of each good in total nominal consumption. With discrete time, equation (6) should be calculated as an equation in growth rates using Fisher or other superlative weights.

We now see how equation (6) applies to the question of the ideal welfare-theoretic measure of productivity in an economy with many sectors experiencing varying rates of productivity growth. The major result is that the ideal measure of productivity growth is a weighted average of the productivity growth rates of different sectors. This formula is very similar to that currently used in constructing superlative indexes of prices and output. The important point is that the indexes used in the appropriate measure are chain indexes of productivity growth rather than differences in the growth rates or indexes of output and inputs.

II. Decomposing Actual Productivity Growth into its Components

In this section, we turn to the question of how productivity growth is actually measured. It will be convenient to begin with aggregate measures of productivity growth and break them into their major components. We will see that the welfare analysis of the previous section fits very neatly into this decomposition.

**Productivity Accounting**

Movements in aggregate productivity are composed of both productivity changes in individual industries and changes in the composition of industries. Research on decomposing the aggregate between the two components goes back many years, but it has not been updated to reflect changes in measurement of output. This section reviews the issues and updates prior work to reflect the shift from fixed-weighted quantity indexes to chain indexes.

*Productivity with fixed weights (“old-style output”)*

We begin with a review of the decomposition of output growth using fixed-weighted quantity indexes (references to the literature will be provided in the
discussion section below). To begin with, we consider productivity accounting where output is measured in the “old style” -- that is, with fixed weights. Consider aggregates of output ($X_t$), composite inputs ($S_t$), and total or partial factor productivity ($A_t = X_t/S_t$). The aggregates are the weighted sums of industry output and inputs ($X_{it}$ and $S_{it}$), where for simplicity we measure output and inputs in base-year prices so that they can be summed to form the aggregates. We can rewrite these as built up from industry values ($i = 1, ..., N$) as follows:

$$X_t = \sum_i X_{it}$$

$$S_t = \sum_j S_{jt}$$

and aggregate productivity is:

$$A_t = \frac{X_t}{S_t} = \frac{\sum_i X_{it}}{\sum_j S_{jt}}$$

For simplicity, I will begin by analyzing productivity growth in continuous time under the assumption of smooth series; the discrete analysis is extremely messy. Taking the logarithmic derivative of (6) yields:

$$g(A_t) = g(X_t) - g(S_t)$$

$$= \sum_i \left( \frac{\dot{X}_{it}}{X_{it}} \right) (X_{it}/X_t) - \sum_i \left( \frac{\dot{S}_{it}}{S_{it}} \right) (S_{it}/S_t)$$

$$g(A_t) = \sum_i g(X_{it})z_{it} - \sum_i g(S_{it}) w_{it}$$

where $z_{it} = X_{it}/X_t$ = the share of real output of industry $i$ in total real output and $w_{it} = S_{it}/S_t$ = the share of inputs of industry $i$ in total inputs.

Next add and subtract $\sum_i g(X_{it})\sigma_{it}$ and $\sum_i g(S_{it})\sigma_{it}$ from the right-hand side of (7):

---

8 So messy that, in the earlier draft deriving an exact discrete formula for the decomposition, I employed an inexact approximation which led to an error when applied to fixed-weight quantity indexes. I am grateful to Thijs ten Raa for pointing out the error.
(8) \[ g(A_t) = \sum_i g(A_{it})\sigma_{it} + \sum_i g(S_{it})[\sigma_{it} - w_{it}] + \sum_i g(X_{it})[z_{it} - \sigma_{it}] \]

Finally, add and subtract \( \sum_i g(A_{it})\sigma_{i,0} \), where \( t = 0 \) is the base period for productivity calculations:

(9) \[ g(A_t) = \sum_i g(A_{it})\sigma_{i,0} + \sum_i g(A_{it})[\sigma_{it} - \sigma_{i,0}] + \sum_i g(S_{it})[\sigma_{it} - w_{it}] + \sum_i g(X_{it})[z_{it} - \sigma_{it}] \]

The four terms in (9) are the fixed-weight average productivity growth, the Baumol effect, the Denison effect, and the fixed-weight drift term. I will discuss each of the terms in a later section.

