Problem Set 4 - SOLUTIONS

Econ 115a

1. Definitions
   
   (a) Iso-Quant: A curve that shows all possible combinations of inputs (such as labor and capital, for example) that can be used to produce the same quantity of output.
   
   (b) Average Variable Cost: Per-unit variable costs, where variable costs total costs minus fixed costs.
   
   (c) Sunk Costs: All costs that cannot be recovered. In the case of a firm, these costs cannot be recovered even if the firm decides to exit the market so they have no influence on the production or exit decisions of the firm. Not to be confused with fixed costs, which do not influence the firm’s production decision but can influence its exit decision.
   
   (d) Increasing Returns to Scale: Term used to describe the downward sloping part of the average cost curve. If there are increasing returns to scale, then it is possible to double the use of inputs and more than double output.

2. Widgets-R-Us
   
   (a) When we plot $K$ on the $y$-axis and $L$ on the $x$-axis, the general formula for the slope of the iso-quant is

   $$\frac{MP_L}{MP_K}$$

   To see why this is so, we can ask how much less capital ($\Delta K$) we need if one extra unit of labor ($\Delta L = 1$) is used and output is required to remain unchanged. The additional output from the extra unit of labor is $MP_L$. Thus we would like the loss in output from the change in capital stock to equal $MP_L$: i.e., $\Delta K * MP_K = MP_L$. Rewriting this expression, we get $\Delta K = MP_L / MP_K$. The slope of the iso-quant is simply $\Delta K / \Delta L$, which is just $\Delta K$ since we set $\Delta L = 1$. Substituting the given expressions for $MP_L$ and $MP_K$, we get that the slope is

   $$\frac{MP_L}{MP_K} = \frac{1}{2} \frac{\sqrt{K}}{\sqrt{L}} = \frac{K}{L}.$$ 

   (b) See graph on last page.
   
   (c) The cost-minimizing input choices of $L$ and $K$ are at the point where the $\overline{Q}$ iso-quant is tangent to the iso-cost line. This point is found by setting the slope of the iso-quant equal to the slope of the iso-cost line

   $$\frac{K}{L} = \frac{w}{r}. $$

   Now, since we are on the $\overline{Q}$ iso-quant, we know the relationship between $K$ and $L$ is $\sqrt{KL} = \overline{Q}$, or alternatively $KL = \overline{Q}^2$. Using this, we can substitute for $K$ in the above expression and get

   $$\overline{Q}^2 \frac{L^2}{L} = \frac{w}{r}.$$ 

   Solving for $L$, we get that the cost-minimizing choice of labor is

   $$L = \overline{Q} \sqrt{\frac{r}{w}}.$$ 

   To find the cost-minimizing choice of capital, simply substitute the above result into the expression for $K$ in (a):

   $$K = \overline{Q} \sqrt{\frac{w}{r}}.$$ 

   Note that as the wage rate $w$ increases, the cost-minimizing choice of labor decreases and capital increases, and vice versa as the rental rate $r$ increases.
(d) The cost function must specify the minimum cost of producing a given level of output $Q$. Since we have calculated the cost-minimizing input choices of $L$ and $K$ when output is $Q$, the cost function is simply the sum of the cost of each input choice:

$$C(Q) = wQ \sqrt{\frac{r}{w}} + rQ \sqrt{\frac{w}{r}} = 2Q \sqrt{rw}.$$ 

Note that the cost function is linear and upward-sloping: as $Q$ increases, so does the total production cost.

3. $C(Q) = 10 + 2Q^2$

(a) Fixed cost is the part of the cost function that does not vary with the level of output. So fixed cost is equal to 10. Note that even when $Q$ equals zero, costs are equal to 10.

(b) If price is 8, we know that the perfectly competitive firm will maximize its profits by producing an output level where price equals marginal cost. Since marginal cost is $4Q$, the output level chosen by the firm is given by $8 = 4Q$, i.e., output is 2. Average cost is thus the total cost of producing 2 units divided by 2:

$$Average\ Cost = \frac{10 + 2 \cdot 2^2}{2} = 9.$$ 

(c) What are the firm’s total profits when it produces 2 units? Since total revenues (price times output level) are 16 and total costs are 18, the firm’s total profit is -2 (loss). Whether the firm can recover its fixed cost upon exiting will be relevant to the stay/exit decision:

i. If all the fixed costs are not sunk (i.e. they can be recovered), then the firm should exit the industry. By staying it makes a loss of -2; but by exiting it can recover its fixed cost and avoid making any loss.

ii. If all the fixed costs are sunk, the firm cannot recover them and so it should not consider it in the stay/exit decision. By producing 2 units, the firm earns a revenue (18) that exceeds its non-sunk costs (8), so it should stay.

4. We know that each firm’s marginal cost function is also its supply function in a perfectly competitive market. At every given price $p$, we know that each firm’s supply will be given by

$$p = 5q.$$ 

Rewriting with $q$ on the left-hand side, we get that each firm will supply $q = p/5$. At every price, the industry supply is simply the sum of all $N$ firms’ supply. Thus, industry supply as a function of price is $q = N(p/5)$. Alternatively this industry supply function can be written as

$$p = \frac{5}{N} q.$$ 

Note that the industry supply function has the same intercept (zero) as the individual firm’s supply function, but has a smaller slope (i.e. it is flatter)—the magnitude of which depends on how many firms are in the industry. The industry supply function is flatter as industry size ($N$) increases.
2 (b) Graph of cost-minimizing input choices when prices are \( r \) and \( w \), output is \( Q' \).

The optimal input choices are found by setting the slope of the iso-cost line equal to the slope of the iso-quant (i.e., the point where the line is tangent to the curve).

To produce \( Q' \) units of output, \( K^* \) and \( L^* \) are the cost-minimizing input choices. On all other iso-cost lines, either (a) it will not be feasible to produce \( Q' \) units, or (b) \( Q' \) units will be produced for a cost higher than \( C \).