Preliminary Results on the Determinants of the Variability of Stock Market Prices

by

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It is a fact that stock market prices vary. Recent work by Robert Shiller, and Stephen LeRoy and Richard Porter, has shown that the variability of stock prices cannot be accounted for by the variability of discounted dividends. These studies assume a constant discount factor. In this paper, we show that the variability of stock prices can, to a large degree, be attributed to the variability of the discount factors (i.e., the real interest rate).

The appropriate discount factor to be applied to dividends which are received k years from today is the marginal rate of substitution between consumption today and consumption k periods from today. We use historical data on per capita consumption from 1890 - 1979 to estimate the realized value of these marginal rates of substitution.

Robert Hall also studied these marginal rates of substitution and concluded that consumption is a random walk. We show that if current consumption is the best predictor of future consumption in Hall's sense, then the discount factor applied to stock prices would not vary. The variability of stock prices implies they do vary, so that we conclude that consumers must have a better method for forecasting future consumption than using only current consumption (e.g., consumers may know when the economy is in a recession).

I. Perfect Foresight Generated Discount Factors

Consider a consumer who can buy or sell asset i at time t with no trans-
action costs. A necessary condition for his holdings of the asset at \( t \) to be optimal, given that the consumer maximizes the expected sum of discounted utility is:

\[
(1) \quad u'(C_t) P_{it} = \beta E[u'(C_{t+1}) (P_{it+1} + D_{it+1}) | I_t],
\]

where \( P_{it} \) is the real price (in terms of the single consumption good \( C_t \)) of asset \( i \) and \( D_{it+1} \) is the real dividend paid at \( t+1 \) to holders of record at \( t \). \( I_t \) is the information about everything in the future which the agent possesses at time \( t \). The left-hand side of (1) is the cost in terms of foregone current consumption of buying a unit of the asset, while the right-hand side of (1) gives the expected future consumption benefit derived from the dividend and capital value of the asset.

By iterating (1), we find that

\[
(2) \quad P_{it} = E_t \sum_{j=1}^{n} \beta^j \frac{u'(C_{t+j})}{u'(C_t)} D_{it+j} + E_t \beta^n \frac{u'(C_{t+n})}{u'(C_t)} P_{in}.
\]

It is useful to define the perfect foresight stock price \( P_{it}^* \), which is the price at \( t \) given that the consumer knows the whole future time path of consumption, dividends and the terminal price \( P_{in}^* \):

\[
(3) \quad P_{it}^* = \sum_{j=1}^{n} \beta^j \frac{u'(C_{t+j})}{u'(C_t)} D_{it+j} + \beta^n \frac{u'(C_{t+n})}{u'(C_t)} P_{it+n}.
\]

Clearly (2) states that \( P_{it} = E[P_{it}^* | I_t] \). Note that (2) is true for every consumer, where the consumer's consumption and utility function are used. In this paper, we will assume that it is true for per capita real consumption in the U.S. Further, we assume the \( u(C) \) is of the constant relative risk aversion form

\[
(4) \quad u(C) = \frac{1}{1-A} C^{1-A},
\]
where $A$ is the coefficient of relative risk aversion.

Note that equation (2) holds for portfolio's of assets, so that it holds for Standard and Poor Monthly Composite Stock Price Index. Figure 1 shows a plot of $P_t$ from 1889-1979, where $P_t$ is the average real price of this Market Index in year $t$, denoted by $P_t^*$. On the same Figure, we plot the perfect foresight real price $P_t^*$ using (3), where we use actual realized real annual dividends, per capita consumption, and the terminal condition $P_n$ = actual real annual stock price in 1979 (i.e., at each time $t$ we use the data from $t$ to 1979). In order to plot $P_t^*$ we must make some assumption about $\beta$, $A$ (which we later estimate). Figure 1 contains plots for $A = 0, 2, 4, 6$. For each $A$, we generate a value

$$\beta_1 = \mathbb{E} \left( \frac{C_t}{C_{t+1}} \right)^A \frac{P_{t+1} + D_{t+1}}{P_t}$$

which is the implied value for $\beta$ from (1). We estimate the expectation by taking the sample mean. The case $A = 0$ is revealing; this is the case of risk neutrality, and of a constant discount factor. Notice that with a constant discount factor, $P_t^*$ just grows with the trend in dividends; it shows none of the short term variation of actual stock prices. As $A$ gets larger, variations in the rate of growth of consumption have an important effect on short-term stock price movements.