**Productivity with chain weights (“new-style output”)**

Decompositions for fixed-weight quantity indexes like those in the prior section have been known for at least a quarter century. Since that time, the United States and some other countries have introduced “superlative” indexes that remove some of the problems that arise with fixed-weight quantity indexes. The U.N. System of National Accounts (SNA) recommends using annual chain indexes,\(^9\) and the U.S. and Canada have adopted chain Fisher indexes for their national accounts.\(^10\)

Consider next the decomposition of productivity growth where output is measured using chain indexes, focusing primarily on the Fisher ideal index. All variables are as in the last section except that output is measured as a chain index. Consider the growth of \( X_t \), dealing now for the moment in discrete time. The exact definition of the aggregate index, \( X_t \), will differ depending upon how the chain index is calculated. If the index is calculated using Fisher’s ideal index, then the growth of output is calculated as a Fisher index (\( F_t \)), which is a geometric average of the growth

---

\(^9\) The SNA summary states, “The System also provides specific guidance about the methodology to be used to compile an integrated set of price and volume indices for flows of goods and services, gross value added and GDP that are consistent with the concepts and accounting principles of the System. It is recommended that annual chain indices should be used where possible, although fixed base indices may also be used when the volume measures for components and aggregates have to be additively consistent for purposes of economic analysis and modelling.” (SNA, 1993, paragraph 1.17). The reference is available at [http://unstats.un.org/unsd/sna1993/](http://unstats.un.org/unsd/sna1993/).

\(^10\) A full discussion of the introduction of Fisher indexes is provided on the web site of the Bureau of Economic Analysis at [http://www.bea.gov/bea/an1.htm](http://www.bea.gov/bea/an1.htm).
rates using Laspeyres (Lt) and the Paasche (Pt) indexes. More precisely, \( X_t = X_{t-1} F_t = X_{t-1} (L_t P_t)^{1/2} \), where \( L_t = \sum_i X_{it} q_{i,t-1} / \sum_i X_{i,t-1} q_{i,t-1} \) and \( P_t = \sum_i X_{it} q_{it} / \sum_i X_{i,t-1} q_{it} \).

Taking logarithms and combining the expressions, we have

\[
\Delta \ln(X_t) = \frac{1}{2} \left\{ \ln \left[ \sum_i \frac{X_{it} q_{i,t-1}}{X_{i,t-1} q_{i,t-1}} \right] + \ln \left[ \sum_i \frac{X_{it} q_{it}}{X_{i,t-1} q_{it}} \right] \right\}
\]

A little manipulation shows that \( L_t = \sum_i [(1 + g_{it}) \sigma_{i,t-1}] \), while \( P_t = \{ \sum_i [(1 + g_{it})^{-1} \sigma_{i,t}] \}^{-1} \), where \( \sigma_{i,t} \) is again the share of industry i in nominal output for period t.

For smooth series and small time intervals, we can take the first-order approximation of these equations, which yields the approximation to the Fisher index as follows:

\[
\Delta \ln(X_t) = \ln(F_t) = \sum_i g(X_{it}) \sigma_{i,tav}
\]

where \( \sigma_{i,tav} \) is the average share of \( \sigma_i \) for the current and previous period. It is useful to note that the approximation formula in (10) is actually the Tornqvist index.

Applying the Fisher formula in (10), we can derive the growth of productivity as:

\[
\Delta \ln(A_t) = \sum_i g(X_{it}) \sigma_{i,tav} - \sum_i g(S_{it}) w_{i,t-1}
\]

Moving to continuous time and then adding and subtracting \( \sum_i g(S_{it}) \sigma_{i,t} \) yields:

\[
\Delta \ln(A_t) = \sum_i g(A_{it}) \sigma_{i,t} + \sum_i g(S_{it}) [\sigma_{i,t} - w_{i,t}]
\]

The interpretation of (11) is that the aggregate rate of productivity growth is equal to the weighted average productivity growth of the individual sectors plus the difference-weighted average of the growth of inputs. The weights on productivity growth are the shares of nominal outputs while the difference-weights on input growth are the differences between output and input shares. (A symmetrical formula could be derived where the roles of input and output shares are reversed.)

