Problem Set 3 Solutions

1. a) Diversification: holding a variety of assets with different characteristics (e.g., different rates of return) to reduce the risk of a portfolio

b) Marginal utility: the additional utility (or happiness) a consumer gets from consuming the last unit of a good

c) Indifference curve: on a graph plotting quantities of two goods, a curve showing all combinations of the two goods that yield the same utility

d) Substitution effect: the reallocation of consumption away from a good that becomes relatively more expensive and towards a good that becomes relatively cheaper, leaving the consumer at the point on the original indifference curve where the slope is equal to the new price ratio; one portion of a consumer’s response to a change in the price of a good

2) a) To determine the present value of the stream of gifts, we need to total the present values of each separate lump sum. Each lump sum’s present value is calculated according to the formula \( \frac{Y}{(1+r)^t} \) where \( Y \) is the amount of money to be received, \( t \) is the number of years in the future when it will be received, and \( r \) is the interest rate. The $2000 is received now, so it won’t be discounted at all. (Or if you prefer you can think of this sum as being discounted according to the formula, with \( t=0 \).) The $4000 will be received one year in the future, and so on. Each of the other amounts is discounted and totaled, yielding:

\[
\text{Present Value} = \frac{2000}{1+r} + \frac{4000}{(1+r)^1} + \frac{6000}{(1+r)^2} + \frac{8000}{(1+r)^3}
\]

b) Although Sarah receives her uncle’s gifts in fixed amounts over the course of four years, she can allocate her consumption differently, since in the earlier years she can borrow (at an interest rate of \( r \)) against her future income stream. Her budget is constrained because her only income source is the uncle’s money. The present discounted value of her total consumption over the next four years must equal the present discounted value of the income. This can be expressed with the following equation, where consumption in a given year \( t \) is denoted \( C_t \) (rather than \( Y_t \), to avoid confusion with income), and \( PV_Y \) is the present value of her income stream (the answer from part a):

\[
PV_Y = C_0 + \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3}
\]

To put it differently, the difference between her discounted future income stream and her discounted future consumption should be zero:

\[
0 = \left( \frac{2000-C_0}{1+r} \right) + \left( \frac{4000-C_1}{1+r} \right) + \left( \frac{6000-C_2}{1+r} \right) + \left( \frac{8000-C_3}{1+r} \right)
\]
3) a) Texts

B₀ and T₀ are the amount of beer and texts consumed. U₀ is the indifference curve, and L₀ is the budget line.

b) Texts

B₁ and T₁ are the new quantities of beer and texts consumed, given the new budget constraint (L₁). Beer and texts are substitutes, since an increase in the price of beer leads to an increase in the consumption of texts.

4) We know that Al’s demand for a good results from the interaction of his income and his preferences. We can derive Al’s demand equations algebraically by combining the following two equations: \( Y = p₁x₁ + p₂x₂ \) (budget constraint) and \( \frac{\text{MU}_1}{p₁} = \frac{\text{MU}_2}{p₂} \) (optimization condition). First we’ll plug the equations for MU into the optimization condition:

\[
\frac{3}{x₁}p₁ = \frac{2}{x₂}p₂
\]

\[
x₂p₂ = \left(\frac{2}{3}\right)x₁p₁
\]

\[
Y = x₁p₁+x₂p₂ \quad \text{or} \quad x₂p₂ = \left(\frac{2}{3}\right)x₁p₁
\]

So Al’s demand for good 1 is given by: \( x₁ = \left(\frac{3}{5}\right)\frac{Y}{p₁} \)

A similar analysis yields Al’s demand for good 2, starting with the relation \( x₁p₁ = \left(\frac{3}{2}\right)x₂p₂ \)

\[
Y = x₁p₁+x₂p₂ \quad \text{or} \quad Y = \left(\frac{3}{2}\right)x₂p₂ + x₂p₂ \quad \text{or} \quad Y = \left(\frac{5}{2}\right)x₂p₂ \quad \text{or} \quad x₂ = \left(\frac{2}{5}\right)\frac{Y}{p₂} \]