Intuitively, if there is an increase in wealth at $t$, celeris paribus, everyone will try to spread that consumption over time by trying to save more. This will drive up the price of assets at time $t$. Note that if there is a permanent increase in consumption at $t$, there is no such effect because there is no need to save more to get more consumption at $t+1$, $t+2$,... To put this more precisely, if the rate of growth in consumption is small, then using (4)

$$\frac{u'(C_{t+1})}{u'(C_t)} = 1 - A \frac{\Delta C_{t+1}}{C_t} \equiv 1 - A \dot{C}_{t+1}$$
Using this, (3) can be written

\[ p_{t+n} = \sum_{j=1}^{n} \left( \Pi_{i=1}^{j} 6(1 - A C_{t+i+1}) \right) D_{t+j} + \beta^n \frac{u'(C_{t+n})}{u'(C_t)} p_{t+n} \, . \]

The right-hand side of (6) shows the effect of changes in consumption on the perfect foresight prices \( p^*_t \). The larger \( A \) is, the larger is the effect of temporary consumption changes on \( p^*_t \).

There need not be very much relationship between \( p_t \) and \( p^*_t \). If consumers have no information about \( p^*_t \), then \( p_t \) will be a constant and \( p^*_t \) will vary. Since \( p_t = E[p^*_t | I_t] \),

\[ p^*_t = p_t + (E[p^*_t | I_t] - p_t) + p^*_t - E[p^*_t | I_t] = p_t + (p^*_t - E[p^*_t | I_t]) \, . \]

Since \( p_t \) is in the information set \( I_t \), (7) implies that \( p^*_t = p_t \) + uncorrelated noise, so that the variance of \( p^*_t \) will be larger than the variance of \( p_t \). The difference in the variability of \( p^*_t \) and \( p_t \) depends on how well \( I_t \) forecasts \( p^*_t \).

The fact that for \( A > 2 \), \( p_t \) and \( p^*_t \) tend to vary together is strong evidence in favor of the fact that short-term movements in consumption are forecastable by consumers. In particular, Robert Hall has found that in the post World War II period consumption is a random walk with drift. If \( \text{Var}(C_{t+1} | C_t) \) is small, then this is not very different from assuming that the logarithm of consumption is a random walk

\[ \log C_{t+1} = \gamma' + \log C_t + \varepsilon_{t+1} \, , \]

where \( E[\varepsilon_t | C_t] = 0 \), and \( E[\varepsilon_{t+1} | \varepsilon_{t-j}] = 0 \).

Alternatively \( C_{t+1}/C_t = \gamma v_{t+1} \), where \( E[v_{t+1} | C_t] = 1 \), and \( E[v_{t+1} | v_{t-j}] = 1 \). We can use this fact to get some insight about consumer information, if we also make the assumption that the log of dividends follows a random walk with
drift, i.e. $D_{t+1} = D_t \delta Z_{t+1}$ where $\delta$ is one plus the dividend growth rate and $Z_t$ is i.i.d. noise with mean 1. Using (2), (4), setting $n = \infty$ to ignore tails, and setting $I_t = \{C_t, D_t\}$, we find