One final formula includes the role of changing shares of output. Add and
subtract \( \sum g(A_{i,0}) \sigma_{i,0} \) from equation (11’) and rearrange terms. This yields the decomposition of productivity growth for chain quantity indexes:

\[
g(A_t) = \sum_i g(A_{i,t}) \sigma_{i,0} + \sum_i g(A_{i,t}) [\sigma_{i,t} - \sigma_{i,0}] + \sum_i g(S_{i,t})[\sigma_{i,t} - \sigma_{i,0}] + \sum_i g(X_{i,t})[\sigma_{i,t} - \sigma_{i,0}] + \sum_i g(X_{i,t})[\sigma_{i,t} - \sigma_{i,0}]
\]

For easy reference, I repeat equation (9) for fixed-weight quantity indexes:

\[
g(A_t) = \sum_i g(A_{i,t}) \sigma_{i,0} + \sum_i g(A_{i,t}) [\sigma_{i,t} - \sigma_{i,0}] + \sum_i g(S_{i,t})[\sigma_{i,t} - \sigma_{i,0}] + \sum_i g(X_{i,t})[\sigma_{i,t} - \sigma_{i,0}]
\]

**Interpretation**

Equations (9) and (12) are identical except for the fourth term of (9), which is the fixed-weight drift term. The other three terms are, in order: the pure (fixed-weight) productivity term using fixed nominal output weights for a given year (\( t = 0 \), which might be different from the base year for a fixed-price-base quantity index); the Baumol effect which reflects the difference between current nominal output weights and base-year nominal output weights; and the Denison effect which reflects the interaction between the growth of inputs and the difference between output and input weights. We now discuss each term in detail.

**Fixed-weight drift term.** The fourth term in (9) represents the “fixed-weight drift term.” The interesting part of the fourth term is \((z_{it} - \sigma_{it})\). This term is the difference between the share of industry \( i \) in total real output and its share in total nominal output. Its properties are well-known: When the relative prices of year \( t \) are very close to those of the price-base period, this term will be close to zero. On the other hand, as we move further from the price-base period, and as relative prices diverge from the price-base-year prices, relative real outputs diverge from relative nominal outputs, and the fourth term will tend to be nonzero. Real output with a Laspeyres fixed-base quantity index tends to grow more slowly than a chain index in periods before the base year and more rapidly in periods after the base year. The divergence of relative real outputs from relative nominal outputs with “old-style” fixed-weight quantity indexes motivates the name “fixed-weight drift term.” This term vanishes with the introduction of chain indexes (or more precisely, well-constructed superlative index numbers) because real output shares used in calculating the growth rates are to a first approximation equal to nominal output shares.

**Pure Productivity Effect.** The first term on the right hand side of equations (9) and (12) is a fixed-weighted average of the productivity growth rates of different sectors.
More precisely, this measures the sum of the growth rates of different industries weighted by base-year nominal output shares of each industry. Another way of interpreting the pure productivity effect is as the productivity effect if there were no change in output composition among industries. This is a pure measure of the underlying trends in technology in different industries that omits the effect of changes in spending patterns.

*The Baumol effect.* The second term captures the interaction between the differences in productivity growth and the changing shares of different industries over time. This effect has been emphasized by William Baumol in his work on unbalanced growth.\(^1\) The Baumol effect occurs when those sectors with relatively slow productivity growth also have rising nominal output shares. This syndrome is often attributed to the service industries, with live performances of Mozart string quartettes being a much-cited example.

The Baumol effect, also sometimes called the “cost disease,” is often misunderstood as saying that low-productivity-growth sectors have higher than average sectoral inflation rates. This proposition is virtually universal and completely unremarkable. The true Baumol effect arises when industries with low productivity growth and rising relative prices also have rising shares of nominal output – in effect, being industries with relatively price-inelastic demand. Whether such cases are the norm or not is in fact a major empirical question. The usual example of health care probably does not prove the case because the price indexes used to estimate the prices of medical care are highly defective. Estimates from a companion paper indicate that the Baumol effect for the United States in the period 1977-2000 was actually slightly positive rather than negative as generally supposed.\(^2\)

*Denison Effect.* The third terms in (9) and (12) capture level effects due to either changing shares of inputs on aggregate productivity. I label these the Denison effect, after Edward Denison who pointed out that the movement from low-productivity-level agriculture to high-productivity-level industry would raise productivity even if the productivity growth in the two industries were zero. Denison showed that this effect was an important component of overall productivity growth in much of the twentieth century.\(^3\)


\(^2\) See the references in footnote 5.

