Q1.
(a) Natural Monopoly:
Any industry where the cheapest way to produce a given output Q is to have one firm produce all of Q. Examples include phone services, cable services etc.

(b) Patent:
A patent gives inventors the exclusive right to produce or to license others to produce their discoveries for limited period of time.

(c) “Per se” anti-trust rule:
Any action that is ‘by itself’ (“per se”) illegal and therefore punishable under the law. For example collusion is “per se” illegal in this country, i.e. the government only needs to show that it happened for the company to be punished, in court.

Q2.
(a) Single-period payoff matrix, with Bertrand competition between two firms in homogeneous goods.

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<th>Collude</th>
<th>Cheat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collude</td>
<td>$\pi^m/2$, $\pi^m/2$</td>
<td>0, $\pi^m$</td>
</tr>
<tr>
<td>Cheat</td>
<td>$\pi^m$, 0</td>
<td>0, 0</td>
</tr>
</tbody>
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[Note: the first payoff in every box is for Firm 1 and the second one is for Firm 2.]

i. If Firm 1 colludes then the best response for Firm 2 should be to cheat, since by cheating Firm 2 gets the entire monopoly profits and not just half as it would get under collusion. Similarly if Firm 1 cheats then Firm 2 is indifferent between colluding or cheating, since in both cases it gets a payoff of zero. Therefore both colluding and cheating may be considered a best response for Firm 2 in this case (if Firm 1 cheats).

ii. It therefore follows that Firm 2 should always cheat given that if Firm 1 colludes it is strictly better off by cheating, and if Firm 1 cheats as well then it is indifferent between cheating and colluding. Cheating is Firm 2’s best response in all situations. Similarly, we can show that cheating is Firm 1’s
best response in all situations. Therefore the equilibrium strategies for this game will involve both firms cheating if the game is played only once.

(b) Firm 1 plays the ‘grim trigger strategy’ i.e. it colludes as long as Firm 2 colludes and if ever Firm 2 cheats it plays the ‘Bertrand’ strategy forever more.

(i) Payoff to Firm 2 of colluding this period is $\frac{\pi^m}{2}$, payoff from colluding next period is $\frac{\pi^m}{2}$ and so on, as long as it colludes it always gets $\frac{\pi^m}{2}$ in every period, since in the grim trigger strategy that Firm 1 is playing it never unilaterally cheats but only retaliates. Then the present value of colluding forever for Firm 2 is the present value of the stream of income of amount $\frac{\pi^m}{2}$ every period forever into the future. I.e.

$$PV = \frac{\pi^m}{2} + \frac{\frac{\pi^m}{2}}{(1+r)} + \frac{\frac{\pi^m}{2}}{(1+r)^2} + \frac{\frac{\pi^m}{2}}{(1+r)^3} + \ldots \ldots \ldots$$

(whence $r$ is the single-period rate of interest)

Or,

$$PV = \frac{\frac{\pi^m}{2}(1+r)}{r}$$

(ii) If the Firm 2 plays the ‘grim trigger strategy’ then it gets the present value calculated above since Firm 1 is playing the same strategy and therefore never deviates unless Firm 2 does so. Alternatively if it decides to cheat in any period given that Firm 1 is playing the ‘grim trigger strategy’ it gets monopoly profits this period and once it`s cheating is detected from next period onwards there is no collusion since Firm 1 always plays ‘Bertrand’ forever under this strategy, therefore Firm 2 gets zero from next period onwards.

Therefore it pays to cheat if and only if the following equation holds:

$$\pi^m > \frac{\pi^m}{2} (1+r) / r$$

i.e. the gain from cheating needs to be greater than the present value of colluding. This simplifies to the following.

$$r > 1$$

Therefore in this setup it pays to cheat only if interest rate is more than 100%.

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1 Note that this situation is similar to the Prisoner’s Dilemma game where the pursuit of self-interest by both players led to a mutually suboptimal outcome.

2 Note that the sum of a series $a + ax + ax^2 + ax^3 + \ldots$ is $a/(1-x)$ and not $a/x$ as given in class. Here $x$ is $1/(1+r)$. 
Q3.
Given \( C(Q) = F + cQ \), the AC curve is as follows:

Note that Producer Surplus is the difference between price and marginal costs until the optimal output. Alternatively, it is equal to the sum of profits and fixed costs. Total surplus is producer surplus added to consumer surplus.

**Case I: MR = MC**

The lighter shaded region (rectangle) is producer surplus and the darker shaded region (triangle) is the consumer surplus, together they form the total social surplus.

The shaded region (rectangle) is the firm’s profits.
Case II: $P = AC$

The darker shaded region (triangle) is the consumer surplus and the lighter shaded region is the producer surplus. Together they form the total surplus, the entire shaded area. The firm makes no profit or loss in this case since price equals average cost.

Case III: $P = MC$

Here the shaded region (triangle) is the consumer surplus, there is no producer surplus since $p = MC$. Therefore the total social surplus is also the shaded triangle. Firm makes a loss operating at this output because of fixed costs, this is given by the rectangle EGIH in the figure above.

The third rule $P = MC$ always maximizes total surplus, but the regulators may not choose this rule since sometimes (as shown above) the firms might be making a loss and will therefore exit the industry, unless they are subsidized. Subsidies bring along with it a whole set of other issues, like political “capture” etc.