\begin{align*}
P_t &= \sum_{j=1}^{\infty} \beta^j E \left[ \left( \frac{u'(C_{t+1})}{u'(C_t)} \frac{u'(C_{t+2})}{u'(C_{t+1})} \cdots \right) D_{t+j} \mid C_t, D_t \right] \\
P_t &= \sum_{j=1}^{\infty} \beta^j D_t E \left[ \frac{u'(C_{t+1})}{u'(C_t)} \delta Z_{t+1} \right] \times E \left[ \frac{u'(C_{t+2})}{u'(C_{t+1})} \delta Z_{t+2} \mid C_{t+1}, D_{t+1} \right] \\
&\quad \times \cdots E \left[ \frac{u'(C_{t+j})}{u'(C_{t+j-1})} \delta Z_{t+j} \mid C_{t+j}, D_{t+j} \right] \mid C_t, D_t \\
(9) \quad P_t &= D_t \sum_{j=1}^{\infty} \left( \beta \delta y^{A} e^{\gamma - \lambda} \right)^j = \frac{\beta}{1-\beta} D_t,
\end{align*}

where $\bar{\beta} = \beta \delta y^{A} e^{\gamma - \lambda} \bar{\delta}$ and we assume $\bar{\beta} < 1$ (to get a finite stock price!).

Equation (9) is a reductio ad absurdum of the idea that consumers only observe current consumption and dividends. For, if it were true, then stock market prices would just be a linear function of dividends, and the variability of stock prices would be completely explained by the variability of dividends. Since (9) is inconsistent with the data, we conclude that at time $t$ consumers know something about the part of $\log C_{t+1}$ which is not forecastable from $\log C_t$. That is, consumers know something about whether a rise in $C_t$ is permanent or temporary at time $t$.

This result does not contradict Robert Hall's observations that (1) to an econometrician who does not know as much as consumers, consumption is a random walk and (2) that income may be a proxy for lagged consumption in econometric models which have shown that consumption is very sensitive to income. The fact that stock prices vary so much with consumption is evidence that the perfect
foresight model \( P_t = P^*_t \) is a better model than \( P_t = E[P^*_t | C_t, D_t] \).

There are some other interesting observations about Figure 1. The model with \( A > 2 \) seems to work very well except for the period 1954 to 1970. There was an enormous rise in stock prices in that period which cannot be explained by changes in realized dividends or in marginal rates of substitution. Preliminary results show that it cannot be explained by taxes. (Irwin Friend and Marshall Blume noticed an extremely high excess return of stocks over bonds in this period relative to all other subperiods from 1890 - 1980. Their estimated market price of risk was twice as high in the decade 1952 - 1961, than the highest of any other decade.) We don't have a very good idea as to why this great price rise occurred. One possibility is that if savings occurs by families in order to make transfers to children, then a rise in the birth rate (the post-war baby boom) would be associated with a rise in desired savings. The rise in desired savings would raise stock prices. Eventually as the children grow up, dissavings would occur and the price of assets would return to historical levels relative to dividends and marginal rates of substitutions. We have not tested this idea. However, it is true that in the 1970's, real stock prices did return to the perfect foresight path \( P^*_t \). In Figure 1, \( P^*_{1979} \) is constrained to equal \( P_{1979} \), so the previous statement cannot be inferred from Figure 1. However, we have computed a terminal condition for \( P^*_t \) by assuming that dividends stay on their historical growth path.\(^2\) When this is done, \( P^*_t \) and \( P_t \) come together again in the 1970's; see Figure 2.

II. Estimation of \( \beta, A \); How Risky are Stocks?

Equation (1) can be written as

\[
\beta^{-1} = E_t \frac{u'(C_{t+1})}{u'(C_t)} R_{t+1}
\]
where \( R_{it+1}^e \) is one plus the (real) rate of return on asset \( i \). If we define \( R_{t+1}^e \) to be the excess return of one asset over another, then (10) implies that

\[
E_t \frac{R_{t+1}^e}{R_{t+1}} = - \text{cov}_t \left( \frac{u'(C_{t+1})}{u'(C_t)}, R_{t+1}^e \right) \cdot \frac{1}{E_t \frac{u'(C_{t+1})}{u'(C_t)}}
\]