\(^3\) A number of studies found this syndrome. See particularly his studies of postwar Europe in *Why Growth Rates Differ*, Brooking, Washington, D.C., 1962. This finding was also identified in Nordhaus,
We can get a more interesting interpretation of the Denison effect by manipulating it as follows. Define $R_{it} = \frac{A_{it}}{A_t} = \text{productivity in sector } i \text{ relative to aggregate productivity.}$ Then, note that $R_{it} \cdot W_{it} = \sigma_{it}$ and that $W_{it} [g(S_{it})- g(S_t)] = \hat{S}_{it}$. Substituting all these where appropriate into equation (12), we have:

$$\text{Denison effect} = \sum_i \sigma_{it} [g(S_{it}) - \hat{S}_{it}] = \sum_i \sigma_{it} \left[ g(S_{it}) - g(S_t) \right] = \sum_i R_{it} \left[ g(S_{it}) - g(S_t) \right]$$

or

$$(13) \quad \text{Denison effect} = \sum_i R_{it} \cdot \hat{S}_{it}$$

The Denison effect is therefore equal to the sum of the changes in input shares of different industries weighted by their relative productivity levels. The classical Denison effect occurred when the share of inputs in agriculture fell and agriculture was a low-productivity-level industry. In this case, the Denison effect was be positive, indicating that the aggregate productivity increase was higher than the weighted average productivity increases in all industries. More generally, when the input share of inputs of high-productivity-level industries rises at the expense of low-productivity-level industries, this will tend to raise aggregate productivity.

**Appropriate Treatment of the Different Effects**

A major question in measuring productivity growth concerns the appropriate construction of indexes. Which of the four components of equation (9) or three in (12) should be included if our productivity measures are to be a useful measure of welfare?

For this discussion, we turn as an application to changes in labor productivity – that is, we interpret the variable $S$ as labor hours worked. The question then becomes what is the ideal measure of labor productivity? This question can be answered by comparing measured productivity growth in equation (9) or (12) with the ideal productivity growth measure shown in equation (6). A comparison of the three equations shows that the ideal index of productivity growth from a welfare-theoretic perspective includes the first two terms in (9) or (12) but excludes the other two terms.

*op. cit.*, 1972, which used a slightly different version of the decomposition for fixed-weight indexes.
This implies that the pure productivity effect and Baumol effect should be included in a welfare-oriented measure of productivity growth. The reason for the pure productivity effect is intuitive. Additionally, the Baumol effect reflects the impact of changing expenditure shares on the overall productivity measure. If spending is increasingly devoted to sectors that have low productivity growth, then this implies that our economic welfare will indeed be growing relatively slowly.

This also implies that the fixed-weight drift term should be excluded from a welfare-theoretic measure of productivity growth. This is fortunate, given the universal movement away from fixed-weight indexes of output growth noted above. The reason that it is appropriate to exclude the fixed-weight drift term is that the welfare weights on growth in outputs are current nominal output, which are the weights used by chain indexes. In economic terms, since idealized consumers set relative marginal utilities each period equal to current relative market prices, the welfare weights should be current relative market prices rather than relative prices of some historical base year.

Less obvious, but equally true, is that the Denison effect should normally be excluded from an ideal productivity index. To understand the reason for its exclusion requires some discussion of the potential sources of the Denison effect. This point is most easily understood using the Denison effect in equation (13). The shows that the Denison effect is equal to the weighted average of the productivity relatives and the change in the shares of inputs. In other words, the Denison effect arises because of differences in the levels of productivity by industry.

