Solutions to Problem Set 6  
Econ 115  
(prepared by Oleg Melnikov)

1.  
(a) Arbitrage – taking an advantage of price disparities by buying an asset at a lower price and simultaneously reselling it at a higher price.  
(b) Reaction function – relates an optimal action of an economic agent to the actions of its rivals (i.e. in the Cournot duopoly reaction function describes the optimal choice of quantity for one firm given the quantity chosen by its competitor).  
(c) Prisoner’s dilemma: a classic game of two players. Each player can either cooperate or compete. Mutual cooperation is better than competition, but unilateral deviation from the cooperative agreement is profitable to the defector. In equilibrium, both parties end up cheating on the cooperative agreement.  
(d) Monopolistic competition – market structure of an industry in which there are many firms and freedom of entry and exit but in which each firm has a product somewhat differentiated from the others, giving it some control over its price.

2. Elasticity of demand, \( e_d = \left| \frac{\Delta Q}{\Delta P} \right| \cdot \frac{P}{Q}. \)

Monopolist’s total revenue \( TR = PQ. \)

Monopolist’s marginal revenue

\[ MR = P + Q \frac{\Delta P}{\Delta Q} = P \left( 1 + \frac{Q}{P} \frac{\Delta P}{\Delta Q} \right) = P (1 - 1/e_d). \]

Inelastic demand means \( e_d < 1. \) Then \( 1/e_d > 1 \) and \( MR < 0. \)

Monopoly produces at \( MR = MC > 0. \) Hence, a monopolist will never choose to produce on the inelastic portion of the demand curve.

3. Above graph illustrates the case when the monopolist’s marginal cost is constant at \( MC. \) Under perfect competition, \( p = mc, \) industry output is at \( Q_c \) and social surplus = consumer surplus = area of the triangle ABC. Under monopoly, industry output is at the
point of intersection between the marginal revenue curve and the marginal cost curve \((Q_m)\). From the demand curve, monopolist would charge the price represented by point D. Under monopoly, consumer surplus is given by the area of the triangle ADE, while monopolist makes profits equal to the area of the rectangle DEFB. Thus, social surplus under monopoly declines by the area of the triangle EFC.

4. Demand curve is \(p=a-bq\); marginal cost equals \(c\).

(a) Monopolist profit \(\pi(q) = (a-bq) q - cq\).

Marginal revenue \(MR = p + q \frac{\Delta p}{\Delta q}\). Since the slope of the inverse demand curve \(\Delta p/\Delta q=-b\), \(MR = p-bq = a-bq-bq = a-2bq\).

\(MR=MC\) implies that \(a-2bq = c\) and it follows that \(q = (a-c)/2b\).
Price \(p = a-b(a-c)/2b = (a+c)/2\).

(b) In Cournot duopoly, for the first firm

\[\pi_1(q_1, q_2) = (a-b(q_1+q_2))q_1 - cq_1.\]

Holding the output of the second firm fixed at \(q_2\), firm’s 1 marginal revenue should equal to its marginal cost, \(c\). Hence,

\[MR_1 = a-2bq_1-bq_2 = c.\]

Similarly for the second firm

\[MR_2 = a-2bq_2-bq_1 = c.\]

One can solve these two equations simultaneously to determine \((q_1, q_2)\). A handy shortcut is to rewrite the first equation as

\[a-bq_1-bq_2 = c+bq_1\]

and similarly for the second firm

\[a-bq_1-bq_2 = c+bq_2.\]

But expression on the left hand side is the market price and both firms must charge the same price at the equilibrium. It follows that \(q_1 = q_2 = q_c = (p-c)/b\). If equilibrium quantities are the same, then from any equation

\[a-3bq_c = c\]

and \(q_c = (a-c)/3b\).

Combined output of two firms is then \(Q_c = 2q_c = 2(a-c)/3b\).

Now we can just read the market price from the demand curve:
\[ p = a - bQ_e = a - 2(a - c)/3 = (a + 2c)/3. \]

(c) Under Bertrand duopoly, as we know from class, \( p = c \). Again, we can read the total market quantity from the demand curve:
\[ c = a - bQ_b \Rightarrow Q_b = (a - c)/b. \]

5. Collusion is easier when…
(a) it is easier to detect cheating (makes punishment easier to implement);
(b) there are fewer firms in the industry (makes benefits from cooperation larger and gains from cheating smaller);
(c) firms are more patient (benefits from future cooperation dominate today’s gains from cheating).

6. Price discrimination occurs when a producer charges different price for identical products for reasons not associated with differences in cost.
   Two examples of price discrimination:
   - adult/children/senior fares in movie theaters or for public transportation => issue different tickets to prevent the possibility of resale.
   - student discounts for software – Yale CIS center often sells software on the subscription basis. The license can be (costlessly) renewed on the annual basis while you remain a student. This makes it difficult to resell your copy of software.

And two examples that are not price discrimination:
- oranges are more expensive in Alaska than in Florida, but this may be due to the higher transportation cost;
- companies may pay different price for electricity (especially in deregulated markets) in different times on the same day, but this is because electricity is more difficult to supply at peak hours than during “normal” hours.