Equation (11) states that the excess return of one asset over another depends on how much more correlated one asset is with the rate of growth in the marginal utility of consumption. That is, the appropriate measure of the riskiness of an asset is that an asset is very risky if its payoff is very negatively correlated with the marginal utility of consumption. (Douglas Breeden has recently persuasively argued for the use of consumption correlatedness as the appropriate measure of risk.) That is, if the asset has a low payoff when consumption is low, then it is not very helpful in smoothing the flow of consumption. This can be seen clearly if we use the approximation in (5) to rewrite (11) as

\[
\frac{E R_{t+1}^e}{\sqrt{\text{var } R_{t+1}^e}} = A \rho_{CRe} \frac{\sqrt{\text{var } C}}{1 - AE_c}
\]

where \( \rho_{CRe} \) is the correlation coefficient between the rate of growth in consumption and excess returns. Note that when everyone is risk neutral (i.e. \( A = 0 \)), all assets have the same mean returns. Thus, the excess return of assets can be used to measure the degree of risk aversion of assets.

Though this formula can be used to estimate \( A \), we will use another formula which avoids the approximation in (5). In particular we use the assumption of log-normality of real returns and consumption to derive estimates of \( A \). In particular, we use our second asset 4-6 month prime commercial paper (which we call bonds) and compute the compounded annual real return for each year in our sample. Let \( R_{1t} \equiv \) one plus the real return on stocks, \( R_{2t} \equiv \) one plus the real return on bonds, \( X_t \equiv \) real per capita consumption at \( t \) divided by real per capita consumption at...
t+1. We assume that \((\log X_t, \log R_{1t}, \log R_{2t})\) is a Normal vector which is i.i.d. over the sample period.

To see how this assumption is used, first note that (irrespective of the assumption), equation (10) holds unconditionally for each asset (just take the unconditional expectation of both sides). Next use (4) and (10) to get

\[
(13) \quad \text{EX}_{t+1t}^{R_1} = \text{EX}_{t+2t}^{R_2}.
\]

Note that if the log of a random variable, say \(\log Y\), is Normal, then \(\text{E}Y = \text{Ee}^{\log Y} = \exp(\text{E} \log Y + \frac{1}{2} \text{Var} \log Y)\). Using this fact

\[
(14) \quad \text{EX}_{t+1t}^{R_1} = \exp\{\text{E}(\log X_t + \log R_{1t}) + \frac{1}{2} \text{Var}(\log X_t + \log R_{1t})\}.
\]

This can be used in (13) to solve for \(A:\)

\[
(15) \quad A = \frac{\log \text{ER}_1 - \log \text{ER}_2}{\text{Cov}(\log X_t, \log R_2) - \text{Cov}(\log X, \log R_1)}.
\]

The maximum likelihood estimator of \(A\) can be found from (15) by substituting the maximum likelihood estimators of \(\text{ER}_1\) and \(\text{cov}(X,R_1)\) into (14). Similarly, the maximum likelihood estimator of \(\beta\) can be found using the fact that \(\beta^{-1} = \text{EX}_{t+1t}^{R_1}\) to equal

\[
(16) \quad \beta = \exp\{-\text{E}(\log X + \log R_1) + \frac{1}{2} \text{Var}(\log X + \log R_1)\}.
\]

Table 1 presents estimates of the coefficients and approximate standard errors.\(^3\) Assuming (correctly) that (real) stock returns and (real) bond returns are uncorrelated, then the approximation in (12) can be written

\[
\sqrt{\frac{\text{Var} R_1}{\text{Var} R_2}} \frac{\text{ER}_1}{\text{Var} R_1} - \sqrt{\frac{\text{Var} R_2}{\text{Var} R_2}} \frac{\text{ER}_2}{\text{Var} R_2} = A(\rho_{\text{CR}1} - \rho_{\text{CR}2}),
\]

where \(R_e = R_1 - R_2\) and \(\text{Cov}(R_1,R_2) = 0\).
Suppose that $\rho^{*}_{CR1} - \rho^{*}_{CR2}$ is small, then the excess (variance) corrected mean return on stocks can be much higher than bonds only if people are very risk averse (i.e., $A$ is large). Thus, in Table 1, $A$ is quite large when stocks do very well relative to bonds. In the most recent subperiod, bonds and stocks are almost equally risky, yet stocks have an enormously higher mean rate of return. For the whole sample, stocks averaged a 7% real return while bonds only yielded 1.8%. This occurs even though stocks are not that risky (returns having only a .38 correlation coefficient with consumption growth). It seems as if the consumers need an enormous premium to take the slightest risks.

