The Agricultural Household Model

Start with conventional preferences over a consumption good and leisure

\[ u(c, l) \] (1)

add a budget constraint that incorporates production

\[ pc + wL^h + rA^h \leq f(L, A) + wL^m + rA^m \] (2)

where
\[ L = L^f + L^h \]  
(3)

\[ A = A^f + A^h \]  
(4)

\[ E^A = A^f + A^m \]  
(5)

\[ E^L = L^f + L^m \]  
(6)

and non-negativity constraints. Substitute (3)-(6) into (2) to get the full income constraint:

\[ pc + wl \leq \Pi + wE^L + rE^A \]  
(7)

where

\[ \Pi = f(L, A) - wL - rA. \]
So the problem becomes to max (1) subject to (7). Since we have local non-satiation, (7) binds at any optimum. Hence we can rewrite the problem as

$$\max_{c,l} u(c, l)$$

subject to

$$pc + wl \leq \Pi^* + wE^L + rE^A$$

where

$$\Pi^* = \max_{L,A} f(L, A) - wL - rA$$

Thus a utility maximizing household maximizes profits when markets are complete.

Now do some pictures.
What happens to output if preferences change? What happens if there is no market for labor (but still one for land)?

Figure 1:
Now mess up both markets. No land market. Constrained labor market:

\[
\max_{c, l, L^H, L^M} u(c, l) \tag{8}
\]

s.t.

\[
p_c = F(L^f + L^h, E^A) - wL^h + wL^m \tag{9}
\]

\[
l + L^f + L^m = E^L \tag{10}
\]

\[
L^m \leq M \tag{11}
\]

No change if (11) not binding.

But when it binds we have

\[
c = F(E^L - M - l, E^A) + wM
\]

and profit maximization no longer holds. In a picture:
Now generalize. These results have nothing to do with the static nature of the discussion so far. Let \( c_{st} \) be a vector of consumption goods in state \( s \) of period \( t \), and \( c \) be the concatenation of all those vectors. Same notation extends to other variables.

\[
\max_{c_{st}, l_{st}, L_{st}, A_{st}} u(c, l) \tag{12}
\]
s.t.

\[
\sum_{s \in S, t \in T} \left[ w_{st} E_{st} + \Pi_{st} - p_{st} c_{st} - w_{st} l_{st} \right] \geq 0 \quad (13)
\]

\[
\Pi_{st} = q_{st} F_{st}(L_{st}, A_{st}) - w_{st} L_{st} - r_{st} A_{st} \quad (14)
\]

and non-negativity and feasibility \((l_{st} \leq E_{st})\) constraints. Precisely the same recursivity lets you write

\[
\begin{aligned}
\max_{c_{st}, l_{st}} u(c, l) \\
\text{s.t.} \\
\sum_{s \in S, t \in T} \left[ w_{st} E_{st} + \Pi_{st}^* - p_{st} c_{st} - w_{st} l_{st} \right] \geq 0 \quad (16)
\end{aligned}
\]

where

\[
\Pi_{st}^* = \max_{L_{st}, A_{st}} q_{st} F_{st}(L_{st}, A_{st}) - w_{st} L_{st} - r_{st} A_{st} \quad (17)
\]
Works in more realistic context as well, which \( L_{st} = L_t \) & \( A_{st} = A_t \). This yields a new b.c.:

\[
\sum_t \left[ w_tE_t + \Pi_t - w_t l_t - \sum_s p_{st}c_{st} \right] \geq 0
\]

and

\[
\Pi_t = \sum_s q_{st}F_{st}(L_t, A_t) - w_t L_t - r_tA_t.
\]