We can identify three major reasons for differences. The first is that differences in productivity levels reflect differences in inputs which are not captured by our productivity measures. For this first case, the Denison effect should be excluded from a welfare-oriented measure of productivity because interindustry shifts produce spurious changes in productivity growth. If in fact the levels of total factor productivity are equal in all industries, then the Denison effect would by construction be zero.14

A second reason for differences in productivity levels would arise because of differences in indirect taxation. A third reason would arise from disequilibrium in input or output markets, because of slow migration of labor from farming to industry, or because of market power. If the second and third reasons were the major source of differences in productivity levels, then the treatment would be more complex and some or all of the Denison effect might be appropriately included in a welfare-theoretic measure of productivity growth. As in other cases where tax wedges or other
distortions apply, the appropriate treatment will usually be somewhere between inclusion and exclusion depending upon the relevant elasticities.

An examination of the actual patterns of labor productivity across sectors suggests that the differences in productivity arise primarily because of the first reason, differences in inputs which are not captured by our productivity measures. In 1998, the level of labor productivity (gross output per hour worked) differed by more than a factor of 100 across major industries. One major source of difference comes from differences in capital intensity or in labor skills across industries. For example, the highest gross output per person employed in 1998 at the two-digit level was in nonfarm housing services, with a productivity level 34 times that of the overall economy. The high productivity arose because this sector is essentially entirely imputed rents. Other high productivity ratios are found in capital-intensive sectors such as pipelines, oil and gas extraction, and telephone services. Similarly, high productivity levels are found in sectors with specialized and highly compensated skills such as security and commodity brokers. At the other end of the spectrum are industries with low-skilled workers, such as private households, personal services, and apparel.

The second and third sources of productivity differences appear less significant today. The industry which gave rise to the Denison effect, farming, had a productivity ratio of 90 percent of the total economy in 1998. This suggests that disequilibrium in labor migration patterns is a relatively unimportant source of productivity differences today. Moreover, there are few two-digit industries where indirect taxes are a large share of gross output. The major case is tobacco products, where indirect taxes were 60 percent of gross output in 1998. Among one-digit industries, the ratio of indirect business taxes to gross output in 1998 ranged from a low of 2 percent in construction to a high of 21 percent in wholesale trade. While these differences are not trivial, they are much smaller than the differences in productivity due to differing capital intensities or labor qualities.

**Numerical examples**

We can illustrate the problems discussed here using a numerical example. For this example, we use a Fisher chain index of the growth of output (so the fixed-weight drift term does not apply). Unfortunately, the calculations are tedious, so I have omitted the underlying details. Table 1 illustrates how the Denison effect can provide misleading estimates of productivity growth if not properly calculated. It shows an economy with two industries with differing levels of productivity. Industry 1 is a high productivity level sector while industry 2 is a low productivity level sector. The bottom of the table shows different ways of calculating productivity growth. Line 1 shows the welfare-theoretic measure of productivity growth from equation (6), which includes the pure productivity effect and the Baumol effect. Lines 5 shows the standard calculation...
(simply output per hour) and line 6 shows the sum of the three effects. Line 7 shows the error, which is due to omitted second order effects. For chain indexes, the direct calculation is also the difference of growth rates methodology, which is the approach used by the BLS and many scholars.

For the example in Table 1, both the aggregate on line 6 and the difference of growth rates methodologies give misleading results because they include the Denison effect. (If they use chain indexes, happily, they do not included the fixed-weight drift term.) Aggregate productivity grows at 0.704 percent. The Denison effect is negative (see line 4), so the aggregate estimate of productivity growth understates the appropriate method which includes only the pure productivity effect and the Baumol effect (shown on line 3). Hence the welfare measure of productivity growth is understated.

Table 2 shows an example with a strong negative Baumol effect (illustrating his sick-services syndrome). Here, the initial levels of productivity in the two sectors are equal, but industry 1 shows a declining share of hours along with strong productivity growth while industry 2 is technologically stagnant with a relatively large growth in hours. For this case, the Baumol effect is strongly negative (-1.33 percent in line 3). The positive pure productivity effect is pulled down by the large negative Baumol effect. In this case, the standard calculation – which includes the Baumol effect -- provides a reasonably accurate estimate of the welfare-theoretic estimate because the Denison effect is small.