Note that the marginal rate of substitution of current for future consumption is larger than 1 for some subperiods. This is a result of real bond returns being so low. From 1949-1978, bonds were risky yet had virtually no real return. At the margin, this meant that agents had a strong preference for future as opposed to current consumption!!

Our estimates of $A$ are far, far higher than those of Friend and Blume (they get about $A = 2$). For that estimate, they use (long-term and short-term) bonds and stocks as a proxy for wealth, which is in turn a proxy for consumption. Stocks are much more highly correlated with their measure of market wealth than they are with consumption. Hence this makes them seem more risky to the econometrician. Hence a lower $A$ is required to justify the high average excess return.

There are at least four problems with the model and estimation technique, some of which we are attempting to correct. First we only have average annual data on consumption going back to 1890. Yet it is the correlation between instantaneous consumption growth and returns which determines the riskiness of assets. The use of annual average data can severely bias downward our estimates of the correlation. We are trying to estimate this bias. To check on the temporal aggregation problem, we used January stock price data for $P_t$ (rather than the average stock
price in year t) to compute stock returns. This leads to a higher correlation between consumption and returns than that of Table 1. It leads to lower estimates of A and β.

Another problem with the model is that we have assumed all individuals are identical. This ignores intergenerational transfers as a motive for savings as well as the effects of demographic changes on equilibrium excess returns. A third problem is that we have ignored taxes. Taxes are not very difficult to incorporate and preliminary results indicate that estimates of A are not significantly changed. Finally, we have used an ex post version of the model. To the extent that the conditional distribution of returns given information at time t varies a lot, our sample will be too small to get very good estimates of the joint distribution of returns and consumption. The standard errors presented for A and β reflect the variability of mean returns, but not the variability of the correlation between returns and consumption.

Subject to the above remarks, it seems clear that consumers are not risk neutral with respect to stock returns. To the extent that they have good and variable information about their future consumption, stock prices can vary a great deal even though dividends do not vary very much. This is because large values of risk aversion amplify the effect of changes in the rate of growth in consumption on stock prices.
The real and nominal consumption series starting in 1929 are the annual average personal consumption expenditure on nondurable goods and services series from the National Income and Product Accounts of the United States 1929-74, updated by the Survey of Current Business. The real and nominal consumption series for 1889 and 1928 are Kuznets [1961] flow of goods to consumers (perishables, semi-durables and services), variant III, adjusted to correspond to Commerce Department accounting practices as described by Kendrick [1961] and multiplied by the ratio of the Commerce Department series to the Kuznets-Kendrick series for the year 1929. The resulting series is divided by the population of the United States to arrive at per capita series. The price index used to deflate the stock price series and convert nominal interest rates into ex post real interest rates is the consumption deflator arrived at by dividing the nominal consumption series by the real consumption series, hence the base year is 1972.

The nominal stock price series is the Standard & Poor Monthly Composite Stock Price Index (Standard & Poor, 1978). The annual average is plotted in figure 1, but for return calculations in Table 1, figures are for January of the year. The Standard & Poor stock price index is a continuation of the Cowles Commission Common Stock Price Index (Cowles, [1938]). The dividend series from 1926 is "Dividends per share ... 12 month moving total adjusted to index" from Standard & Poor [1978]. For 1889 to 1925 total dividends are Cowles [1938] series Dα-1 multiplied by .1264 to correct for change in base year.

The short-term interest rate is the return from investing for six months at the 4-6 month prime commercial paper rate in January and reinvesting for six months at the 4-6 month prime commercial paper rate for July. The prime
commercial paper rate for 1890-1924 was from the Board of Governors of the Federal Reserve System ([1943], Table 120) and for 1925-70 from the Federal Reserve Bulletin.
Perfect Foresight Prices $P^*_t$, with $A=0$, are plotted with dashed line.

