1. Credit and the Separation Property:

A household has initial liquid wealth \( y_0 \), and landholdings \( A_0 \). Suppose there is no quality variation in land, no land market, but that labor markets work smoothly. But production takes time, so labor and land are combined in period 0 to produce output in period 1. Consumption takes place in both periods, but there is no work to do in the second period (so you can write utility as \( u(c_0, l_0, c_1) \)).

(a) Suppose that credit markets work smoothly and the production function \( q_1 = f(L_0, A_0) \) is crts. Show that yield is independent of landholdings.

(b) Show that with imperfect credit markets, yield now may depend on landholdings. There are several ways you could proceed – no way of transferring resources across periods at all, borrowing constraints, transaction costs. Choose whatever seems to make the point most directly.

2. Risk Sharing in Households (thanks to P-A Chiappori).

A household has two members. Each member \( i \) receives an exogenous random income \( y_i \) from a distribution \( f_i \). There is only one commodity, which is privately consumed by each. Let \( \rho(y_1, y_2) \) be member 1’s consumption as a function of the realization of each income; 2 obviously gets \( y_1 + y_2 - \rho \). Chiappori calls \( \rho \) the ‘sharing rule’. Each member has a VNM utility function and is strictly risk averse. Suppose that the household is efficient: risk is fully pooled in the household.

(a) Show that there is a function \( \tilde{\rho} \) such that \( \rho(y_1, y_2) = \tilde{\rho}(y_1 + y_2) \). To be clear: show that the optimal consumption of 1 can depend only on aggregate \( y_1 + y_2 \).
(b) Suppose that each member’s utility is CARA with risk aversion parameters $\sigma_1$ and $\sigma_2$. Show that $\bar{\rho}(y_1 + y_2)$ is linear. Calculate its slope as a function of $\sigma_1$ and $\sigma_2$. How does the Pareto weight of individual 1 affect $\bar{\rho}$?

3. Gender and Agricultural Productivity

We’ll use the Ghana data collected by Goldstein and Udry (http://www.econ.yale.edu/~cru2//ghanadata). The relevant file can be downloaded from the class website. This is plot level data for all plots that were cultivated with a standard maize-cassava intercropped mixture. Plots are uniquely identified by the combination vill-hhn-id-plot-year.

(a) First replicate the findings of Udry (1996) for this context (the closest analogue for this data would be Table 3, columns 2 or 3). Show that within households in which both men and women cultivate maize/cassava, women achieve lower yields conditional on observable characteristics of the plot. Provide some context from the data, what’s the magnitude of this difference?

(b) A plausible explanation for this finding is that there are unobserved differences between the plots of men and women. Some of these differences might be highly spatially-correlated at the scale of a few hundred meters. This suggests a spatial fixed effects approach. For each plot, define a neighborhood of an appropriate scale (using Xcoord and Ycoord), and difference out a mean spatial effect (exactly as you would for a standard household fixed effect, for example). Don’t forget that household-year effects must remain in the model. How does this influence the estimated gender effect on yield?