**Application to aggregate U.S. data**

We can illustrate the procedures using actual data on labor productivity for the United States. These data are derived from two companion papers on the subject. The first companion paper presents a new data set on aggregate and industrial productivity derived from income-side data.\(^\text{15}\) The second companion paper applies the concepts in this paper and the data in the first paper to estimating productivity growth and the role of the new economy in the recent productivity upsurge.\(^\text{16}\)

Figure 1 and Table 3, which are drawn from the second paper, show a comparison of two measures of labor productivity for the overall economy over the period 1978-2000. The series called “ideal measure” is the welfare-theoretic index derived in a manner defined in equation (6) above. This shows the rate of productivity growth that best measures the growth in average living standards. The measure labeled

\(^{15}\) See footnote 4.

\(^{16}\) See footnote 5.
“GDI productivity” is the growth of total labor productivity, measured as income-side GDP per hour worked.

The results show a significant difference between the two concepts. The ideal or welfare-theoretic measure is higher than standard labor productivity in two of the three subperiods. The differences are substantial in the most recent period, where the ideal index exceeds the standard measure by 0.58 percentage points per year. On average, the ideal or welfare-theoretic measure over the entire period was 0.13 percentage points per year higher than total income-side productivity growth.

**Current Approaches to Constructing Productivity Measures**

Given the vast literature on productivity growth, there is surprisingly little discussion of the welfare-theoretic interpretation of alternative measures. There is of course a vast literature on the construction of ideal indexes of prices and output. One of the few studies to apply these to productivity is by Caves, Christensen, and Diewert. This study does not address the issue of differing levels of productivity in different industries, however. One important study of the relationship between alternative measures of productivity and welfare theory was by Baumol and Wolff, which recommended the use of what they called a “deflated index of total factor productivity.” This index is constructed by deflating nominal labor productivity by the economy-wide average of the real wage. While this formula does correct for the Denison effect, it does not appear to identify the need for a chain index to measure welfare improvements of productivity growth.

Early applied analyses of total factor productivity and labor productivity did not analyze the appropriate index from a welfare-theoretic point of view. Alternative approaches were used in these studies, but the central approaches were generally ones which weighted productivity growth by fixed output weights. This approach is clearly inappropriate and can give misleading results if productivity growth differs by sector.

---


The work of Jorgenson and Griliches and later work by them and co-authors derive measures of productivity growth from general transformation functions.\(^{20}\) This approach led to the suggestion, pioneered by Griliches and Jorgenson, that productivity growth be estimated as the difference between Divisia indexes of output growth and input growth (these are essentially Tornqvist indexes). This approach, which we call the “difference of growth rates” approach, will be close to the ideal approach if productivity levels in different industries are close, but it will not in general provide the correct result from a welfare theoretic point of view if the Denison effect is present. (See Tables 1 and 2.)

The Bureau of Labor Statistics currently uses the difference of growth rates approach in its productivity measures.\(^{21}\) The output measures are currently chain indexes of output using Fisher or Tornqvist weights. Labor inputs are either hours or weighted hours at work. Measures of productivity are calculated as the differences between the growth in output and the growth of inputs. The BLS measures therefore suffer from the deficiency that it includes the Denison effect. More generally, it is not a chain index of productivity growth, which is the preferred measure.

In summary, none of the current approaches to estimating productivity growth appear to be well-grounded in welfare economics. Rather, assuming that differences in productivity growth are due to differences in unmeasured inputs, the appropriate measure would be a chain index of productivity growth of different sectors weighted by current expenditure or current-value inputs shares. In terms of the decomposition derived above, the appropriate measure would be total productivity growth after removing both the fixed-weight drift term and the Denison effect. Alternatively, the appropriate measure would be the pure productivity effect plus the Baumol effect. It is useful to note, that none of the current measures of productivity follow the appropriate procedure for measuring productivity growth.