Perfect Foresight Prices $P^*_t$, with $A=2$, are plotted with dot-dashed line.

Actual Prices $p_t$ are plotted with solid line.

Plots use the actual stock price in 1979 as terminal value.
Perfect Foresight Prices $P_t^*$, with $A=0$, are plotted with dashed line.

Perfect Foresight Prices $P_t^*$, with $A=4$, are plotted with dotted line.

Actual Prices $P_t$ are plotted with solid line.

Figure 1.2 plots the actual stock price in 1979 as terminal value.
Perfect Foresight Prices $P^*_t$, with $A=0$, are plotted with dashed line.

Perfect Foresight Prices $P^*_t$, with $A=6$, are plotted with spaced line.

Actual Prices $P_t$ are plotted with solid line.

Figure 1.3 plots use the actual stock price in 1979 as terminal value.
Perfect foresight prices P_t, with A=0, are plotted with dashed line.
Actual prices P_t are plotted with solid line.

Forecast P_t = 65
Terminal condition derived from Perfect Forecasts Dividend

Figure 2.1
Perfect Foresight Prices $p_t$, with $A_t$, are plotted with dashed line.

Actual Prices $p_t$ are plotted with solid line.

Terminal Condition derived from Perfect Foresight Dividend

$\frac{1979}{2.2} = 65$
Perfect foresight prices \( P_t \) with \( A=0 \) are plotted with dashed line. Actual prices \( P_t \) are plotted with solid line.

Terminal Condition: Derived from Perfect Foresight Dividend

\[ \text{Forecast of} \quad P_{1979} = 65 \]
Table 1

<table>
<thead>
<tr>
<th>Subperiod</th>
<th>Average Annual Returns on Stocks</th>
<th>Bonds</th>
<th>Correlation Coefficient between consumption rate (i.e., $\hat{C}$) and Stock returns</th>
<th>Bond returns</th>
<th>MRS</th>
<th>Estimates for A, using approximations</th>
<th>Estimates for A, using approximations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1890–1918</td>
<td>1.051 (.026)</td>
<td>1.032 (.011)</td>
<td>.479</td>
<td>.091</td>
<td>.962</td>
<td>7.40 (9.28)</td>
<td>1.053 (1.93)</td>
</tr>
<tr>
<td>1919–1948</td>
<td>1.082 (.038)</td>
<td>1.016 (.014)</td>
<td>.403</td>
<td>-.024</td>
<td>.992</td>
<td>13.55 (8.45)</td>
<td>1.022 (.237)</td>
</tr>
<tr>
<td>1949–1978</td>
<td>1.077 (.026)</td>
<td>1.006 (.004)</td>
<td>.169</td>
<td>.180</td>
<td>31.8</td>
<td>259.55 (90.5)</td>
<td>21.24 (1.35)</td>
</tr>
<tr>
<td>1890–1933</td>
<td>1.071 (.027)</td>
<td>1.044 (.010)</td>
<td>.381</td>
<td>-.051</td>
<td>.96</td>
<td>7.43 (7.63)</td>
<td>1.000 (.201)</td>
</tr>
<tr>
<td>1934–1978</td>
<td>1.069 (.023)</td>
<td>0.993 (.005)</td>
<td>.451</td>
<td>.004</td>
<td>2.78</td>
<td>64.1 (18.2)</td>
<td>2.13 (.80)</td>
</tr>
<tr>
<td>1890–1978</td>
<td>1.070 (.017)</td>
<td>1.018 (.006)</td>
<td>.374</td>
<td>-.083</td>
<td>1.05</td>
<td>20.83 (7.13)</td>
<td>1.09 (.226)</td>
</tr>
</tbody>
</table>

Average returns are arithmetic means of real returns, the numbers in parenthesis are the standard errors of the estimate in the above row (e.g. for 1890–1918, .026 is the std. err. of the mean of stock returns). MRS is the average marginal rate of substitution between future and present consumption

$$
MRS = EB \frac{u'(C_{t+1})}{u'(C_t)} \approx \beta (1 - \hat{\Delta C} + \frac{(\hat{\Delta} + 1) \hat{\Delta} \hat{\Delta}^2}{2})
$$

where $\hat{\Delta} C$ is the real rate of growth in consumption. The standard errors for $A$ and $\beta$ are estimated using the method in footnote 3. The estimates using the approximate model involve using $(u'(C_{t+1}))/u'(C_t) = 1-\Delta \hat{C}$ as described in footnote 3.
Footnotes

1/ In our sample period, we find

\[
\log C_t = \frac{.017 + .900 \log C_{t-1} + .275 \log C_{t-2} - .216 \log C_{t-3}}{(2.65)} + .034 \log C_{t-4}
\]

\[
C.W. = .9926, \, R^2 = .9926, \, S.E. = .035, \, \text{Annual observations from 1889-1979.}
\]

The same regression in levels yields

\[
C_t = -.011 + 1.036 C_{t-1} + .082 C_{t-2} - .122 C_{t-3} + .027 C_{t-4}
\]

\[
D.W. = 2.01, \, R^2 = .9955, \, S.E. = .049, \, N = 87
\]

where the numbers in parenthesis are t statistics.

- Since consumption is an average annual figure, there are serious problems in determining whether quarterly consumption (which Hall used) is a random walk. The standard error of the log \( C_{t-1} \) coefficient is .11, so that 1.00 is less than one standard error away from .900.

2/ The terminal condition was computed using equations (8) and (9). In particular, since we are looking for the perfect foresight terminal condition, we set \( z_t = v_t = 1 \), so \( \bar{\beta} = \beta \delta \gamma^{-A} \). The average real growth rate of dividends in our subperiod was 1.99%. Note that \( \gamma^{-A} \equiv 1 - A \hat{C} \), where in our subperiod the annual rate of growth in consumption was 1.83%. If we use the values of \( A \) and \( \beta \) estimated in the next section for the whole sample period, we find \( \beta > 1 \). As we explain in that section, the model doesn't work very well for the second half of the sample. If we use the estimated value of \( \beta, A \) for the first half of the data 1889-1954, then we get \( P\hat{t}_{1979} = 65 \). That is, we get a dividend price ratio of 5.26%.

Note that \( \beta \equiv 1 - (1 - \bar{\beta}) \) is very sensitive to the estimates of \( A, \beta, \delta, \gamma \) used. We have not yet computed an approximate standard error for \( P\hat{t}_{1979} \). We have not yet corrected for taxes.

3/ Using the approximation in (5), we get \( 0 = E(1-A\hat{C})R_e \) so \( A = E\hat{R} / E\hat{C}R_e \). If we define the estimator \( \hat{A} \) using the sample moments for \( E\hat{R} \) and \( E\hat{C}R_e \), then an approximate standard error of \( \hat{A} = (1/\hat{C}R_e) \text{stderr} (\hat{R}_e) = (1/\hat{C}R_e) (\text{std err}(\hat{R}_e))^{1/2} \), where \( \hat{R}_e \) refers to the sample mean excess return. To get a standard error for \( \beta \), note that using the same approximation \( \beta^{-1} = E(1-A\hat{C})R_1 \). So we can estimate \( \beta^{-1} \) by \( \hat{\beta}^{-1} = \hat{R}_1 - \hat{A}\hat{C}R_1 \). An approximate standard error for \( \hat{\beta}^{-1} \) can be gotten by treating \( \hat{C}R_1 \) as exactly \( E\hat{C}R_1 \). So the \( \text{Var}(\hat{\beta}^{-1}) = \text{Var}(\hat{R}_1) + (\hat{C}R_1)^2 \times \text{Var}(\hat{A}) - 2 \hat{C}R_1 \text{Cov}(\hat{A}, \hat{R}_1) \). We then use the result that if \( \hat{a} \equiv f(\hat{b}) \), then the asymptotic standard error of \( \hat{a} = |f'(\hat{b})| \text{stderr} \hat{b} \). So the stderr \( \hat{\beta} = 1/(\hat{\beta}^{-1})^2 \times \text{stderr}(\hat{\beta}^{-1}) \).