Table 1
Example of Alternative Measures of Productivity Growth
Showing Strong Denison Effect

Illustration of Denison Effect

<table>
<thead>
<tr>
<th>Period</th>
<th>Industry 1</th>
<th>Industry 2</th>
<th>Sum</th>
<th>Fisher Index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real output (period 1 prices)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>100.00</td>
<td>50.00</td>
<td>150.00</td>
<td>150.00</td>
</tr>
<tr>
<td>2</td>
<td>101.00</td>
<td>51.00</td>
<td>152.00</td>
<td>152.00</td>
</tr>
<tr>
<td></td>
<td>Hours</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>60.00</td>
<td>100.00</td>
<td>160.00</td>
<td>160.00</td>
</tr>
<tr>
<td>2</td>
<td>59.00</td>
<td>102.00</td>
<td>161.00</td>
<td>161.00</td>
</tr>
</tbody>
</table>

Components of productivity growth

<table>
<thead>
<tr>
<th></th>
<th>Welfare productivity growth</th>
<th>Pure productivity effect (early weights)</th>
<th>Baumol effect</th>
<th>Denison effect</th>
<th>Sum (2 + 3 + 4)</th>
<th>Direct calculation</th>
<th>Error (5 - 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1.773%</td>
<td>0.000%</td>
<td>1.773%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Pure productivity effect</td>
<td>1.784%</td>
<td>0.000%</td>
<td>1.784%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Baumol effect</td>
<td>-0.011%</td>
<td>0.000%</td>
<td>-0.011%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Denison effect</td>
<td>-0.490%</td>
<td>-0.578%</td>
<td>-1.068%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Sum (2 + 3 + 4)</td>
<td>1.283%</td>
<td>-0.578%</td>
<td>0.705%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Direct calculation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.704%</td>
</tr>
<tr>
<td>7</td>
<td>Error (5 - 6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.001%</td>
</tr>
</tbody>
</table>
Table 2
Example of Alternative Measures of Productivity Growth
Showing Strong Baumol Effect

Illustration of Baumol Effect

<table>
<thead>
<tr>
<th>Period</th>
<th>Industry 1</th>
<th>Industry 2</th>
<th>Sum</th>
<th>Fisher Index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real output (period 1 prices)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>100.00</td>
<td>100.00</td>
<td>200.00</td>
<td>200.00</td>
</tr>
<tr>
<td>2</td>
<td>105.00</td>
<td>100.00</td>
<td>205.00</td>
<td>204.57</td>
</tr>
<tr>
<td>Hours</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>100.00</td>
<td>100.00</td>
<td>200.00</td>
<td>200.00</td>
</tr>
<tr>
<td>2</td>
<td>85.00</td>
<td>115.00</td>
<td>200.00</td>
<td>200.00</td>
</tr>
</tbody>
</table>

Components of productivity growth

1 Welfare productivity growth 9.741% -7.494% 2.247%
2 Pure productivity effect (early weights) 10.565% -6.988% 3.577%
3 Baumol effect -0.825% -0.506% -1.330%
4 Denison effect 0.000% 0.000% 0.000%
5 Sum (2 + 3 + 4) 9.741% -7.494% 2.247%
6 Direct calculation 2.257%
7 Error (5 - 6) -0.010%
Figure 1

Source: Output_Create_062002.xls
### Table 3
**Alternative Measures of Labor Productivity for Overall Economy, 1978-98**

<table>
<thead>
<tr>
<th></th>
<th>(1) 1978-89</th>
<th>(2) 1989-95</th>
<th>(3) 1996-2000</th>
<th>(2) - (1)</th>
<th>(3) - (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal measure</td>
<td>1.44%</td>
<td>0.72%</td>
<td>2.37%</td>
<td>-0.72%</td>
<td>0.92%</td>
</tr>
<tr>
<td>GDI measure</td>
<td>1.20%</td>
<td>1.11%</td>
<td>1.79%</td>
<td>-0.09%</td>
<td>0.59%</td>
</tr>
</tbody>
</table>

Note: The “ideal measure” is an index of productivity growth that includes the fixed-weight and Baumol effects but excludes the Denison effect. The “GDI measure” is productivity growth calculated as the growth in output less the growth in inputs.

Source: Output_Create_062002.xls